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FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN PBS-17, UNDER THE FEDERAL GOVERNMENT, 2003

PURE MATHEMATICS, PAPER-I

TIME ALLOWED: THREE HOURS **MAXIMUM MARKS: 100**

NOTE: Attempt **FIVE** questions in all, including question **NUMBER- 8** which is **COMPULSORY**. Select at least **TWO** questions from each of the **SECTIONS I** and **II**. All question carry **EQUAL MARKS**.

SECTION -I

1. (a) Let H, K be subgroup of a group G and $HK=\{hK \mid h\in H,\ k\in K\}$. Show that HK is a subgroup of G if and only if HK=KH. **(08)**

   (b) Let H be a normal subgroup and K a subgroup of group G. Show that the factor groups HK/H and $K/H\cap K$ exist and are isomorphic to each other. Also give the famous name of this result. **(12)**

2. (a) Give definition of normalizer of a set in group G. Prove that the index of normalizer of an element g in G is equal to the number of elements in conjugacy class C_g of g in G. **(10)**

   (b) State the famous Pigeonhole principle. Use this principle to justify the claim "every integral domain is a field". **(10)**

3. (a) What is meant by a basis of vector space V over field F. If $x_1, \ldots, x_m$ are m linearly independent vectors in n – dimensional vector space V over field F then show that $n \geq m$. **(08)**

   (b) Give definition of finite extension of a field. If L is a finite extension of field K and K is a finite extension of field F, Then show that L is a finite extension of F. **(12)**

4. (a) Let S and T be linear transformations of finite – dimensional vector space V into itself. Define the rank r(s) of s. Then show that $r(TS)\leq \min\{r(s), r(T)\}$ and that r(ST)=r(TS)=r(T) whenever S is invertible. **(10)**

   (b) Let V be an n-dimensional vector space over field F. Let T be a linear transformation from V into itself having all its characteristic roots in F. Show that T satisfies a polynomial of degree n over F. **(10)**

SECTION –II

5. (a) How would you differentiate between hyperbola and parabola? Prove that the lines $y = \frac{b}{a}x$ and $y = \frac{-b}{a}x$ are asymptotes of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. **(10)**

   (b) Give significance of the pedal equation of a plane curve. Show that $p^2 = ar$ is the Pedal equation of the parabola $y^2=2a(x+a)$. **(10)**

6. (a) Express the equation $P = 7 \sin\theta \sin\phi$ in cylindrical and rectangular coordinates. **(10)**

   (b) What kind of surfaces in $\mathbb{R}^3$ are called ellipsoids? Identify the standard name of the surface $x^2 + 9y^2 - 4z^2 - 6x + 18y + 16z + 20 = 0$. **(02+08)**

7. (a) What is the osculating plane of a curve at point P: Show that the osculating planes at any three points of the cubic curve $\bar{r} = (u, u^2, u^3)$ meet at a point lying in the plane determined by the three points. **(10)**

(b) Find the curvature and torsion of the curve of intersection of the following two quadric surfaces: $a_1x^2+b_1y^2+c_1z^2 = 1$, $a_2x^2+b_2y^2+c_2z^2=1$. **(10)**

COMPULSORY QUESTION

8. Write only the correct choice in the Answer Book. Do not reproduce the question.

(1) The number of identity elements in a group is:
(a) 0 (b) 1
(c) 2 (d) None of these.

(2) A field must contain at least:
(a) one element (b) Two elements
(c) Three elements (d) None of these.

(3) A basis of Vector space contains:
(a) only the zero vector (b) no zero vectors
(c) zero as well as non-zero vectors (d) None of these.

(4) Every vector space is:
(a) a group (b) a ring
(c) a field (d) None of these.

(5) Matrix A is nilpotent iff:
(a) $A^n \neq 0, \forall n$ (b) $A^n = 0$ for some n
(c) $A^n = 0, \forall n$ (d) None of these.

(6) A unit matrix of order n has the rank:
(a) 0 (b) 1
(c) n (d) None of these.

(7) The matrix equation AX = B has unique solution if:
(a) 0 (b) A is singular
(c) A is not invertible (d) None of these.

(8) The determinant of a triangular matrix is the product of its entries on:
(a) first row (b) second row
(c) main diagonal (d) None of these.

(9) In any conic, the harmonic mean between the segments of focal chord is:
(a) the geometric mean (b) zero
(c) semi-latus-rectum (d) None of these.

(10) $a = r\cos\theta$ is an asymptot of the curve:
(a) $r = a\cos\theta$ (b) $r = a\sin\theta$
(c) $r = a\tan\theta$ (d) None of these.

(11) The radius of curvature of $y = \sqrt{r^2 - x^2}$ for $x \in [-r, r]$ is:
(a) $\frac{1}{r}$ (b) r
(c) 2r (d) None of these.

(12) The distance from the origin to the plane $x + 2y - z - 4 = 0$ is:
(a) $\frac{1}{\sqrt{6}}$ (b) $\frac{\sqrt{6}}{4}$

(c) $\frac{4}{\sqrt{6}}$ (d) None of these.

(13) The rectangular coordinates of the point with spherical coordinates $(5, .5\pi, .5\pi)$ are:
(a) (5,0,0) (b) (0,5,0)
(c) (0,0,5) (d) None of these.

(14) $a^{-2} x^2 + b^{-2} y^2 - c^{-2} z^2 = -1$ is hyperboloid of:
(a) 1 sheet (b) 2 sheets
(c) 3 sheets (d) None of these.

(15) The principal normal at point P on a curve is the intersection of normal plane at P and:
(a) the curve (b) tangent plane
(c) osculating plane (d) None of these.

(16) A curve is not a straight line iff its curvature is:
(a) zero (b) non-zero
(c) one (d) None of these.

(17) The relations $t' = kn$, $n' = \tau b$, $b' = -\tau n$ are known as
(a) Gauss-Bonnet equations (b) serret – Frenet formulae
(c) Tissot equations (d) None of these.

(18) A set of n+1 vectors in n-dimensional vector space:
(a) must be linearly independent (b) must be linearly dependent
(c) must be a basis (d) None of these.

(19) Which of the following terms is not used in algebra?
(a) homomorphism (b) homeomorphism
(c) epimorphism (d) None of these.

(20) No group of order 28 can have subgroup of order:
(a) 7 (b) 11
(c) 14 (d) None of these.

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PURE MATHEMATICS, PAPER-II

TIME ALLOWED: THREE HOURS **MAXIMUM MARKS: 100**

NOTE: Attempt FIVE questions in all, including question NUMBER- 8 which is COMPULSORY. Select at least TWO questions from each of the SECTIONS I and II. All questions carry EQUAL MARKS.

SECTION -I

1. (a) For every positive integer n, show that $\lim_{x\to 0} \frac{\sin nx}{nx} = 1$. (05)

(b) Discuss the continuity of function f given by

$$f(x) = \begin{cases} x \text{ if } x \text{ is irrational number} \\ 1 - x \text{ if } x \text{ is rational number, at } x = \frac{2}{3} \end{cases}$$ (05)

(c) Show that any real function $f(x)$ which is differentiable at point x_0 must be continuous at x_0. Further show that the converse generally is not true. (10)

2. (a) Find $\frac{dy}{dx}$ of $(\tan x)^y + y^{\cot x} = b$. (06)

(b) Find the volume of the solid region bounded above by the sphere $x^2 + y^2 + z^2 = 4$ and below by the upper nappe of the cone $z^2 = x^2 + y^2$. (06)

(c) Show that radius of the base of an open cylinder of given surface s and greatest volume V is equal to its height. (08)

3. (a) Let A be any set in a metric space X and $x \in X$. Show that x is a closure point of A iff every open sphere about x intersects A. (10)

(b) Let f be a function from metric space X into a metric space Y and $x_0 \in X$. Prove that f is continuous at x_0 iff $\lim_{n\to\infty} f(x_n) = f(x_0)$ wherever (x_n) is a sequence in X converging to x_0. (10)

4. (a) Examine the series $\sum_{m=1}^{\infty} \frac{\arctan m}{1+m^2}$ for convergence. (09)

(b) Let $f(x)$ be Riemann integrable function on [a,b] and let there be a differentiable function F on [a,b] such that $F' = f$. Show that $\int_a^b f(x)dx = F(b) - F(a)$. Also give the famous name of this result. (11)

SECTION -II

5. (a) Prove that every complex number has n nth roots, for all positive integer n. (08)

(b) Deduce the famous Cauchy – Riemann conditions as a necessity for analytic functions. Show also that these conditions are not sufficient to guarantee the analyticity. (12)

6. (a) Give the standard construction of arc tan z and then discuss its analyticity in detail. (08)

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(9) Every Riemann integrable function is:
(a) differentiable (b) analytic
(c) Riemann-steltye's integrable (d) None of these.

(10) Every subset of a finite metric space is closed because:
(a) there exists no closed set
(b) you can not find any limit point of such sets.
(c) such set have no limit points (d) None of these.

(11) Interior of a set A is:
(a) smallest closed superset of A (b) proper open subset of A
(c) largest open subset of A (d) None of these.

(12) Every set is a metric space w.r.t the metric known as:
(a) indiscrete metric (b) discrete metric
(c) normable metric (d) None of these.

(13) A metric space:
(a) is always complete (b) can never be complete
(c) may be complete (d) None of these.

(14) The function $f: X \rightarrow IR$ is continuous if the metric space X is:
(a) complete (b) discrete
(c) incomplete (d) None of these.

(15) $e^{i\theta} = \cos\theta + i\sin\theta$ is called:
(a) Cauchy formula (b) Gauss formula
(c) Euler formula (d) None of these.

(16) $ln(z+\sqrt{z^2+1})$ is equal to:
(a) $\sin^{-1}z$ (b) $\cos h^{-1}z$
(c) $\sin hz$ (d) None of these.

(17) The converse of the cauchy's integral theorem is also known as:
(a) Jordan Theorm (b) Goursat Theorem
(c) Morera's Theorem (d) None of these.

(18) $1-z+\frac{z^2}{2!}-\frac{z^3}{3!}+\frac{z^4}{4!}-\ldots$ converges to:
(a) e^z (b) e^{-z}
(c) $-ze^z$ (d) None of these.

(19) $\Gamma(z+1)$ equals:
(a) $\Gamma(z)$ (b) $z^{-1}\Gamma(z)$
(c) $z\Gamma(z)$ (d) None of these.

(20) For Beta function B(m,n) is equal to:
(a) $\frac{\Gamma(m+n)}{\Gamma(m)\Gamma(n)}$ (b) $\frac{\Gamma(n)\Gamma(m-n)}{\Gamma(m)}$
(c) $\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ (d) None of these.

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