FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BPS-17, UNDER THE FEDERAL GOVERNMENT, 2005

PURE MATHEMATICS, PAPER-I

TIME ALLOWED: THREE HOURS MAXIMUM MARKS: 100

NOTE: Attempt **FIVE** questions in all, including **QUESTION NO.8** which is **COMPULSORY**. Select **TWO** question from each **SECTION**, all questions carry **EQUAL** marks.

SECTION - I

1- (a) If f is a homomorphism of a group G into a group G with kernel K,prove that k is a normal subgroup of G. 10

(b) If G is a group, then A(G), the Set of all automorphisms of G,is also a group. 10

2- (a) If D is a commutative integral domain with unity and has finite characteristic n, prove that n is prime number. 08

(b) If R in a commutative ring with unity and M is an ideal of R, prove that M is a maximal ideal of R if and only if R/M is a field. 12

3- (a) If W is a subspace of finite-dimensional vector space V, prove that $\dim W \leq \dim V$ *and* $\dim V/W = \dim V - \dim W$. 08

(b) Let A be an $n \times n$ matrix prove that det A=0 if and only if rank A is less than n. 12

4- (a) For what values of K the equations 08

$(5-K)x_1 + 4x_2 + 2x_3 = 0$

$4x_1 + (5-k)x_2 + 2x_3 = 0$

$2x_1 + 2x_2 + (2-k)x_3 = 0$

have non-trivial solutions. Find the solutions.

(b) Let V be finite dimensional vector space over a field F and A(V) the algebra of linear Transformations on V. prove that $\lambda \in F$ is an eigen value of $T \in A(V)$ if and only if $vT = \lambda v$ for some $v \neq 0$ in V. 12

Contd......

PURE MATHEMATICS, PAPER-I:

SECTION – II

5- (a) Find the pedal equation of the cardiode $r = a(1+\cos\theta)$.

(b) Find the center of curvature of the parabola $x^2 = 4y$ at the point (4,1). 10

6- (a) Find the volume of a tetrahedron whose vertices are (1,-1,2), (2,0,1) (0,-2,1) and (-2,2,1) 10

(b) The normal at a point P of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ meets the coordinates planes in G_1, G_2, G_3 respectively. Prove that the ratios $PG_1 : PG_2 : PG_3$ are constant. 10

7- (a) Define the involute and evolute of a space curve. Prove that the tangent to the involute is parallel to the principal normal to the given cutve. 10

(b) If the curve of intersection of two surfaces is a line of curvature on both, prove that the surfaces cut at a constant angle. 10

8- (a) Write only the correct choice in the answer book. Do not reproduce the less than n.

(i) The additive group of all rational number is:

(a) Torsion free
(b) Finitely generated
(c) Cyclic
(d) None of these.

(ii) Every group of order 25 must be

(a) Cyclic
(b) Nonabelian
(c) Abelian
(d) None of these.

(iii) The order of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 3 & 2 & 5 & 1 \end{pmatrix}$ is

(a) 5
(b) 6
(c) 7
(d) None of these.

Contd......

(x) The equation of the surface of revolution obtained by rotating the curve $x^2+2y^2=8$, $z=0$ about y axis is.

(a) $x^2+2y^2+2z^2=8$
(b) $x^2+2y^2+z^2=8$
(c) $x^2+2z^2=8$
(d) None of these.

(xi) If the tangent at a point P on a parabola meets the directrix in K, then angle KSP (S focus) is

(a) Right angle
(b) Straight angle
(c) Obtuse angle
(d) None of these

(xii) The sum of the focal distances of a point P on an ellipse is

(a) Variable with P
(b) Greatest when P is at an end of major axis
(c) Constant
(d) None of these.

(xiii) If field F has finite order q, then for every $a \in F$,

(a) $a^{q-1}=0$
(b) $a^q=a$
(c) $a^q=0$
(d) None of these.

(xiv) Let A be an n x n matrix. Then det A=0 if and only if

(a) Rank $A \langle n$
(b) Rank A$\rangle$ n
(c) Rank A=n
(d) None of these.

(xv) A square matrix A such that $A^n=0$ for some positive integer n is called

(a) Idempotent
(b) Involutory
(c) Nilpotent
(d) None of these.

Contd.....

(iv) If a group G has finite order divisible by n then

(a) G contains a subgroup of order n
(b) G contains an element of order n
(c) G need not contain an element of order n.
(d) None of these.

(v) The multiplicative group of non zero elements of a finite field is

(a) Of prime order
(b) Of prime power order
(c) Cyclic
(d) None of these.

(vi) The envelope of the normal plane of a twisted curve is called ________developable

(a) Osculating.
(b) Polar.
(c) Rectifying.
(d) None of these.

(vii) The Gauss curvature of a surface at any point is the ________ of the principal curvatures.

(a) Difference
(b) Sum
(c) Product
(d) None of these.

(viii) The theorem Kn = Kcos θ connecting normal curvature in any direction with the curvature of any other section through the same tangent is called.

(a) Meunier's theorem.
(b) Euler's theorem.
(c) Dupin's theorem.
(d) None of these.

(ix) The envelope of the family $x^2 + y^2 - 4az + 4a^2 = 0$ is

(a) $x^2 + y^2 = yx$
(b) xyz = 1
(c) $x^2 + y^2 = z^2$
(d) None of these.

Contd.....

(xvi) Let $f: V \rightarrow W$ be a linear map where v is finite-dimensional, then

(a) dim W=dim V + dim (Kerf)
(b) dim V=dim(Kerf) + dim (imf)
(c) dim V = dim W+dim (imf)
(d) None of these.

(xvii) The perpendicular distance of the point (2,2,1) from the line

$\frac{x-1}{2} = \frac{y+1}{3} = Z$ is

(a) 2
(b) 3
(c) $\sqrt{\frac{5}{7}}$
(d) None of these.

(xviii) The cylindrical coordinates of a point with spherical polar coordinates $\left(3, \frac{\pi}{6}, \frac{\pi}{4}\right)$ are

(a) $\left(\frac{3}{\sqrt{2}}, \frac{\pi}{6}, \frac{3}{\sqrt{2}}\right)$
(b) $\left(\frac{3}{4}, \frac{\sqrt{2}}{3}, 6\right)$
(c) $\left(2, \frac{\pi}{2}, 1\right)$
(d) None of these.

(xix) A set of 4 vectors in a 3-dimensional vector space must be

(a) Linearly independent
(b) A basis
(c) Linearly dependent
(d) None of these

(xx) If A,B are matrices such that AB exists and is the zero matrix, then

(a) A must be zero matrix.
(b) B must be zero matrix.
(c) Neither A nor B need be zero matrix
(d) None of these.

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PURE MATHEMATICS, PAPER–II

TIME ALLOWED: THREE HOURS MAXIMUM MARKS: 100

NOTE: Attempt **FIVE** questions in all, including **QUESTION NO.8** which is **COMPULSORY**. Select **TWO** question from each **SECTION**, all questions carry **EQUAL** marks.

SECTION - I

1 (a) Evaluate : $\lim_{x\to 0} \frac{e^{2x}-e^{-2x}}{\ln(1+x)}$ 06

(b)If f is continuous on [a,b] and differentiable on (a,b), then there exist a number C in (a,b) such that f(b) - f(a) = b-a f' (c). 06

(c) If ∑ an converges absolutely, prove that ∑ an converges. Give an example to show that the converse is not true. 08

2 (a) By evaluating both repeated integrals show that: 08

$$\int_0^1\left[\int_0^1 \frac{x-y}{(x+y)^3}\,dy\right]dx \neq \int_0^1\left[\int_0^1 \frac{x-y}{(x+y)^3}\,dx\right]dy$$

(b) Find the whole length of the cardioid r = a (1+sinθ). 06

(c) Let $\sum_{n=1}^{\infty} Mn$ be a convergent series of positive term, and let | f_n(x) |≤ 06

M_n for all x in [a,b] and all n. prove that $\sum_{x=1}^{\infty} fn(x)$

converges uniformly in [a,b].

3 (a) Let f be Riemann integrable on [a,b] and let, F(x) = 10
Prove that F is Continuous on[a,b].if f is continuous at a point c in (a,b), prove that $f'(c) = f(c)$.

(b) let $f(x,y) = \frac{xy}{\sqrt{x^2+y^2}}$ $when (x,y) \neq (0,0) and f(0,0) = 0$ 10

prove that f is continouse possesses partial dervatives but is not differentiable at (0,0).

Contd……

PURE MATHEMATICS, PAPER-II:

4 (a) Let x,y be metric spaces, $f : x \to y$ a function and $C \in X$.
Prove that f is continous at C if and if $\lim_{n\to\infty} f(xn) = f(C)$
whenever (x_n is a sequence in x coverging to C.

(b) let (x,d) be a metric space . Define the term:Cauchy sequence and completeness. 10
Prove that if (x,d) is complete and A is a closed subset then (A,d) is also complete.
If A is a compact subset of X, is (A,d) complete ? justify your answer.

Section –II

5 (a) Use De Moivre's Theorm to prove that 08

$$\sum_{k=0}^{8} \cos(\frac{2k\pi}{9}) = 0$$

(b) Let $f(Z) = \begin{cases} 0, & z = 0 \\ u(x,y) + iv(x,y), & z \neq 0, \end{cases}$ 12

where $u(x,y) = (x^3 - y^3)/(x^3 + y^2)$
$v(x,y) = (x^3 + y^3)/(x^2 + y^2)$
Show that the cauchy-Riemann equations are satisfied at the origin
but $f'(0)$ does not exit.

6 (a) State and prove Liouville's theorem. 06

(b) Use cauchy integral formula to evaluate 08
$\int_c \frac{9z^2 - iz + 4}{z(z^2+1)} dz$, where c is the circle |z|=2 in the positive direction .

(c) Use Taylor's series, prove that: 06

$$\frac{1}{z^2} = \frac{1}{4}\sum_{n=0}^{\infty}(-1)^n (n+1)(\frac{z-2}{2})^n \ for (|z-2| < 2).$$

7 (a) Find the residues of tan z at its poles. 10

(b) Use the method of residues to evaluate $\int_0^{2\pi} e^{\cos\theta} \cos(\sin\theta - 3\theta) d\theta$ 10

Contd......

(vii) $f(x)=\frac{\sin x}{x}, x\in(0,\frac{\pi}{2})$, is

(a) strictly increasing
(b) strictly decreasing
(c) unbounded.
(d) None of these

(iii) $\lim_{n\to\infty}(1+\frac{x}{n})^n$ *equals*

(a) 1
(b) Dose not exit
(c) e^x
(d) None of these.

(ix) Suppose $f(x)=\sum_{n=0}^{\infty}Cnx^n$, where the series is convergent for all $|x|<R$. then f is

(a) continous but not differentiable
(b) differentiable
(c) monotonic
(d) None of these.

(x) The interval (0,1) is

(a) A countable set.
(b) A copmpact set
(c) An uncountable set
(d) None of the above.

(xi) Let $e=\lim_{n\to\infty}(1+\frac{1}{n})^n$ then e is

(a) Rational
(b) Irrational
(c) Algebraic
(d) None of these

(xii) The series $\sum_{n=1}^{\infty}\frac{1}{n^{1+\frac{1}{n}}}$ *is*

(a) convergent
(b) oscillating
(c) divergant
(d) None of these

(xiii) the function $f(z)=z^2e^{-z}$ is

(a) Entire .
(b) meromorphic
(c) bounded
(d) None of these.

Contd......

PURE MATHEMATICS, PAPER-II:

COMPULSORY QUESTION

8. write only the correct choice in the answer book. Do not reproduce the question.

(i) for all real number a, Limit a/n equals
   (a) 0
   (b) ∝
   (c) 1
   (d) none of these.

(ii) the series $\sum_{x=1}^{\infty}\frac{(-1)^x}{x}$ *is*
   (a) divergent.
   (b) Convergent.
   (c) Absolutely convergent.
   (d) None of these.

(iii) If f is Riemanns integrable on [a,b], the f must be
   (a) Continous on [a,b].
   (b) Differentiable on [a,b].
   (c) Monotonic on [a,b].
   (d) None of these.

(iv) Every closed subset of R, the real line, is
   (a) Complete.
   (b) Compact.
   (c) Bounded.
   (d) None of these.

(v) The series $\sum_{n=1}^{\infty}\frac{\sin n}{n^2}$ *is*
   Convergent but not absolutely.
   (a) Absolutely convergent.
   (b) Divergent
   (c) None of these.

(vi) $\lim_{x\to\infty} x^n e^{-x}$ $(x = 1,2,3,......)$ equals
   (a) 0
   (b) 1
   (c) ∝
   (d) None of these.

Contd......

(xiv) The converse of cauchy integral theorem is known as

(a) Goursat theorm
(b) Morera theorm
(c) Cauchy's inequality.
(d) None of these.

(xv) A simple closed curve divides the complex palne into__________ disjoint domains

(a) Two.
(b) Three
(c) Four
(d) None of these

(xvi) If a series of complex numbers $\sum_{n=1}^{\infty} z_n$ converges, then $\lim_{n\to\alpha}(-1)^n zn$ is

(a) -1
(b) Zero
(c) 1
(d) None of these.

(xvii) $\lim_{n\to\infty}\frac{(n-i)^3}{2n^3+n+2}$ equals

(a) $\propto$
(b) $\frac{1}{2}$
(c) zero
(d) None of the

(xviii) Log (-1+i) = 1/2log2 + iQ, where Q equals
(a) ¾
(b) -3/4
(c) -1/4
(d) none of these

(xix) Every compact subset of the complex plane is

(a) Open.
(b) Closed and bounded.
(c) Open and unbounded.
(d) None of these.

(xx) If z is not an integer, then $\pi(z)\pi(1-z)$ equals
(a) π
(b) $z\pi(z)$
(c) $\frac{\pi}{\sin \pi z}$
(d) None of these.