Free-Standing Mathematics Qualification
June 2008
Advanced Level

MODELLING WITH CALCULUS 6992/2
Unit 12

Wednesday 14 May 2008 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- a calculator
- a clean copy of the Data Sheet (enclosed)
- a ruler.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is 6992/2.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- You may **not** refer to the copy of the Data Sheet that was available prior to this examination. A clean copy is available for your use.

Information
- The maximum mark for this paper is 60.
- The marks for questions are shown in brackets.

There are no questions printed on this page

SECTION A

Answer **all** questions.

Use **Temperatures** *on page 2 of the Data Sheet.*

1 For part of the day, the temperature, F ° Fahrenheit, can be modelled by the equation

$$F = t^2 - 5t + 71$$

where t is the time in hours after 3 am.

(a) **Use this model and calculus** to predict the minimum temperature which occurred that night. *(6 marks)*

(b) Find $\frac{d^2F}{dt^2}$; what can you deduce from its value? *(2 marks)*

(c) Sketch a graph of the equation for $0 \leqslant t \leqslant 8$. *(2 marks)*

(d) Compare your sketch in part (c) with the graph on the Data Sheet. *(2 marks)*

Turn over for the next question

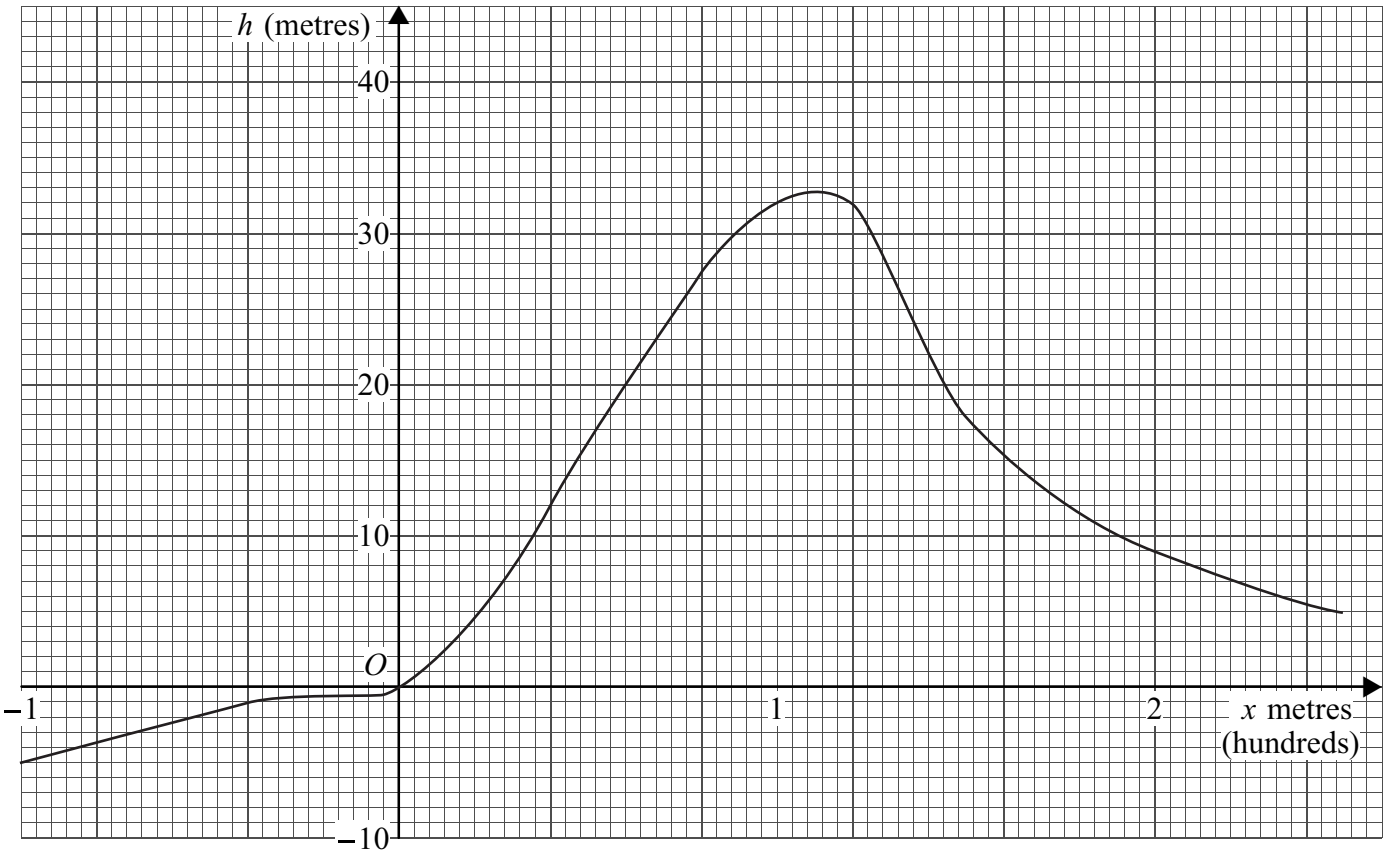
SECTION B

Answer **all** questions.

Use **Sand dunes** *on page 3 of the Data Sheet.*

2 The cross section of a typical sand dune is shown below.

The height of the sand dune, h metres, is the vertical height above O.
The horizontal distance, x, is measured in hundreds of metres from O.



The height may be modelled by the function

$$h = 12x + 60x^2 - 40x^3$$

for values of x from -0.2 to 1.6.

(a) **Use this model and calculus** to answer the following questions.

(i) Find x when the height is a maximum. *(6 marks)*

(ii) Find this maximum value. *(2 marks)*

(iii) Find $\dfrac{\mathrm{d}^2h}{\mathrm{d}x^2}$. *(2 marks)*

(iv) Use your answer to part (a) (iii) to confirm that the value found in part (a) (ii) is a maximum value. *(2 marks)*

(v) Find the value of x when $\dfrac{\mathrm{d}^2h}{\mathrm{d}x^2} = 0$ and state what is the physical relevance of this fact. *(3 marks)*

(vi) Find the value of $\dfrac{\mathrm{d}h}{\mathrm{d}x}$ when $x = 0.7$.

Hence find the gradient of the sand dune 70 metres from O.

State the units in your answer. *(2 marks)*

(b) The mean height of the sand dune from $x = 0$ to $x = 1.5$ is given by

$$\bar{h} = \frac{1}{1.5}\int_0^{1.5} (12x + 60x^2 - 40x^3)\,\mathrm{d}x$$

(i) Use the trapezium rule with three strips to find an estimate for the mean height. *(5 marks)*

(ii) Use integration to find the value of

$$\frac{1}{1.5}\int_0^{1.5} (12x + 60x^2 - 40x^3)\,\mathrm{d}x$$ *(4 marks)*

(iii) Comment on your answers to parts (b) (i) and (b) (ii). *(1 mark)*

SECTION C

Answer **all** questions.

Use **Ice cream manufacturer** *on page 4 of the Data Sheet.*

3 The number of employees, y, may be modelled by the function

$$y = 90 - 20\cos\frac{\pi}{6}t$$

where t is the number of months after 1 January.

(a) Find the number of employees predicted by the model on 1 January. *(2 marks)*

(b) Find the number of employees predicted by the model on 1 July (ie when $t = 6$). *(2 marks)*

(c) Find an expression for $\frac{dy}{dt}$. *(3 marks)*

SECTION D

Answer **all** questions.

Use **Balloon** *on page 4 of the Data Sheet.*

4 After t minutes, the radius of the balloon, r m, satisfies the differential equation

$$\frac{\mathrm{d}r}{\mathrm{d}t} = -0.2r$$

(a) (i) Find the general solution of this differential equation. *(5 marks)*

(ii) Given that, when $t = 0$, the radius was 4 m, show that $r = 4\mathrm{e}^{-0.2t}$. *(1 mark)*

(b) Find the value of t when the radius is 2 m. *(4 marks)*

5 The radius of the balloon, r m, is given by

$$r = 4\mathrm{e}^{-0.2t}$$

(a) Find r, when:

(i) $t = 3$;

(ii) $t = 3.1$. *(2 marks)*

(b) Using your answers to part (a), find an estimate for $\frac{\mathrm{d}r}{\mathrm{d}t}$ when $t = 3$. *(2 marks)*

END OF QUESTIONS

There are no questions printed on this page