FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BPS-17 UNDER THE FEDERAL GOVERNMENT, 2001.

STATISTICS

TIME ALLOWED: THREE HOURS   MAXIMUM MARKS: 100

NOTE: Attempt **FIVE** questions in all, including question No.8 which is **COMPULSORY**. All questions carry **EQUAL** marks.

1. (a) Explain classical, axiomatic and relative frequency definitions of Probability with one example in each case. Which definition you prefer in day to day problems solving in a Chaotic Situations.

   (b) Define law of total probability. Three facilities supply micro processors to a manufacturer of telemetry equipment. All are supposedly made to the same specifications. However, the manufacturer has for several years tested microprocessors, and records indicate following numerical facts:

Supply Facility	Fraction Defective	Fraction Supplied by
1	0.02	0.15
2	0.01	0.80
3	0.03	0.05

   The director of manufacturing randomly selects a microprocessor, takes it to the test department, and finds that it is defective. If we let A be the event that an item is defective and Bi be the event that the item came from facility i(i=1,2,3). Compute $P(B_i \mid A)$ i=1,2,3 and comment.

2. (a) The daily demand of Computer diskettes in a office follows the probability distribution:

X = x	0	1	2	3	4
P(X=x)	0.2	0.25	0.25	0.2	0.1

   Compute E(X) and Var (X).

   (b) State Chebyshev's inequality. Estimate the demand interval such that the probability is at least $\frac{8}{9}$ that the demand will remain or lie in that interval.

3. Define binomial, Poisson and negative binomial random variables and find their mean and variance respectively. Comment on relation between mean and variance for each random variable.

4. (a) What do you understand by maximum likelihood estimation of parameter θ, if X follows pdf $f_x(x, \theta)$ and a random sample of size n is given on X. Discuss with an example.

   (b) Find maximum likelihood estimator of λ when r.v.x follows exponential distribution given by: $f(x, \lambda) = \lambda e^{-\lambda x}, x > 0$.

5. (a) Define regression line of Y on X and regression line of X on Y. How regression coefficients are related with correlation between X and Y.

   (b) For regression line of Y on X is $y = \alpha + \beta X + \epsilon$. Give complete procedure for testing Ho: β=0 where ϵ follows $N(0, \sigma^2)$ and σ^2 is not known.

6. In a large city of Pakistan, we are interested to study socio-economic conditions of the citizens. It is known that lower middle class, middle class, higher middle class and affluent people are living in the city. Discuss the sampling technique which is most suitable in such situations. How the sample size will be determined?

7. In an organization married and unmarried individuals are working and we are interested to study the following null and alternative hypothesis using statistical methods. A sample of 500 employees was selected and following results in the tabular form are obtained under the Hypothesis:

H_0 : Absentee behaviour is independent of marital status.

H_1 : Absentee behaviour is dependent of marital status.

Marital Status vs Absentee Rate

Marital Status ↓	Absentee Rate → Zero	1-5	over 5	Row Total
Single	84	82	34	200
Married	50	64	36	150
Divorced	50	34	16	100
Widow	16	20	14	50
Column Total	200	200	100	500

Test H_0 against H_1 as stated above and write conclusion.

COMPULSORY QUESTION

8. Select the correct answer by writing (a), (b), (c) or (d) (for each part of the question in the answer book. Don't reproduce questions:

(1) Statistic is used for:
(a) Subject statistics (b) a number
(c) random number (d) (b) and/or (c).
(e) None of these.

(2) If a random variable X is measurable then the probability $P(X = o)$ is
(a) 1 or 0 (b) less than 1
(c) 1 or less than 1 (d) zero.
(e) None of these.

(3) If $A_1, A_2 \subsetneq S$ and S is sample space then $P(A_1 \cup A_2 \mid S) \leq P(A_1) + P(A_2)$ if:
(a) $A_1 \subset A_2$ (b) $A_1 \subsetneq A_2$
(c) $A_1 \wedge A_2 \neq \Phi$ (d) $A_1 \wedge A_2 = \Phi$
(e) None of these.

(4) If a frequency distribution is normal then:
(a) $\beta_1 = 3, \beta_2 = 0$ (b) $\beta_1 = 0, \beta_2 = 3$
(c) $\beta_1 = 1, \beta_2 = 2$ (d) None of these.

(5) If α = Prob (Reject $H_0 \mid H_0$ true), β = Prob (Accept $H_0 \mid H_1$ true):
(a) If α increases then β decreases.
(b) If α increases then β remains unchanged.
(c) If α decreases then β decreases.
(d) If α increases then β increases.
(e) None of these.

(6) In binomial distribution:
(a) Number of successes are fixed.
(b) Number of successes are random.
(c) Number of trials are random.
(d) Number of trials and successes are random.
(e) None of these

(7) If events A and B are not mutually exclusive then:
(a) $P(A|B) = 0$
(b) $P(A|B) = P(A)$
(c) $P(A|B).\ P(B) = P(A \cap B)$
(d) $P(A|B) = P(B)$
(e) None of these.

(8) The joint density function of X_1 and X_2 is given by $f(x_1, x_2) = \frac{1}{500}$; $0 \le x < 0.25;\ 0 \le x \le 2000$ then:
(a) X_1 and X_2 independent
(b) X_1 and X_2 are not independent
(c) X_1 depends on X_2
(d) X_2 depends on X_1
(e) None of these.

(9) If X be a r.v. with pdf $p(x) = k.q^{x-1}$, $x = 1, 2, \ldots$ and k is constant then
(a) $k = 1$
(b) $k = \frac{1}{p}$
(c) $k = p$
(d) $k = q$
(e) None of these.

(10) For a negative binomial distribution, if p = 2 then for r=50th success needs on average:
(a) 50 trials
(b) 100 trials
(c) 150 tirals
(d) 200 trials
(e) None of these.

(11) For normal distribution, pdf $f(x;\ \mu,\ \sigma^2)$ is:
(a) $f(x + \mu,\ \sigma\ ) = f(x - \mu;\ \sigma\ )$
(b) $f(x + \mu,\ \sigma\ ) = f(-x + \mu;\ \sigma\ )$
(c) $f(x + 2\mu,\ \sigma\ ) = f(-x + \mu;\ \sigma\ )$
(d) $f(2x + \mu,\ \sigma\ ) = f(2x - \mu;\ \sigma\ )$
(e) None of these.

(12) Equality of two population means is tested by:
(a) Z-test with $\sigma_1^2 = \sigma_2^2$ is known.
(b) t-test with $\sigma_1^2 = \sigma_2^2$ is known.
(c) chi-square test.
(d) None of these.

(13) If $n \to \infty$ and p is fixed then binomial probabilities can computed using:
(a) normal (np, npq)
(b) Poisson (np)
(c) hypergeometrix
(d) χ^2 - distribution
(e) None of these.

(14) If x has a binomial distribution with parameters p and n then $\frac{x}{n}$ has variance:

(a) npq (b) n^2pq

(c) $\frac{pq}{n}$ (d) $\frac{pq}{n^2}$

(e) None of these.

(15) If x is $n(\mu, \sigma^2)$ then $Z = \frac{x-\mu}{2\sigma}$ is :

(a) $n(0,1)$ (b) $n(0, \frac{1}{4})$

(c) $n(0, 2\sigma)$ (d) $n(1, \sigma^2)$

(e) None of these.

(16) $Y_i = \alpha + \beta x_i + \epsilon_i$, $i = 1,2,\ldots, n$ if $H_0: \beta = 0$ is:

(a) rejected then there is linear relationship between x & y.
(b) accepted then there is linear relationship between x & y.
(c) rejected then there is no linear relationship between x & y.
(d) accepted then there is no linear relationship between x & y.
(e) None of these.

(17) The mean square error of an estimator $\hat{\theta}$ of θ is:

(a) $V(\hat{\theta})$ if $\hat{\theta}$ is biased estimator of θ.
(b) $V(\hat{\theta})$ if $\hat{\theta}$ is unbiased estimator of θ.
(c) $V(\hat{\theta})$ if $\hat{\theta}$ is unbiased or biased estimator of θ.
(d) None of these.

(18) If $\hat{\theta}_1$ estimates θ with $V(\hat{\theta}_1)$ and $\hat{\theta}_2$ estimates θ with $V(\hat{\theta}_2)$ then:

(a) $\hat{\theta}_1$ is better than $\hat{\theta}_2$ if $V(\hat{\theta}_1) < V(\hat{\theta}_2)$
(b) $\hat{\theta}_1$ is better than $\hat{\theta}_2$ if $V(\hat{\theta}_1) > V(\hat{\theta}_2)$
(c) $\hat{\theta}_1$ is unbiased and minimum variance estimator.
(d) $\hat{\theta}_1$ is biased and minimum variance estimator.

(19) Sample corelation coefficient between x & y is γ then:

(a) $|\gamma| < 1$ (b) $|\gamma| > 1$

(c) $1 < |\gamma| \neq 0$ (d) None of these.

(20) The variance of sampling distribution of mean is:

(a) σ_{β}^2 (b) $n\sigma^2$

(c) $n^2\sigma^2$ (d) $\frac{\sigma^2}{n}$

if $v(x_i) = \sigma^2$ and $\bar{x} = \frac{x_1 + x_2 + \ldots + x_n}{n}$.

****