Free-Standing Mathematics Qualification
June 2007
Advanced Level

MODELLING WITH CALCULUS **6992/2**
Unit 12

Thursday 17 May 2007 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- a calculator
- a clean copy of the Data Sheet (enclosed).

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is 6992/2.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of a calculator should normally be given to three significant figures.
- You may **not** refer to the copy of the Data Sheet that was available prior to this examination. A clean copy is available for your use.

Information
- The maximum mark for this paper is 60.
- The marks for questions are shown in brackets.

There are no questions printed on this page

SECTION A

Answer **all** questions.

Use **Shares** *on page 2 of the Data Sheet.*

1 The value, £V, of Primary Pathfinder shares during the first ten months of the year 2006 may be modelled by the function

$$V = t^2 - 7t + 25$$

where t is the time in months after 1 January 2006.

(a) (i) **Use this model and calculus** to predict the time at which the minimum value of the shares in this ten-month period occurred. *(4 marks)*

(ii) Hence find the value of the shares at this time. *(2 marks)*

(b) Find $\dfrac{d^2V}{dt^2}$; what can you deduce from its value? *(2 marks)*

Turn over for the next question

SECTION B

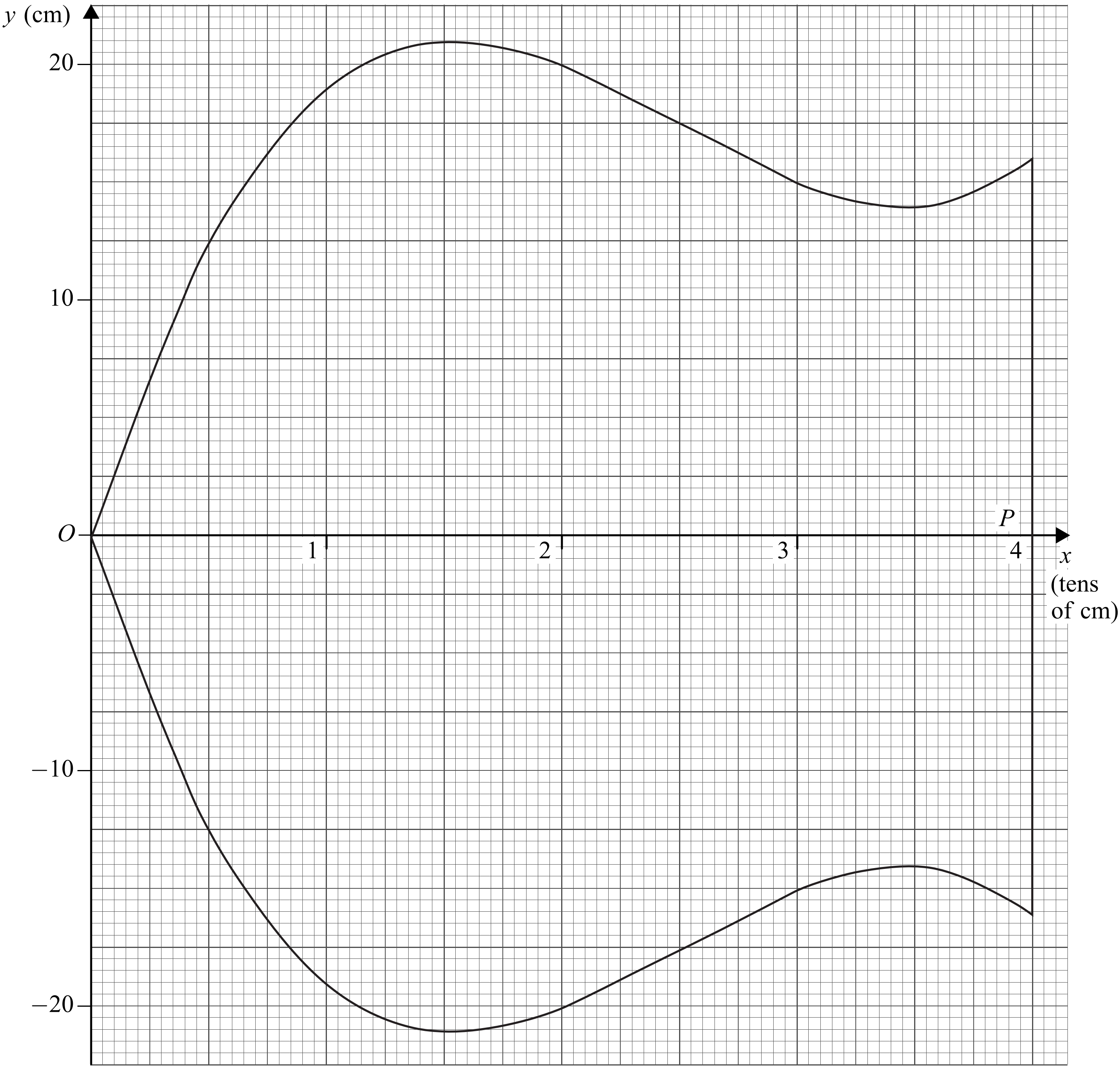
Answer **all** questions.

Use **Table mat** *on page 3 of the Data Sheet.*

2 The plan of the mat is reproduced below.

The distance of a point on the upper edge of the fish from the line OP is y centimetres, and x, measured in tens of centimetres, is the distance from O along the line of symmetry.

The length of the mat is 40 cm, so that the point P is actually 40 cm from O.



The distance y may be modelled by the function

$$y = 2x^3 - 15x^2 + 32x$$

for values of x from 0 to 4.

Use this model and calculus to answer the following questions.

(a) (i) Find x when y is a maximum. *(6 marks)*

(ii) Find this maximum value. *(2 marks)*

(b) Write down the value of x when y is a minimum. *(2 marks)*

(c) (i) Find $\frac{d^2y}{dx^2}$. *(2 marks)*

(ii) Find the value of x when $\frac{d^2y}{dx^2} = 0$ and state the physical relevance of this point. *(3 marks)*

(d) Find the width of the tail of the fish, that is when $x = 4$. *(2 marks)*

(e) Anne Marie is making the fish from a rectangular sheet of wood. Find the minimum rectangular area of wood which she needs to make each fish. *(2 marks)*

Turn over for the next question

3 The area of the mat is given by

$$20\int_0^4 (2x^3 - 15x^2 + 32x)\,\mathrm{d}x$$

(a) Use the trapezium rule with four strips to find an estimate for the area of the mat. *(5 marks)*

(b) Use integration to find the value of $20\int_0^4 (2x^3 - 15x^2 + 32x)\,\mathrm{d}x$. *(4 marks)*

(c) Anne Marie wants to find the weight of a mat and needs to choose the better estimate for the area of the fish.

Which of the answers in parts (a) and (b) above is likely to be the better and why? *(2 marks)*

(d) Each square centimetre of wood that Anne Marie is using weighs 1.5 grams.

Find the expected weight of each mat. *(2 marks)*

(e) The area between the function and the x-axis is given by

$$\int_0^4 (2x^3 - 15x^2 + 32x)\,\mathrm{d}x$$

Explain why the area of the mat is 20 times this value. *(1 mark)*

SECTION C

Answer **all** questions.

Use **Elastic string** *on page 4 of the Data Sheet.*

4 The distance, y cm, of the weight below the equilibrium position may be modelled by the function

$$y = 2\cos\frac{\pi}{3}t$$

where t is the number of seconds after the weight is released.

(a) Show that, when $t = 3$, $y = -2$. *(1 mark)*

(b) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}t}$, the velocity of the weight. *(3 marks)*

(c) (i) Find the maximum value of $\frac{\mathrm{d}y}{\mathrm{d}t}$.

You may leave your answer as a multiple of π or as a decimal to three significant figures. *(2 marks)*

(ii) Find the value of t when this occurs. *(1 mark)*

Turn over for the next question

Turn over ▶

SECTION D

Answer **all** questions.

Use **Bacteria** *on page 4 of the Data Sheet.*

5 After t days, the mass of the bacteria, m grams, satisfies the differential equation

$$\frac{dm}{dt} = \lambda m, \quad \text{where } \lambda \text{ is a positive constant.}$$

(a) (i) Find the general solution for m of this differential equation. *(5 marks)*

(ii) The initial mass which Chloe used was 60 g.

Show that $m = 60e^{\lambda t}$. *(2 marks)*

(b) Chloe observes that after 4 days the mass of the bacteria has doubled, so that the mass of the bacteria is now 120 g.

(i) Find λ. *(3 marks)*

(ii) Find the mass of the bacteria after 10 days. *(2 marks)*

END OF QUESTIONS