



FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION FOR
RECRUITMENT TO POSTS IN BPS-17 UNDER
THE FEDERAL GOVERNMENT, 2010

Roll Number

PURE MATHEMATICS, PAPER-I

TIME ALLOWED: 3 HOURS

MAXIMUM MARKS:100

NOTE:	(i) Attempt FIVE questions in all by selecting at least THREE questions from SECTION-A and TWO questions from SECTION-B . All questions carry EQUAL marks. (ii) Use of Scientific Calculator is allowed.
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SECTION – A

- Q.1.** (a) Let W be a subspace of a finite dimensional vector space V , then W is finite dimensional and $\dim(w) \leq \dim(v)$. Also if $\dim(w) = \dim(V)$, then $V = W$. (10)
 (b) Let V & W be vector space and let $T : V \rightarrow w$ be a linear if V is finite dimensional, then $\text{nullity}(T) + \text{rank}(T) = \dim v$ (10)
- Q.2.** (a) Show that there exist a homomorphism from S_n onto the multiplication group $\{-1,1\}$ of 2 elements ($n \geq 1$). (7)
 (b) If H is the only subgroup of a given finite order in a group G . Prove that H is normal in G . (7)
 (c) Show that a field K has only two ideals (namely K & (0)). (6)
- Q.3.** (a) Find all possible jordan canonical forms for 3×3 matrix whose eigenvalues are $-2,3,3$ (10)
 (b) Show that matrix $\begin{bmatrix} 1 & 3 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ (10)
 is diagonalizable with minimum calculation
- Q.4.** (a) Every group is isomorphic to permutation group (7)
 (b) Show that for $n \geq 3$ $Z(S_n) = I$ (6)
 (c) Let A, B be two ideal of a ring, then $\frac{A+B}{A} = \frac{B}{A \cap B}$. (7)
- Q.5.** (a) Verify Cayley – Hamilton theorem for the matrix (7)

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 2 & -3 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

 (b) Prove that ring $A = \mathbb{Z}$, the set of all integers is a principal ideal ring. (7)
 (c) Under what condition on the scalar, do the vectors $(1,1,1)$, $(1,\xi,\xi^2)$, $(1,-\xi,\xi^2)$ form basis of \mathbb{C}^3 ? (6)

SECTION – B

- Q.6.** (a) Show that $T.N. = 0$ for the helix (10)
 $R(t) = (\cos wt) \hat{i} + (\sin wt) \hat{j} + (bt) \hat{k}$
 (b) The vector equation of ellipse $r(t) = (2 \cos t) \hat{i} + (3 \sin t) \hat{j}$; ($0 \leq t \leq 2\pi$)
 Find the curvature of ellipse at the end points of major & minor axes. (10)
- Q.7.** (a) Discuss & sketch the surface (12)
 $x^2 + 4y^2 = 4x - 4z^2$
 (b) Show that an equation to the right circular cone with vertex at 0 , axis oz & semi-vertical angle α is $x^2 + y^2 = z^2 \tan^2 \alpha$ (8)
- Q.8.** (a) Show that hyperboloids of one sheet and hyperbolic parabolas are ruled surface. (6+6)
 (b) Find an equation of the plane which passes through the point $(3,4,5)$ has an x – intercept equal to -5 and is perpendicular to the plane $2x+3y-z = 8$. (8)
