FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BPS-17, UNDER THE FEDERAL GOVERNMENT, 2004

PURE MATHEMATICS, PAPER-I

TIME ALLOWED: THREE HOURS — MAXIMUM MARKS: 100

NOTE: Attempt **FIVE** questions in all, including **QUESTION NO. 8** which is **COMPULSORY**. Select **TWO** questions from each **SECTION**. All questions carry **EQUAL** marks.

SECTION – I

1 (a) If G is a finite group and H is a subgroup of G, prove that the order of H is a divisor of the order of G. (10)

(b) Let G be a group, H a normal subgroup of G, T an automorphism of G. Let $T(H) = \{ T(h) : h \in H \}$. Prove that T(H) is a normal subgroup of G. (10)

2.(a) Let R be a commutative ring with unit element shoes only ideals are (0) and R itself. Prove that R is a field. (10)

(b) Let F be a finite field with q elements and suppose that FCK, where K is also a finite field with [K:F] = n. Prove that K has q^n elements. (10)

3.(a) If $S = \{x_1, x_2, \ldots\ldots\ldots x_n\}$ is a set of non zero vectors spanning a vector space V, prove that S contains a basis of V. (10)

(b) Let $T : U \to V$ be a linear transformation from an n – dimensional vector space U to a vector space V over the same field F. If $N(T) = \{u \in U : T(u) = 0\}$ and $R(T) = \{ v \in V : T(u) = v$ for same $u \in U\}$, Prove that dim N(T) + dim R(T) = n. (10)

4.(a) Let A be a n x n matrix. Prove that A.adj A = det A. I_n (8)

(b) Let V be a finite dimensional vector space over F, A(V) the algebra of all linear transformations V to V. For $T \in A(V)$, r (T) denotes rank of T. (12)
If $S, T \in A(V)$,
Prove:
(i) $r(ST) \leq r(T)$
(ii) $r(TS) \leq r(T)$
(iii) $r(ST) = r(TS) = r(T)$, if S maps V onto V.

SECTION-II

5. (a) Prove that the intrinsic equation of the cardiode $r = (1 - \cos\theta)$ is $8\sin^2(\psi/6)$. (10)

(b) Prove that the normal to a given curve is tangent to its evolute (10)

6. (a) Find the equations of tangent plane and the normal to the hyperboloid $x^2 - 3y^2 - z^2 + 3 = 0$ at (2, 1, -2). (10)

(b) Find the envelope of the family of planes $3a^2x - 3ay + z = a^3$, and show that its edge of regression is the curve of intersection of the surfaces $xz = y^2$, $xy = z$. (10)

7.(a) Prove that a space curve whose curvature and torsion are in a constant ratio is a helix. (10)

(b) Find the curvature and torsion of the curve (10)
$x = 3u - u^3$, $y = 3u^2$, $z = 3u + u^3$.

COMPULSORY QUESTION

(8) Write only the correct choice in the Answer Book. Do not reproduce the questions.

(1) Let G be a cyclic group of order 12. Then G has:
(a) 3 distinct subgroup (b) 4 distinct subgroup
(c) 6 distinct subgroup (d) None of these

(2) Let Q and Z be the additive groups of rationals and integers respectively. Then:
(a) The Group Q/Z is cyclic
(b) Every element of Q/Z is of infinite order
(c) Every element of Q/Z is of finite order.
(d) None of these.

(3) Suppose A,B are matrices such that the product AB exits and is zero matrix, then:
(a) A must be zero matrix (b) B must be zero matrix
(c) Neither A nor B need be zero matrix (d) None of these

(4) Let A be an n x n matrix, with rank A < n. Then:
(a) determinant A may be positive (b) determinant A must be p
(c) determinant A may be negative (d) None of these

(5) A square matrix A such that $A^2 = A$ is called:
(a) involuntary (b) idempotent
(c) nilpotent (d) None of these

(6) Let V be the real vector space of all functions on R to R, and let A = {x, Cos x}. Then:
(a) A is linearly independent (b) A spans V
(c) A is linearly dependent (d) None of these.

(7) The additive group of integers has:
(a) 6 quotient groups of order 6 each (b) 2 quotient groups of order 3 each
(c) 1 quotient group of order 6 (d) None of these

(8) The determinant of a triangular matrix is the product of its entries on:
(a) last row (b) main diagonal
(c) first row (d) None of these.

(9) Every elementary matrix is :
(a) non singular (b) singular
(c) involuntary (d) None of these

(10) The equation $x^2 + y^2 - z^2 = 0$ represents:
(a) quadric cone (b) a hyperbolic cylinder
(c) a hyperbolic paraboloid (d) None of these

(11) Let A be matrix. Then its :
(a) row rank my be greater than its column rank.
(b) Row rank may be less than its column rank.
(c) Row rank = column rank
(d) None of these

(12) A system of m homogeneous linear equations AX = 0 in n variables has a non – trivial solution if and only if:
(a) rank A = n (b) ran A < n
(c) rank A > n (d) None of these.

(13) M_2, R denote all 2 x 2 real matrices and real numbers. Let F: $M_2 \to R$, $f(A) = \det A$, for $A \in M_2$. Then:
(a) f is onto R (b) f is one-to-one
(c) f is neither onto nor one-to-one (d) None of these

(14) If J_n denotes the ring of integers mod n, then:
(a) J_7 is a field (b) J_6 is a field
(c) J_8 is an integral domain (d) None of these

(15) The rectangular coordinates of appoint with spherical coordinates $\left(3, \frac{\pi}{6}, \frac{\pi}{4}\right)$ are:
(a) (3, 1, -2) (b) $\left(\frac{3\sqrt{6}}{4}, \frac{3\sqrt{2}}{4}, \frac{3\sqrt{2}}{2}\right)$
(c) $\left(\sqrt{3}, \frac{1}{2}, 2\right)$ (d) None of these

(16) The distance of the point (3, 2, 3) from the plane 2x + 3y – z = 5 is :
(a) $\frac{5}{\sqrt{14}}$ (b) $\frac{3}{\sqrt{14}}$ (c) $\frac{4}{\sqrt{14}}$ (d) None of these

(17) Monge's form of the equation of a surface is:
(a) $f(x, y, z) = 0$ (b) $f\left(\frac{x}{y}, \frac{y}{z}\right) = 0$
(c) $z = f(x, y)$ (d) None of these

(18) The only space curve whose curvature and torsion are both constant is:
(a) a parabola (b) a circular helix
(c) a circle (d) None of these

(19) If torsion is zero at all points of a curve, the curve is:
(a) a helix (b) a straight line
(c) all on one plane (d) None of these

(20) Let G be a group of order 13. Then:
(a) G is non cyclic (b) G is non abelian
(c) G is commutative (d) None of these.

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PURE MATHEMATICS, PAPER-II

TIME ALLOWED: THREE HOURS **MAXIMUM MARKS: 100**

NOTE: Attempt **FIVE** questions in all, including **QUESTION NO. 8** which is **COMPULSORY.** Select TWO questions from each of the **SECTIONS I AND II**. All questions carry **EQUAL** marks.

SECTION – I

1. (a) Evaluate: $\lim_{x \to 0} \dfrac{e^x - e^{\sin x}}{x - \sin x}$ (8)

(b) If f is continuous on [a,b], f ′exists on [a,b] and f (a) =f(b), prove that there is a point C in (a,b) such that $f'(c) = 0$. (8)

(c) Find the inclined asymptotes of the curve $x^3 - y^3 + 6xy = 0$. (4)

2. (a) Evaluate $\iint_D xy^2\,dxdy$, where D is the region bounded by the x-axis, the ordinate at x = 4 and arc of the parabola $x^2 = 8y$. (6)

(b) If f (x,y) is continuously differentiable and homogeneous of degree n in a region R, prove that $x f_x(x,y) + y f_y(x,y) = n f(x,y)$. (6)

(c) Find all the maxima and minima of $f(x,y) = x^3 + y^3 - 63(x + y) + 12xy$ (8)

3. (a) Show that the function f in [0,1], where
$f(x) = 1$, x is irrational
$= 0$, x is rational, is not Riemann integral (6)

(b) Prove that:

$$\int_0^{\pi/2} \ln \cos x\, dx = -\frac{\pi}{2} \ln 2$$ (6)

(c) Prove that $\int_\pi^0 \frac{\sin x}{x}\,dx$ converges. (8)

4. (a) Prove that every compact subset of a metric space is closed. (8)

(b) Set Q be the space of all rational numbers with metric $d(x,y) = |x - y|$ for x, y in Q. Show that Q is not complete. (6)

(c) Prove that $\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$ is a number e, such that $2 < e < 3$. (6)

SECTION – II

5. (a) Let $x_n + iy_n = (1 + i)^n$, n is a positive integer. Using DeMoivre's theorem, Prove:

(i) $x_{2n}^2 + y_{2n}^2 = 4^n$ (5)

(ii) $X_{n-1}\, y_n - X_n\, y_{n-1} = 2^{n-1}$ (5)

(b) Let $f(z) = u(x,y) + iv(x,y)$ be analytic in a domain D. Using Cauchy – Riemann Conditions, Prove:

$$\left[\frac{\partial}{\partial x} |f(z)|\right]^2 + \left[\frac{\partial}{\partial x} |f(z)|\right]^2 = |f'(z)|^2 \text{ for all z in D.}$$ (10)

6. (a) Let C be a circle with center Z_0 and radius r and let f be analytic in an open set D containing C and its interior. Prove:

$$\left|f^{(n)}(z_0)\right| \le \frac{M\,n!}{r^n} \quad (n = 0,1,2,\ldots\ldots\ldots)$$

where M is the least upper bound of $|f(z)|$ on C.

(b) Show that $\int_c \frac{e^{3z} + 3\cosh z}{\left(z - i\frac{\pi}{2}\right)^4}\,dz = 8\pi$ (6)

where C is a simple closed contour containing $i\frac{\pi}{2}$ in its interior, and the integral is in the positive direction.

(c) Find the Laurent series expansion, in powers of z, for $\frac{1}{(z-1)(z-3)}$ in the annulus $1<|z|<3$. (6)

7. (a) Find the residues of $\frac{\cosh z}{z^2(z+i\pi)^3}$ at its poles. (10)

(b) Use the method of residues to evaluate $\int_c \frac{e^z\,d^z}{\sin hz}$, where C is the circle $|z| = 4$ in the positive direction. (10)

COMPULSORY QUESTION

8. Write only the correct choice in the Answer Book. Do not reproduce the question.

(1) The set of all ……..number forms a sequence:
(a) Real (b) Rational (c) Irrational (d) None of these

(2) $f(X) = x$, x rational = 0, x irrational in [0,1]:
(a) f is discontinuous at $x = \frac{1}{2}$ (b) f is discontinuous at x = 0
(c) f is continuous at $x = \frac{1}{3}$ (d) None of these

(3) The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is converges for:
(a) p = 1 (b) $p = \frac{1}{2}$ (c) p > 1 (d) None of these

(4) $\Gamma(\frac{1}{2})$ equals to:
(a) π (b) $\sqrt{\pi}$ (c) $\frac{1}{2}$ (d) None of these

(5) If f is homogeneous of degree n, $x f_x(x,y) + y f_y(x,y) = n f(x,y)$ is called:
(a) Lagrange's formula (b) Euler's formula
(c) Goursat's formula (d) None of these

(6) Every function X → Y between metric spaces is continuous if:
(a) X is discrete (b) Y is complete
(c) X is complete (d) None of these

(7) If each f_n is continuous and $f_n \to f$ Uniformly on E, then:
(a) f is differentiable on E (b) f is continuous on E
(c) f is discontinuous on E (d) None of these

(8) Every real –valued continuous function on open interval (0,1) is:
(a) bounded (b) Unbounded (c) monotonic (d) None of ...

(9) When n is large, $n! = \sqrt{2\pi n}\ n^n\ e^{-n}$ is called:
(a) Hermite's formula (b) Stirling's formula
(c) Euler's formula (d) None of these

(10) $\Gamma(x)\ \Gamma(1-x) = \dfrac{\pi}{\sin n\pi}$, for:
(a) $0 < x < 1$ (b) $x = 1,2,3,4$.............
(c) $x = \frac{1}{2}$ only (d) None of these

(11) If $\sum_{n=1}^{\infty} A_n$ converges absolutely to A, then any rearrangement of the series:
(a) diverges (b) Converges but not necessary to A
(c) Converges absolutely to A (d) None of these

(12) Every Riemann integrable function is:
(a) Continuous (b) differentiable
(c) monotonic (d) None of these

(13) Every compact metric space is:
(a) discrete (b) complete
(c) Infinite (d) None of these

(14) The set of all points z satisfying $|(z-1)| + |z+1| = 4$ lies on:
(a) a circle (b) a parabola
(c) an ellipse (d) None of these

(15) Let $\sum_{n=1}^{\infty} Z_n$ be a series of Complex numbers:
(a) if $\lim_{n\to\infty} Z_n = 0$ then series converges to zero
(b) if the series converges, then $\lim_{n\to\infty} Z_n = 0$
(c) if the series converges, it converges absolutely
(d) None of these

(16) $(-i)^i$ equal to:
(a) $e^{\pi/2}$ (b) i (c) $\pi/2$ (d) None of these

(17) If C is the circle $|z| = 1$, $\int_c \dfrac{\sin z\,dz}{z^2+4}$ equals to:
(a) 1 (b) 0 (c) $2\pi i$ (d) None of these

(18) Log (-1-i) equal to:
(a) $1/2 \log z - i\frac{3\pi}{4}$ (b) $1/2 \log z + i\frac{3\pi}{4}$
(c) $-1/2\ \log z - i\frac{3\pi}{4}$ (d) None of these

(19) $f(z) = y + i x$ is:
(a) Analytic inside the circle $|z| = 1$ (b) Not analytic in any domain
(c) Is analytic everywhere. (d) None of these

(20) Every meromorphic function of Z is:
(a) monogenic (b) holomorphic
(c) has only poles as singularities (d) None of these

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