Free-Standing Mathematics Qualification
June 2006
Advanced Level

WORKING WITH ALGEBRAIC AND GRAPHICAL TECHNIQUES **6991/2**
Unit 11

Wednesday 17 May 2006 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- an answer sheet for Questions 1, 4 and 5 (enclosed)
- a calculator
- a clean copy of the Data Sheet (enclosed)
- a ruler

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book **and** on the top of the answer sheet for Questions 1, 4 and 5. The *Examining Body* for this paper is AQA. The *Paper Reference* is 6991/2.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of a calculator should normally be given to three significant figures.
- You may **not** refer to the copy of the Data Sheet that was available prior to this examination. A clean copy is available for your use.
- At the end of the examination, remember to hand in both your answer book and the answer sheet for Questions 1, 4 and 5.

Information
- The maximum mark for this paper is 60.
- The marks for questions are shown in brackets.
- You may use either a scientific or a graphics calculator.

SECTION A

Answer **all** questions.

Use **Trees** *on page 2 of the Data Sheet.*

1 The equation $y = ax^2 + b$, where a and b are constants, can be used to model the height, y cm, of a fir tree in terms of the circumference, x cm.

(a) On the answer sheet, complete the table of values. *(2 marks)*

(b) Use the grid on the answer sheet to plot y against x^2.
Draw a line of best fit on your graph. *(3 marks)*

(c) Use your graph to predict the height of a fir tree with circumference 60 cm. *(2 marks)*

(d) Use your graph to predict the circumference of a fir tree with height 300 cm. *(2 marks)*

2 The equation $y = 0.2x^2 + 130$ can be used to model the height, y cm, of a birch tree in terms of the circumference, x cm.

(a) A birch tree with a circumference of 28 cm actually has a height of 305 cm.

Find the percentage error in using the model to predict the height of this birch tree.
(4 marks)

(b) Explain why this model is not appropriate when a birch tree has just started to grow.
(1 mark)

(c) Another model for the height of a birch tree is

$$y = 0.08x^2 + 9x$$

(i) Explain why this is a better model for a birch tree that has just started to grow.
(1 mark)

(ii) Find the two values of x for which the two models for the height of a birch tree give the same prediction.

The solutions of $ax^2 + bx + c = 0$, where $a \neq 0$, are given by $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
(4 marks)

SECTION B

Answer **all** questions.

Use **Life expectancy** *on page 2 of the Data Sheet.*

3 (a) A possible model for the life expectancy, y years, at age x years is given by the equation

$$y = 75e^{(-x/50)}$$

Use this model to find:

(i) the life expectancy at age 60 years; *(2 marks)*

(ii) the age at which the life expectancy is 30 years. *(3 marks)*

(b) A second model for the life expectancy is given by the equation

$$y = 76 - x$$

Why is this model not appropriate for ages greater than 76 years? *(1 mark)*

(c) (i) Sketch the graph of $y = k\sqrt{x}$ where k is a positive constant. *(1 mark)*

(ii) Why is this **not** a suitable model to use for life expectancy? *(1 mark)*

(d) (i) Sketch the graph of $y = \frac{k}{x}$ where k is a positive constant. *(1 mark)*

(ii) Why is this **not** a suitable model to use for life expectancy? *(2 marks)*

Turn over for the next question

SECTION C

Answer **all** questions.

Use **Organic food** *on page 3 of the Data Sheet.*

4 A model for a supermarket's average daily sales, £S, of organic food is given by the equation

$$S = 1300\sin(5t)^{\circ}$$

where t is the number of years since 1995.

(a) Use the organic food graph on the answer sheet to complete the graph of S against t for $0 \leq t \leq 30$. *(4 marks)*

(b) (i) What is the maximum daily sales of organic food predicted by this model? *(1 mark)*

(ii) In what year does this maximum occur? *(1 mark)*

(c) (i) Use the organic food graph on the answer sheet to find the gradient of the graph when $t = 5$. *(2 marks)*

(ii) State the units of the gradient. *(1 mark)*

(d) Find the first time when the average daily sales is £1200. *(2 marks)*

(e) Describe fully the transformations that map the graph of the function $S = \sin(t)^{\circ}$ onto the graph of the function $S = 1300\sin(5t)^{\circ}$. *(2 marks)*

SECTION D

Answer **all** questions.

Use **Bacteria** *on page 3 of the Data Sheet.*

5 (a) The number of bacteria, N, can be modelled by the equation $N = kt^c$ where t is the number of hours since the start of the experiment, and k and c are both constants.

(i) For this model, show that $\ln N = \ln k + c \ln t$. *(2 marks)*

(ii) On the answer sheet, complete the table of values for $\ln t$ and $\ln N$, giving the values to 3 significant figures. *(2 marks)*

(iii) Use the grid on the answer sheet to plot $\ln N$ against $\ln t$.
Draw a line of best fit on your diagram. *(2 marks)*

(iv) Find the equation for the line of best fit in terms of $\ln N$ and $\ln t$. *(3 marks)*

(v) Hence express N in terms of t. *(2 marks)*

(b) In another experiment, the number of bacteria can be modelled by the equation

$$N = 50t^{3.6}$$

(i) Use this model to calculate the number of bacteria after 3 hours. *(2 marks)*

(ii) Use this model to calculate when the number of bacteria is 1000 . *(4 marks)*

END OF QUESTIONS

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Centre Number			Candidate Number				
Candidate Signature									

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ASSESSMENT and QUALIFICATIONS ALLIANCE

WORKING WITH ALGEBRAIC AND GRAPHICAL TECHNIQUES **6991/2**
Unit 11

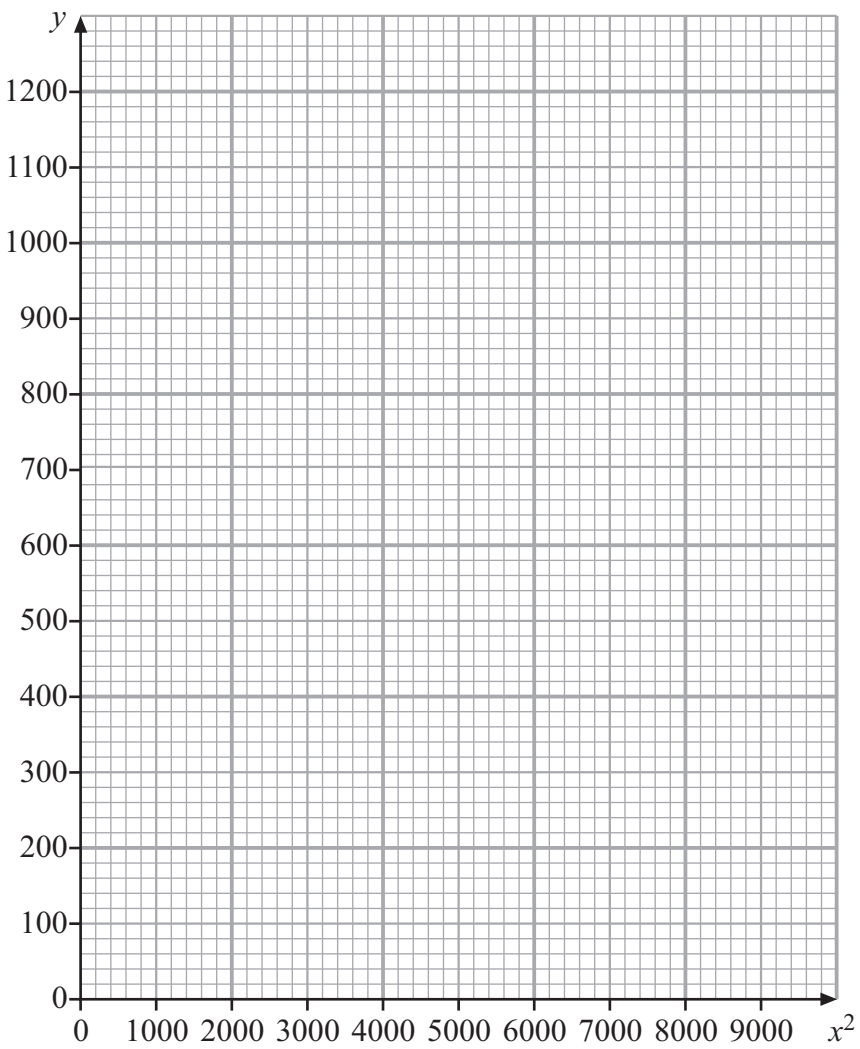
This answer sheet is to be used when answering Questions 1, 4 and 5, as indicated. Fasten this sheet securely to your answer book.

This table and graph are to be used when answering Question 1.

(a)

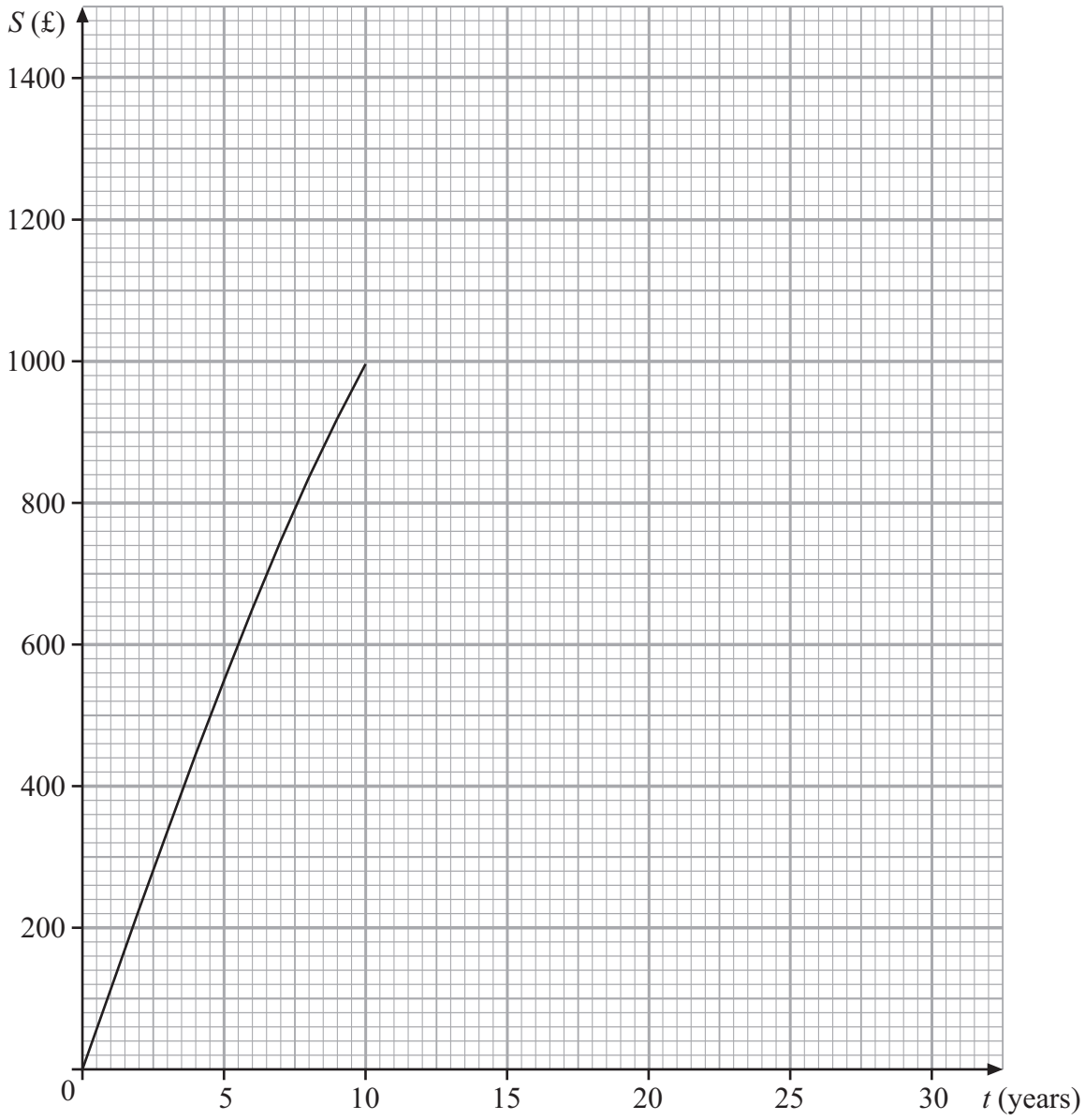
Circumference (cm), *x*	10	38	55	68	81	94
x^2						
Height (cm), *y*	130	260	440	580	760	1000

(b)



This graph is to be used when answering Question 4.

(a)



This table and graph are to be used when answering Question 5.

(a) (ii)

t	1	2	3	4	5	6
N	70	390	1100	2240	3900	6200
ln *t*						
ln *N*						

(a) (iii)