FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BPS-17 UNDER THE FEDERAL GOVERNMENT, 2009

S.No.	
R.No.	

PURE MATHEMATICS, PAPER-II

TIME ALLOWED: 3 HOURS	**MAXIMUM MARKS:100**

NOTE:	(i) Attempt **FIVE** questions in all by selecting at least **THREE** questions from **SECTION–A** and **TWO** question from **SECTION–B**. All questions carry **EQUAL** marks. (ii) Use of Scientific Calculator is allowed.

SECTION – A

Q.1. (a) Let the function f = [–2, 2] → R be defined by f(x) = |x|. Show that f is continuous at x = 0 but it is not differentiable at x = 0. Will there exist a point c in]–1, 1[such that $f'(c) = 0 \; or \; f(1) - f(-1) = 2\, f'(c)$? **(10)**

(b) Evaluate $\underset{x \to o}{Lim} \frac{(1+x)^{\frac{1}{x}} - e}{x}$ **(10)**

Q.2. (a) Find the asymptotes of the curve defined by the equation $(\mathbf{x} - y)^2 (x^2 + y^2) - 10(x - y)\, x^2 + 12y^2 + 2x + y = 0$ **(10)**

(b) Test the convergence of the series

$$\sum_{n=1}^{\infty} \frac{1}{n^k}, k > 0$$

How do we call this series? **(10)**

Q.3. (a) Find the area enclosed by the parabola $y^2 + 16x + 6y - 71 = 0$ and the line $4x + y + 7 = 0$ **(10)**

(b) Find the volume of the solid generated by revolving about the y-axis the area of the triangle with vertices at (2,1), (6,1) and (4,5). **(10)**

Q.4. (a) If $u = \text{are}\; Sin \frac{(x^2 + y^2)}{x + y}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$. **(10)**

(b) Integrate $F(x, y) = \frac{1}{y^4 + 1}$ over the region $R: o \le x \le 8, \sqrt[3]{x} \le y \le 2$ **(10)**

Q.5. (a) Let X be the set of all (bounded or unbounded) sequences of complex numbers. If d: X × X → R is defined as

$$d(x, y) = \sum_{j=1}^{\infty} \frac{1}{2^j} \frac{|\xi_j - \eta_j|}{1 + |\xi_j - \eta_j|}$$

$where\; x = (\xi_j)$ and $y = (\eta_j)$, then show that d is a metric on X. **(10)**

(b) Prove that the mapping:

$T: (X, d_x) \to (Y, d_y)$

is continuous at a point $x_o \varepsilon\; X \Leftrightarrow x_n \to x_o$ implies $Tx_n \to Tx_o$. **(10)**

SECTION – B

Q.6. (a) If $Z = \frac{(1+i)+(3+2i)t}{1+it}$, then show that the locus of Z is a circle. Also calculate the mi[n] and maximum distance of Z from the origin. **(10)**

(b) Find the complex number Z satisfying the equation
$Z^2 + (2i - 3)Z + (5 - i) = 0$ **(10)**

Q.7. (a) Show that the function

$$u(x,y) = 4xy - 3x + 2$$

is harmonic. Construct the corresponding analytic function

$$f(z) = u(x,y) + iv(x,y)$$ **(10)**

(b) Find the Fourier Series of the function

$$f(x) = \begin{cases} x, & 0 < x \le \pi \\ 2\pi - x, & \pi < x < 2\pi \end{cases}$$

period 2π **(10)**

Q.8. (a) Evaluate the following integral by using Canchy Integral Formula:

$$\int_c \frac{4-3z}{z(z-1)(z-2)}\,dz$$

where C is the circle $|z| = \frac{3}{2}$ **(10)**

(b) Prove that

$$\int_o^{2\pi} \frac{d\theta}{1 - 2p\,Cos\theta - p^2} = \frac{2\pi}{1-p^2}$$

where o < p < 1. **(10)**

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