Free-Standing Mathematics Qualification
June 2007
Advanced Level

WORKING WITH ALGEBRAIC AND
GRAPHICAL TECHNIQUES
Unit 11

6991/2

Wednesday 16 May 2007   9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- an answer sheet for use in Questions 1, 3, 4 and 5
  (enclosed)
- a calculator
- a clean copy of the Data Sheet (enclosed)
- a ruler.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book **and** on the top of the answer sheet for Questions 1, 3, 4 and 5.
- The *Examining Body* for this paper is AQA. The *Paper Reference* is 6991/2.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should normally be given to three significant figures.
- You may **not** refer to the copy of the Data Sheet that was available prior to this examination. A clean copy is available for your use.
- At the end of the examination, remember to hand in both your answer book **and** the answer sheet for Questions 1, 3, 4 and 5.

Information
- The maximum mark for this paper is 60.
- The marks for questions are shown in brackets.
- You may use either a scientific or a graphics calculator.

6991/2

SECTION A

Answer **all** questions.

*Use **Receivers** on page 2 of the Data Sheet.*

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1 When the telescope is pointing vertically upwards, a mathematical model for the cross-section of the bowl is produced. The equation of the cross-section is

$$y = \frac{x^2}{92} + 50$$

where the x-coordinate is the horizontal distance in metres from the centre of the bowl, and the y-coordinate is the vertical distance in metres above the ground.

(a) On the answer sheet, draw the graph of $y = \frac{x^2}{92} + 50$ for $-40 \le x \le 40$. *(4 marks)*

(b) Write down the minimum height of the bowl above the ground in this position.

(1 mark)

(c) The value $x = 38$ gives a point on the rim of the bowl.

Calculate the height of this point above the ground. *(1 mark)*

(d) Describe fully the transformations that map the graph of $y = x^2$ onto the graph of $y = \frac{x^2}{92} + 50$. *(2 marks)*

2 A TV satellite dish is fastened to a wall using a bracket.

The cross-section of the dish and bracket is shown on the Data Sheet. The x and y axes are shown and the distances are measured in centimetres.

The equation of the cross-section is

$$y = 350 \pm \sqrt{120(x - 30)}$$

(a) Find the two values of y when $x = 40$. *(2 marks)*

(b) The bracket is horizontal.

Find the height at which the bracket is fastened to the wall. *(1 mark)*

(c) Explain why you cannot use a value of x less than 30 in the equation. *(1 mark)*

(d) Find x when $y = 370$. *(2 marks)*

SECTION B

Answer **all** questions.

*Use **Height** on page 3 of the Data Sheet.*

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3 (a) The answer sheet shows a scatter graph for the data for boys between the ages of 0 and 20.

    (i) On the answer sheet, draw a line of best fit for the data for boys between the ages of 6 and 16. *(1 mark)*

    (ii) Use your line of best fit to predict the average height of boys aged 11. *(1 mark)*

(b) Possible models to fit the data between the ages of 0 and 20 are given by the equations

$$h = \frac{A}{x} + B , \qquad h = A\sqrt{x} + B , \qquad h = Ae^{x} + B$$

where x is the age, h is the height, and A and B are positive constants.

Sketch a graph for each of these three possible models.

Label each graph clearly.

State which of these three possible models could best be used to model the data.
(5 marks)

(c) Between which two ages does a boy have the greatest rate of increase in height?
(1 mark)

(d) The model $h = 122\sin(4.5x)° + 56$ can be used to model the data between the ages of 0 and 20.

Explain why this model is not appropriate for values of x greater than 20. *(1 mark)*

(e) State an appropriate model for the height of an adult male between the ages of 25 and 45. *(1 mark)*

Turn over ▶

SECTION C

Answer **all** questions.

Use **Population** *on page 3 of the Data Sheet.*

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4 (a) The population, *P* million, of England and Wales can be modelled by the equation

$$P = Ae^{kt}$$

where *t* is the time in years since 1911, and *A* and *k* are constants.

   (i) For this model, show that $\ln P = \ln A + kt$. *(2 marks)*

  (ii) On the answer sheet, complete the table of values, giving the values of $\ln P$ correct to three significant figures. *(2 marks)*

 (iii) Use the grid on the answer sheet to plot $\ln P$ against *t*.

       Draw the line of best fit on your graph. *(3 marks)*

 (iv) Use your graph to find the values of *A* and *k*. *(3 marks)*

(b) Since 1981, the population has been growing more slowly and the population can be modelled by the equation

$$P = 49.1e^{0.002t}$$

where *t* is the time in years since 1981.

   (i) What information does the number 49.1 give you? *(1 mark)*

  (ii) Use this model to predict the population in 2011. *(2 marks)*

 (iii) Use this model to predict the year in which the population will be 55 million. *(3 marks)*

 (iv) The actual census result for 2001 was 52 million.

       Calculate the percentage error in using this model for 2001. *(3 marks)*

SECTION D

Answer **all** questions.

*Use **Fairground** on page 3 of the Data Sheet.*

5 A model for the height, *h* metres, of a seat above the ground at time *t* seconds after the start of the ride is given by the equation

$$h = 9.7 - 8\cos(20t)°$$

(a) Use the grid on the answer sheet to complete the graph of *h* against *t* for $0 \leq t \leq 20$.

(4 marks)

(b) (i) What is the maximum height of a seat predicted by the model? *(1 mark)*

(ii) What is the value of *t* at the maximum height? *(1 mark)*

(c) (i) Find the gradient of the graph when $t = 7$. *(2 marks)*

(ii) State the units of the gradient. *(1 mark)*

(iii) What information does the gradient give? *(1 mark)*

(d) For the curve with equation $h = 9.7 - 8\cos(20t)°$, state:

(i) the amplitude; *(1 mark)*

(ii) the period. *(1 mark)*

(e) The vertical speed, *v* m/s, of a seat can be modelled by the equation

$$v = \frac{160\pi}{180}\sin(20t)°$$

(i) Find the first two values of *t* when *v* is a maximum. *(2 marks)*

(ii) Find the two values of *h* when *v* is zero. *(3 marks)*

END OF QUESTIONS

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Surname		Other Names	
Centre Number		Candidate Number	
Candidate Signature			

Free-Standing Mathematics Qualification
June 2007
Advanced Level

WORKING WITH ALGEBRAIC AND   **6991/2AS**
GRAPHICAL TECHNIQUES
Unit 11

AQA
ASSESSMENT and
QUALIFICATIONS
ALLIANCE

This answer sheet is to be used when answering Questions 1, 3, 4 and 5 as indicated.
Fasten this sheet securely to your answer book.

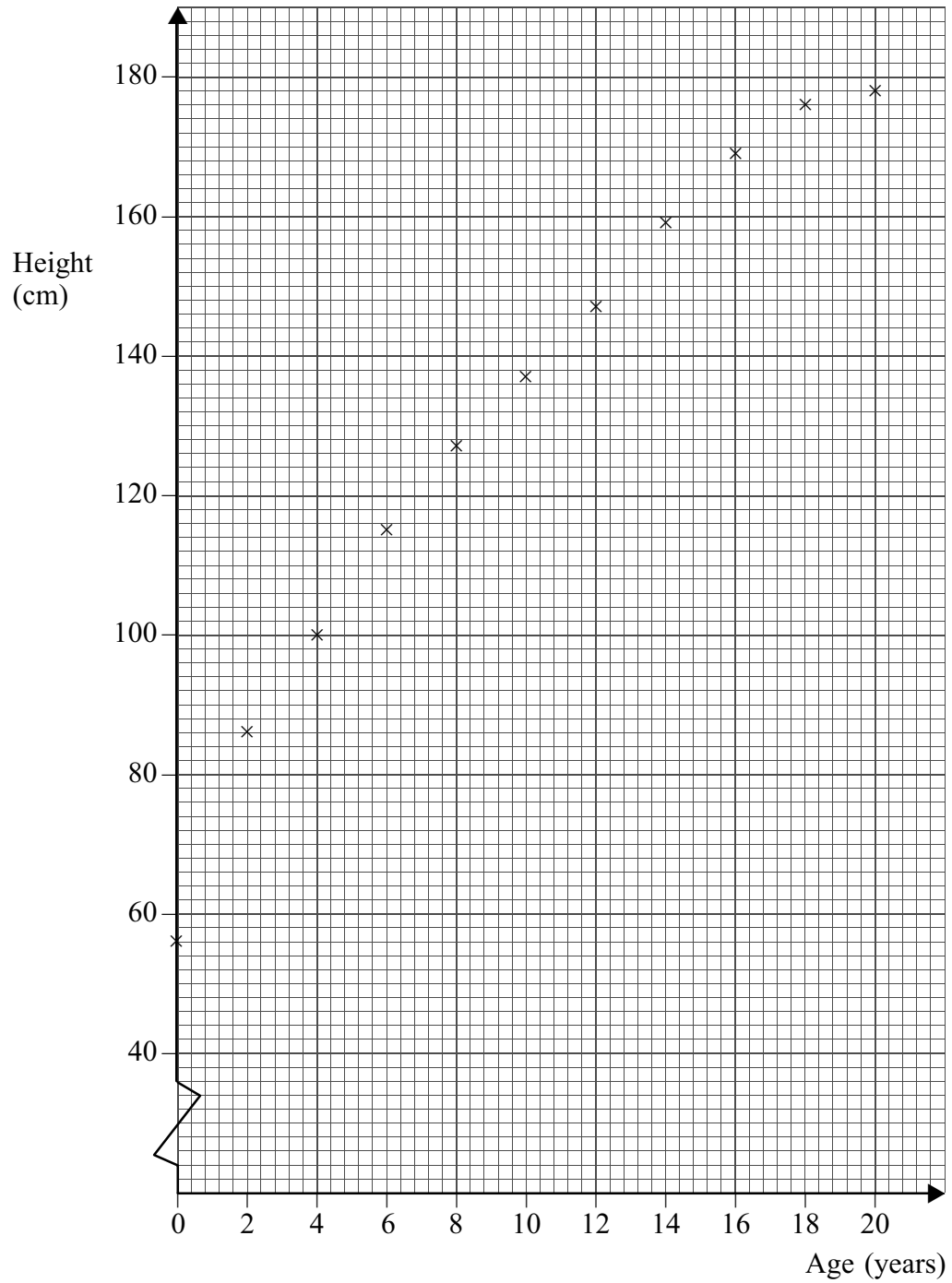**This graph is to be used when answering Question 1.**

(a)

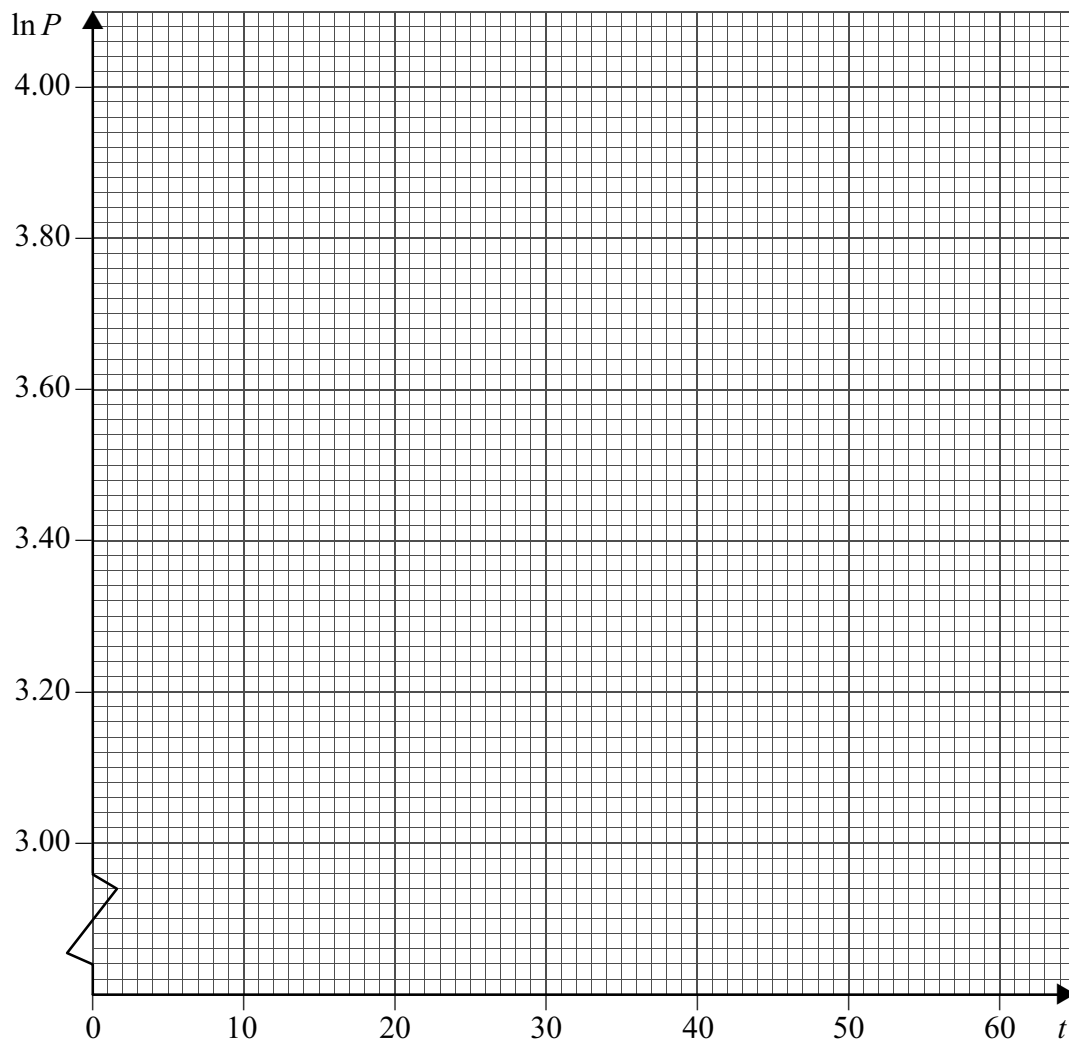**6991/2AS**
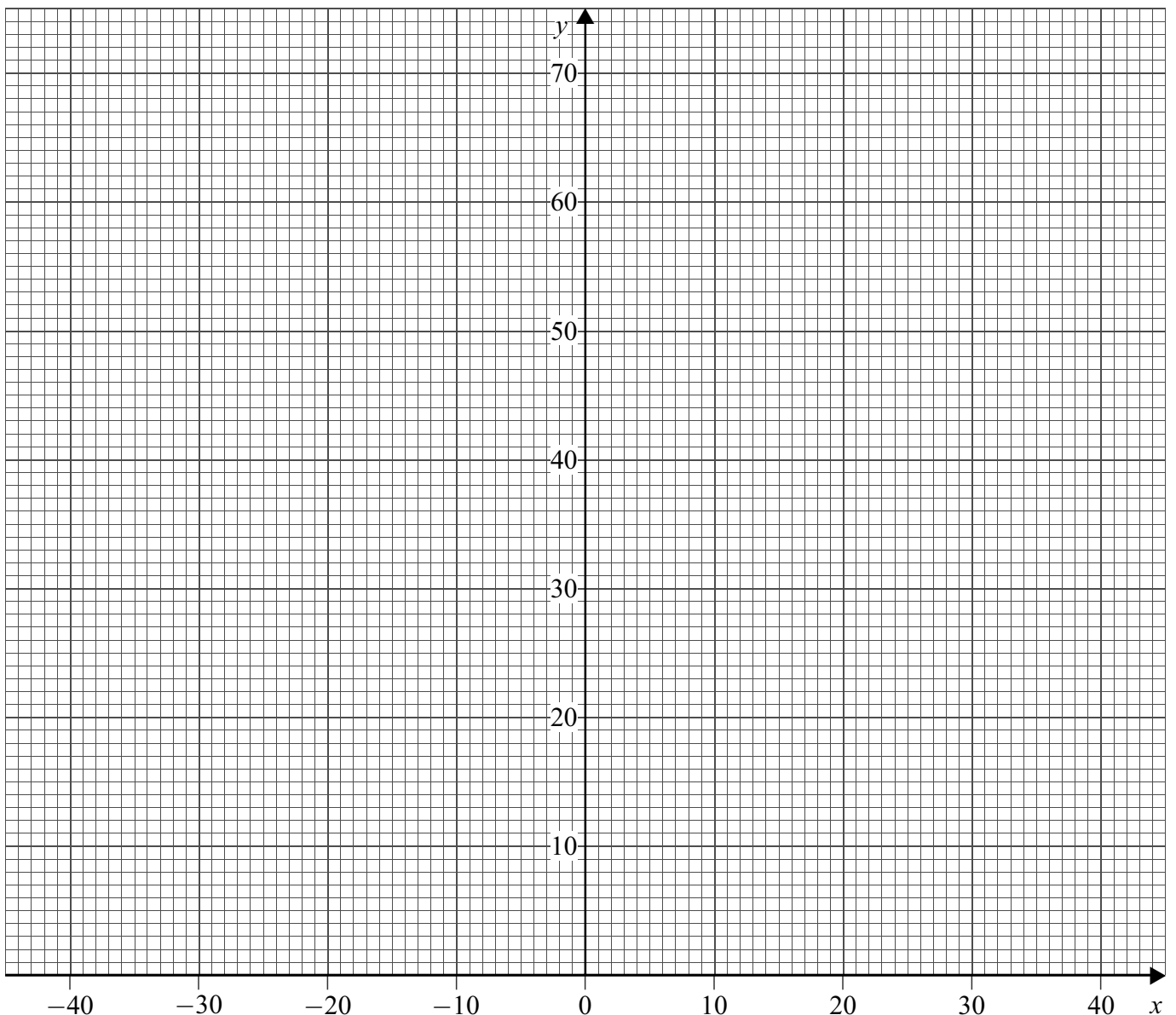
This graph is to be used when answering Question 3.

(a)

This table and graph are to be used when answering Question 4.

(a)(ii)

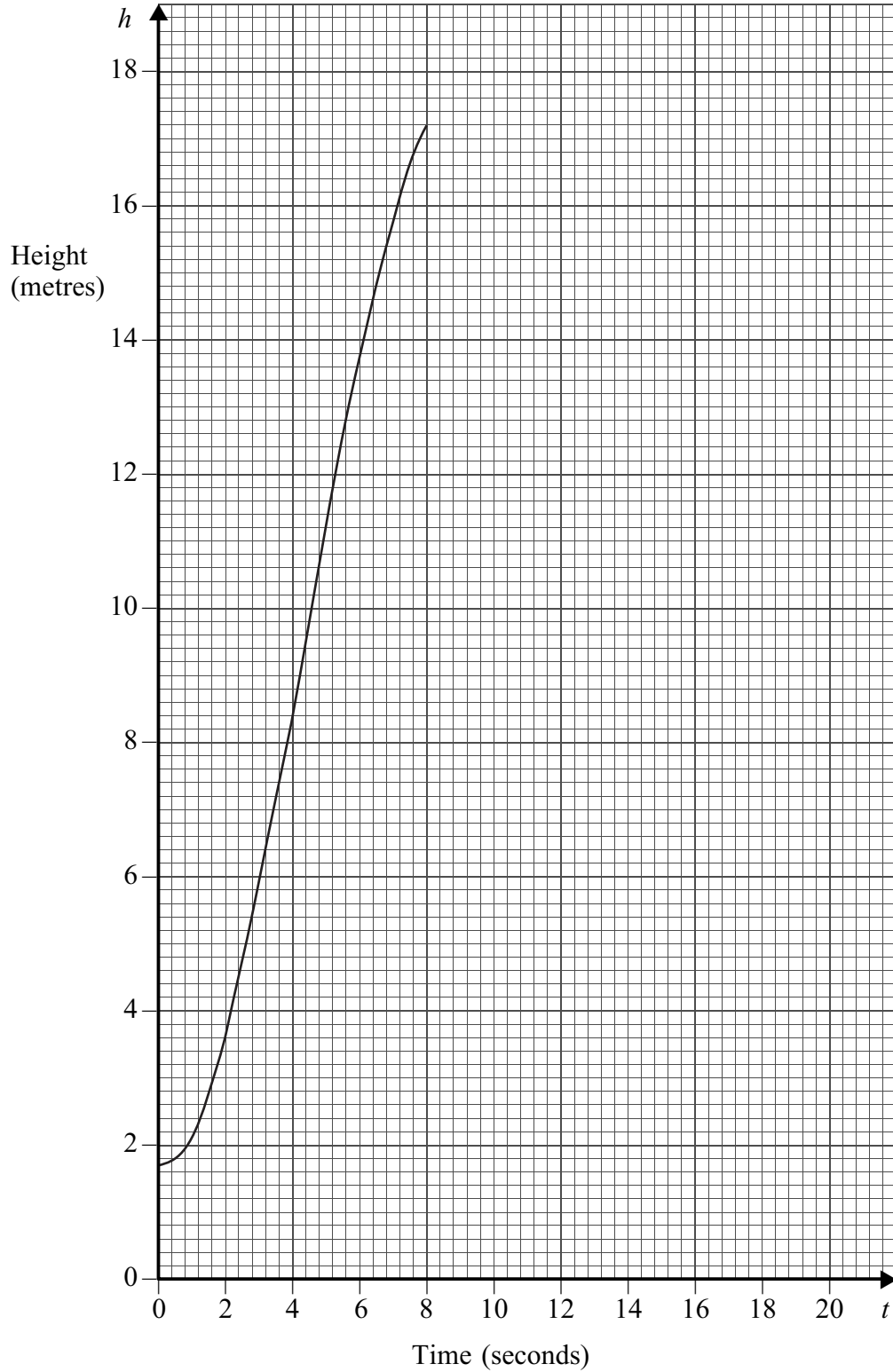| t	0	10	20	40	50	60
P	36.1	37.9	39.9	43.7	46.1	48.7
$\ln P$						

(a)(iii)

Turn over ▶

This graph is to be used when answering Question 5.

(a)

END OF ANSWER SHEET