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ASSESSMENT and QUALIFICATIONS ALLIANCE

Free-Standing Mathematics Qualification

Modelling with Calculus 6992/2

Mark Scheme

2007 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
–*x* EE	deduct *x* marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

June 07

Free-Standing Mathematics Qualification

Advanced Level – Modelling with Calculus (6992/2)

Answers and Marking Scheme

Question 1

(a)(i)	$\frac{dV}{dt} = 2t - 7$	**M1A1**	Can be seen in (b) Needs calculus for (a)(i)
	$\frac{dV}{dt} = 0 \Rightarrow$ $2t - 7 = 0$ $t = \frac{7}{2}$	 **M1** **A1**	
(a)(ii)	When $t = \frac{7}{2}$, $V = \left(\frac{7}{2}\right)^2 - 7\left(\frac{7}{2}\right) + 25$ Value is £12.75	 **M1** **A1**	 Accept 1275p and 12.75
(b)	$\frac{d^2V}{dt^2} = 2$ Thus the value in (a) was a minimum as $\frac{d^2V}{dt^2}$ is positive	**B1** **B1**	 Thus the value of $\frac{dV}{dt}$ is increasing for all t
	TOTAL	**8**	

Question 2

(a)(i)	$\frac{dy}{dx} = 6x^2 - 30x + 32$	**M1A1**	M1 for 2 correct Can be seen in (c)(i)
	$\frac{dy}{dx} = 0 \Rightarrow$	**M1**	
	$6x^2 - 30x + 32 = 0$		
	$3x^2 - 15x + 16 = 0$	**A1**	
	$x = \frac{15 \pm \sqrt{225-192}}{6}$	**M1**	
	$= \frac{20.745}{6}$ or $\frac{9.255}{6}$		
	For maximum $x = 1.5426$		5 unless state which is required
	$= 1.54$	**A1**	Needs 3 significant figures
(a)(ii)	When $x = 1.54$, $y = 2(1.54)^3 - 15(1.54)^2 + 32 \times 1.54$ $= 21.0105\ldots$	**M1**	
	Maximum value of y is 21.0	**A1**	Accept 20.9
(b)	When y is a minimum, $x = \frac{20.745}{6}$	**M1**	
	$= 3.4575$		
	$= 3.46$	**A1**	Accept 3.45 Don't accept 3.5
(c)(i)	$\frac{d^2y}{dx^2} = 12x - 30$	**M1A1**	
(c)(ii)	$\frac{d^2y}{dx^2} = 0$ when $12x - 30 = 0$	**M1**	
	$x = \frac{5}{2}$ or 2.5	**A1**	
	The slope of edge of the fish is steepest when $x = \frac{5}{2}$	**E1**	
(d)	$y = 2(4)^3 - 15(4)^2 + 32 \times 4 = 16$	**M1**	For substitution
	Width is 32 cm	**A1**	Accept 32
(e)	Minimum area is $40 \times 2 \times 21.01\ldots$	**M1**	Accept 80×21
	$= 1680\ \text{cm}^2$	**A1**	Needs units. Accept 1680.84 and 1681
	TOTAL	**19**	

Question 3

(a)	Four strips $\Rightarrow$ values of x are 0, 1, 2, 3 and 4 When $x = 0$, $y = 0$ $x = 1,\ y = 380$ $x = 2,\ y = 400$ $x = 3,\ y = 300$ $x = 4,\ y = 320$ Area $\approx \frac{1}{2} \times 1 \{0 + 320 + 2(380 + 400 + 300)\}$ $= \frac{1}{2}(320 + 2 \times 1080)$ $=1240$	**B2** **M1A1** **A1**	B1 for any three Delete A1 if not $\times 20$ (eg B2, M1, A1 for 62)
(b)	$\int_0^4 (2x^3 - 15x^2 + 32x)\,dx$ $= \left(\frac{1}{2}x^4 - 5x^3 + 16x^2\right)_0^4$ $= \left(\frac{1}{2}(4)^4 - 5(4)^3 + 16(4)^2\right)$ $= 64$ Area is 1280	**B1B1** **M1** **A1**	Need integration above
(c)	Integral in (b) The cubic curve is a much better fit to the shape than the trapezium rule which just uses straight edges	**B1** **E1**	
(d)	Weight is $1280 \times 1\frac{1}{2}$ = 1920 grams	**M1** **A1**√	**FT** from (b) **SC2** 960 grams
(e)	Scale of x is 10 and there are two halves of the fish	**E1**	
	TOTAL	**14**	

Question 4

(a)	When $t = 3$, $x = 2 \cos \pi$ $= -2$	**B1**	
(b)	$\frac{dx}{dt} = -2 \times \frac{\pi}{3} \times \sin \frac{\pi}{3} t$	**B3**	B1 for $\frac{\pi}{3}$ B1 for $\sin \frac{\pi}{3} t$ B1 for – sin
(c)(i)	Maximum value of $\frac{dx}{dt}$ is $\frac{2\pi}{3}$	**M1A1**	**SC1** for $-\frac{2\pi}{3}$ M1 for max of $\sin \frac{\pi}{3} t$ is 1
(c)(ii)	$\frac{9}{2}$	**B1**	Accept $\frac{3}{2}$ or $-\frac{3}{2}$
	TOTAL	**7**	

Question 5

(a)(i)	$\frac{dm}{dt} = \lambda m$ $\int \frac{dm}{m} = \int \lambda dt$ $\ln m = \lambda t + c$ $m = C\ e^{\lambda t}$	 **M1** **A1A1** **M1** **A1**	 M1 for + c
(a)(ii)	When $t = 0$, $m = 60$, $\therefore C = 60$ $m = 60\,e^{\lambda t}$	**M1** **A1**	
(b)(i)	When $t = 4$, $m = 120$, $\therefore 120 = 60\,e^{4\lambda}$ $e^{4\lambda} = 2$ $4\lambda = \ln 2$ $\lambda = \frac{1}{4}\ln 2$ or 0.173	**M1** **M1** **A1**	
(b)(ii)	When $t = 10$, $m = 60\ e^{1.73}$ = 339 g (accept 338)	**M1** **A1**	Or, ten days is 2 $\frac{1}{2}$ doubling periods trial and improvement 0 unless 339 or 338 Accept 338 Hence $60 \times 2^{2.5} = 339$ g M1 only unless used 3 significant figures in (i) and answer within 20g
	TOTAL	**12**	
	TOTAL MARK FOR PAPER	**60**	