



## Free-Standing Mathematics Qualification

# Modelling with Calculus *6992/2*

## Mark Scheme

### *2006 examination – June series*

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

## Key to mark scheme and abbreviations used in marking

M	mark is for method		
m or dM	mark is dependent on one or more M marks and is for method		
A	mark is dependent on M or m marks and is for accuracy		
B	mark is independent of M or m marks and is for method and accuracy		
E	mark is for explanation		
✓ or ft or F	follow through from previous		
	incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	OE	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
-x EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

### Application of Mark Scheme

**No method shown:**

Correct answer without working  
 Incorrect answer without working

mark as in scheme  
 zero marks unless specified otherwise

**More than one method / choice of solution:**

2 or more complete attempts, neither/none crossed out

mark both/all fully and award the mean  
 mark rounded down

1 complete and 1 partial attempt, neither crossed out

award credit for the complete solution only

**Crossed out work**

do not mark unless it has not been replaced

**Alternative solution** using a correct or partially correct method

award method and accuracy marks as  
 appropriate

**Free-Standing Mathematics Qualification****Advanced Level – Modelling with Calculus (6992/2)****Answers and Marking Scheme****Question 1**

(a)(i)	$y = 2 - \frac{5}{4} = 0.75$	<b>B1</b>	
(ii)	$y = 4 - 5 = -1$	<b>B1</b>	
	The ball is below the point where it was hit	<b>B1</b>	
(b)	$\frac{dy}{dt} = 4 - 10t$	<b>B1</b> <b>B1</b>	4 – 10t
(c)	$\frac{dy}{dt} = 0 \Rightarrow$ $4 - 10t = 0$ $t = 0.4$	<b>M1</b> <b>A1</b>	
(d)	When $t = 0.4$ , $y = 4 \times 0.4 - 5 \times (0.4)^2$ $= 0.8$	<b>M1</b> <b>A1</b>	CAO
(e)(i)	$\frac{d^2y}{dt^2} = -10$	<b>B1</b>	
(ii)	This is negative which means the turning point is a maximum	<b>B1</b>	
(f)	Hits ground when $y = -2.4$ $-2.4 = 4t - 5t^2$ $5t^2 - 4t - 2.4 = 0$ $25t^2 - 20t - 12 = 0$ $(5t - 6)(5t + 2) = 0$ $t = \frac{6}{5}$ or 1.2 seconds	<b>B1</b> <b>M1</b> <b>M1</b> <b>A1</b>	M1 $2.4 = 4t - 5t^2$
	<b>TOTAL</b>	<b>15</b>	

**Question 2**

(a)	$\frac{dh}{dt} = 6t - 0.9t^2$ $= 0 \text{ when}$ $6t - 0.9t^2 = 0$ $20t - 3t^2 = 0$ $t = 0, 6\frac{2}{3}$ <p>When <math>t = 6\frac{2}{3}</math>, <math>h = 87.4</math></p>	<p><b>M1A1</b></p> <p><b>M1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>A1</b></p>	<p>Condone omission of zero</p> <p>Accept 87, 87.5</p>
(b)	<p>When <math>t = 0</math> or 8am</p>	<p><b>B1</b></p>	<p>Accept 10.30 pm (<math>\pm 10</math> minutes)</p>
(c)(i)	<p>When <math>t = 0</math>, <math>h = 43</math></p> <p>When <math>t = 2</math>, <math>h = 52.6</math></p> <p>When <math>t = 4</math>, <math>h = 71.8</math></p> <p>When <math>t = 6</math>, <math>h = 86.2</math></p> <p>When <math>t = 8</math>, <math>h = 81.4</math></p> <p>Integral <math>\approx \frac{1}{2} \cdot 2(43+81.4+ 2[52.6+71.8+86.2])</math></p> <p><math>= 545.6</math></p> <p>Mean value is 68.2</p>	<p><b>B2</b></p> <p><b>M1A1</b></p> <p><b>A1</b></p>	<p>B1 for 3 correct.</p> <p>Allow <math>\pm 2</math> for each</p>
(ii)	<p>Integral is <math>[43t + t^3 - 0.075t^4]_0^8</math></p> <p><math>= 344 + 512 - 307.2</math></p> <p><math>= 548.8</math></p> <p><math>\bar{h} = \frac{548.8}{8} = 68.6</math></p>	<p><b>M1A1</b></p> <p><b>A1</b></p> <p><b>A1</b></p>	<p>M1 At least 2 correct</p> <p>A1 At least 2 correct</p> <p>CAO</p>
(d)	<p>When <math>t = 6</math>, <math>h = 86.2</math></p> <p>Error is 3.8</p> <p>Percentage error is <math>\frac{3.8}{90} \times 100</math></p> <p><math>= 4.22\%</math></p>	<p><b>B1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p>Accept <math>4\frac{2}{9}</math> or 4.2</p> <p>SC2 4.4 from rounding</p>
(e)	<p><math>\frac{d^2h}{dt^2} = 6 - 1.8t</math></p> <p><math>= 0</math> when <math>6 = 1.8t</math></p> <p><math>t = 3\frac{1}{3}</math> or 3.33</p> <p>This means that the rate of increase of the height of the water, <math>h</math> is the greatest</p>	<p><b>B1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>B1</b></p>	
	<p><b>TOTAL</b></p>	<p><b>23</b></p>	

**Question 3**

(a)(i)	5	<b>B1</b>	
(ii)	7.12	<b>B1</b>	Accept 7.1 or 7.2
(b)(i)	$\frac{dV}{dt} = \frac{3\pi}{4} \cos \frac{\pi}{4}t$	<b>B1</b>	Differential of 5 is 0
(ii)	When $t = 2$ , $\frac{dV}{dt} = 0$ , $\therefore$ Turning point	<b>B1</b>	$\frac{3\pi}{4}$ B1
(c)(i)	Maximum value of $\frac{dV}{dt}$ is $\frac{3\pi}{4}$	<b>B1</b>	B1 Diff of 5 is 0
(ii)	Shares are rising in value the fastest	<b>B1</b>	$\cos \frac{\pi}{4}t$ B1
	<b>TOTAL</b>	<b>9</b>	Or, use properties of trig curves B1 for $V = 8$

**Question 4**

(a)(i)	$\frac{dc}{dt} = \frac{3}{8}$ or 0.375	<b>B1</b>	
(ii)	The rate of change of the temperature is $\frac{3}{8}$ °C per minute	<b>B1</b>	Needs units
(b)	$\int \frac{dc}{20-c} = \int \frac{1}{40} dt$	<b>M1</b>	
	$-\ln c-20  = \frac{1}{40}t + k$	<b>A1</b>	
	When $t = 0, c = 2$		
	$\therefore k = -\ln 18$	<b>A1</b>	Condone use of $\ln -18$ and cancelling of minus signs
	$\therefore \frac{1}{40}t = \ln \frac{18}{20-c}$	<b>A1</b>	
(c)	$e^{\frac{1}{40}t} = \frac{18}{20-c}$	<b>M1</b>	
	$20-c = 18e^{-\frac{1}{40}t}$	<b>M1</b>	
	$c = 20 - 18e^{-\frac{1}{40}t}$	<b>A1</b>	
d	When $t = 10, c = 20 - 18e^{-\frac{1}{4}}$	<b>M1</b>	
	$= 5.98$	<b>A1</b>	Accept 6
d(i)	20	<b>B1</b>	
(ii)	0	<b>B1</b>	
	<b>TOTAL</b>	<b>13</b>	
	<b>GRAND TOTAL</b>	<b>60</b>	