

**PURE MATHEMATICS, PAPER-I**



**FEDERAL PUBLIC SERVICE COMMISSION  
COMPETITIVE EXAMINATION FOR  
RECRUITMENT TO POSTS IN BPS-17 UNDER  
THE FEDERAL GOVERNMENT, 2009**

**PURE MATHEMATICS, PAPER-I**

S.No.	
R.No.	

**TIME ALLOWED: 3 HOURS**

**MAXIMUM MARKS:100**

<b>NOTE:</b>	(i) Attempt <b>FIVE</b> questions in all by selecting at least <b>THREE</b> questions from <b>SECTION-A</b> and <b>TWO</b> question from <b>SECTION-B</b> . All questions carry <b>EQUAL</b> marks. (ii) Use of Scientific Calculator is allowed.
--------------	--

**SECTION – A**

**Q.1. (a)** Prove that the set  $S_n$  of all permutations on a set  $X$  of  $n$  elements is a group under the operation ‘o’ of composition of permutations. Will  $(S_n, o)$  be an abelian group? How do we call this group? **(10)**

(b) If  $G$  is a group,  $N$  a normal subgroup of  $G$ , then show that the set  $G/N$  of right cosets of  $N$  in  $G$  is also a group. How we call this group? Also, if  $G$  is finite then show that

$$o\left(\frac{G}{N}\right) = \frac{o(G)}{o(N)}. \quad (10)$$

**Q.2. (a)** Let  $\phi$  be a homomorphism of a group  $G$  onto another group  $H$  with kernel  $K$ . Prove that  $G/K$  is isomorphic to  $H$ , that is  $G/K \approx H$ . **(10)**

(b) Let  $Z_n$  be the set of the congruence classes modulo  $n$ , that is,  
 $Z_n = \{[0], [1], [2], \dots, [n-1]\}$   
 Define the two binary operations on  $Z_n$  under which it is a ring. Prove that the ring  $Z_n$  is an integral domain  $\Leftrightarrow n$  is a prime number. **(10)**

**Q.3. (a)** Let  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$  be the linear mapping defined by:  
 $T(x,y,z) = (x+2y - z, y + z, x+2y - z)$   
 Verify that  
 $\text{Rank}(T) + \text{Nullity}(T) = \dim D(T)$   
 Also find a basis for each Rank  $(T)$  and Nullity  $(T)$  **(10)**

(b) If  $U$  and  $W$  are finite – dimensional subspaces of a vector space  $V$  over a field  $F$  then prove that  
 $\dim(U+W) + \dim(U \cap W) = \dim U + \dim W$  **(10)**

**Q.4. (a)** Let  $H$  and  $K$  be two subgroups of a group  $G$ . Prove that  $HK$  is a subgroup of  $G \Leftrightarrow HK = KH$ . **(10)**

(b) Let  $v_1, v_2, \dots, v_n$  be non-zero eigen vectors of an operator  $T:V \rightarrow V$  belonging to distinct eigen values  $\lambda_1, \lambda_2, \dots, \lambda_n$ . Show that the vectors  $v_1, v_2, \dots, v_n$  are linearly independent. **(10)**

**PURE MATHEMATICS, PAPER-I**

- Q.5.** (a) Let  $V$  be the vector space of  $n$ -square matrices over the field  $\mathbb{R}$ . Let  $U$  and  $W$  be subspaces of symmetric and antisymmetric matrices, respectively. Show that  $V = U \oplus W$ .  
 (b) Diagonalize the following matrix:

$$M = \begin{bmatrix} -4 & -4 & -8 \\ 4 & 6 & 4 \\ 6 & 4 & 10 \end{bmatrix}$$

**SECTION – B**

- Q.6.** (a) Find the lengths of the following curves: **(10)**  
 (i)  $9y^2 = 4x^3$  from  $x = 3$  to  $x = 8$   
 (ii)  $r = \sin^2 \frac{\theta}{2}$  from  $\theta = 0$  to  $\theta = \Delta$

- (b) Find the radius of curvature of the given curve at the designated point. **(10)**

$$y = \frac{a}{2} \ln \frac{\sqrt{a^2 + x^2 + a}}{\sqrt{a^2 + x^2 - a}} - \sqrt{a^2 + x^2}; (x, y)$$

- Q.7.** (a) Show that the two lines  
 $L_1 : x = 4 - t, y = -2 + 2t, z = 7 - 3t$   
 $L_2 : x = 3 + 2s, y = -7 - 3s, z = 6 + 4s$   
 are skew. Also find the points on the lines such that the segment joining these points is perpendicular to both lines and hence find the shortest distance between the given lines. **(10)**  
 (b) Find the equation of the sphere through the circle  $x^2 + y^2 + z^2 = 1, 2x + 4y + 5z = 6$  and touching the plane  $z = 0$ . **(10)**

- Q.8.** (a) At a point on a curve  $\underline{r} = \underline{r}(t)$  at which  $k \neq 0$ , show that  

$$\tau = \frac{[\underline{r}' \ \underline{r}'' \ \underline{r}''']}{|\underline{r}' \times \underline{r}''|^2}$$
  
 where  $\underline{r}' = \frac{d\underline{r}}{dt}$  **(10)**

- (b) Find the First Fundamental Form and fundamental magnitudes of first order for the sphere  
 $\underline{r} = (a \cos u \cdot \cos v, a \cos u \cdot \sin v, a \sin u)$   
 Also prove that parametric curves are orthogonal. **(10)**

\*\*\*\*\*