FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN PBS-17, UNDER THE FEDERAL GOVERNMENT, 2002

PURE MATHEMATICS, PAPER-I

TIME ALLOWED: THREE HOURS **MAXIMUM MARKS: 100**

NOTE: Attempt **FIVE** questions in all, including **QUESTION NO. 8** which is **COMPULSORY**. Select at least **TWO** questions from each of the **SECTIONS I and II**. All questions carry **EQUAL** marks.

Q.No.		Question	Marks
		SECTION – I	
1	(a)	Let G be a finite group and H be its Subgroup. Then prove that the order of H divides the order of G.	10
	(b	State and prove Fundamental theorem of Homomorphism in groups.	10
2	(a)	Define (i) Commutator Sub group G' of a group G. (ii) Subrings. (iii) Integral Domain.	9
	(b	Show that the correspondence $a + ib \rightarrow \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$, $a, b \in R^*$ is an isomorphism of the field C of complex numbers into the ring of 2×2 matrices over R*.	11
3	(a)	Let V be a vector space over F and W a non-empty subset of V. Prove that W is a subspace of V iff it is closed under the operation of addition and scalar multiplication.	08
	(b	Show that the yz plane in R^3 is spanned by (0,1,2), (0,2,3) and (0,3,1).	06
	(c)	Let V=R[x] be the vector space of all polynomials in x over R. Show that the mapping: $I : V \rightarrow R$ defined by $(v)I = \int_0^1 v dx$, is linear.	06
4	(a)	If A is an idempotent matrix then prove that (i) $B = I - A$ is an Idempotent matrix, (ii) $AB = BA = 0$.	06
	(b	Find the eigen values and eigen vectors of $A = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$	06
	(c)	Investigate for what values of a, b the simultaneous equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + az = b$, have: (i) No solution (ii) A unique solution (iii) an infinite number of solutions.	08
		SECTION – II	
5	(a)	Find the length of one arc of the cycloid $x = b(\theta - \sin\theta)$, $y = b(1 - \cos\theta)$.	07
	(b)	Find the pedal equation of $r^m = a^m \cos m\theta$.	07

	(c)	Show that the equation of tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the point (x_1, y_1) is $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$.	06
6	(a)	Find the equation of the plane through the points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3).	06
	(b	Find the Cartesian and spherical polar coordinates of the point P with cylindrical coordinates (4, arc Cos $\frac{4}{5}$, 3).	06
	(c)	Find the equation of the sphere having the straight line joining the points (2,3,4) and (-2,-3,-4) as a diameter.	08
7	(a)	Define the curvature, the unit principal normal vector and the unit binormal vector of a curve C.	06
	(b	Find the torsion of the curve C: r(t) = [a cos t, a sin t, ct].	07
	(c)	Prove the Serre-Frenet's formula $b' = -\tau \wedge \rho$.	07

COMPULSORY QUESTION

8. Write only the correct choice in the Answer Book. Don't reproduce the statement.

1	0,0,1 are the direction Cosines of:			
	(a)	x-axis	(b)	y-axis
	(c)	z-axis	(d)	None of these.
2	AB=AC $\Rightarrow$ B=C when			
	(a)	A is Non Singular	(b)	$\|A\| = 0$
	(c)	A^{-1} exists	(d)	None of these.
3	The angle between the planes x –y – 2z + 3 = 0 and 2x + y – z = 5 is			
	(a)	0	(b)	$\frac{\pi}{2}$ radians
	(c)	$\frac{\pi}{3}$ radians	(d)	None of these.
4	If AB = BA, when A and B are square matrices, the multiplication is said to be:			
	(a)	Associative	(b)	Reflexive
	(c)	Commutative	(d)	None of these.
5	The perpendicular distance of the point (3,-1,2) from the plane 2x + y – z = 4 is:			
	(a)	2	(b)	4
	(c)	$\frac{1}{\sqrt{6}}$	(d)	None of these.

PURE MATHEMATICS, PAPER-I

6	$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1$ represents:			
	(a)	Sphere	(b)	Ellipsoid
	(c)	Hyperboloid of one sheet	(d)	Hyperboloid of two sheets.
	(e)	None of these.		
7	The radius of the sphere $x^2+y^2+z^2-4x+2y-6z=1$ is:			
	(a)	1	(b)	5
	(c)	10	(d)	None of these.
8	The equation of surface of revolution obtained by revolving the curve $x=z^2, y=0$ about the x-axis is:			
	(a)	$x^2+y^2=z^4$	(b)	$x=y^2+z^2$
	(c)	$x^2=z^4$	(d)	None of these.
9	$ax^2+by^2+2hxy+2gx+2fy+c=0$ represents a parabola when:			
	(a)	$h^2<ab$	(b)	$h^2>ab$
	(c)	$h^2=ab$	(d)	None of these.
10	$\underline{u}$, $\underline{p}$, $\underline{b}$ constitute a triple of orthogonal unit vectors which is:			
	(a)	Right Handed	(b)	Left Handed
	(c)	Orthonormal	(d)	None of these.
11	An equivalence relation satisfies the following three properties:			
	(a)	Reflexive, symmetric, transitive	(b)	Reflexive, Anti symmetric, transitive
	(c)	Not Reflexive, symmetric, transitive	(d)	Reflexive, symmetric, Not transitive
	(e)	None of these.		
12	If M and N are any two $n \times n$ square matrices, then det (MN) equals:			
	(a)	det M + let N	(b)	det M det N
	(c)	Matrix MN	(d)	None of these.
13	If A is a square matrix, then:			
	(a)	det 3A = det A	(b)	det 3A = 3 det A
	(c)	det $A^t \neq$ det A	(d)	det A = A
	(e)	None of these.		

14	Which of the following mapping is a Linear Transformation?			
	(a)	$T(a_1,a_2,a_3) = (a_1,a_2)$	(b)	$T(a,b,c) = (a,1)$
	(c)	$T(x,y,z) = (x+1,y,z)$	(d)	$T(x,y) = (x+1,y+1)$
	(e)	None of these.		
15	$(R,+,.)$: where R is the set of all real numbers, is a			
	(a)	Field	(b)	Commutative Ring
	(c)	Ring with Identity	(d)	Division Ring
	(e)	None of these.		
16	Which of the following are subspaces of R^2?			
	(a)	$\{(a,a): a \in R\}$	(b)	$\{(a,a^2): a \in R\}$
	(c)	$\{(a,a+1): a \in R\}$	(d)	$\{(a^2,a): a \in R\}$
	(e)	None of these.		
17	Matrix A is called Involuntory if:			
	(a)	$A^2 = A$	(b)	$A^2 = I$
	(c)	$A^{K+1} = A$	(d)	$A' = A$
	(e)	None of these.		
18	Which of the following statements for groups is wrong?			
	(a)	$(g^{-1})^{-1} = g$, for every g in G.	(b)	The inverse of the identity element e is e itself in G.
	(c)	A group contains at least the identity element.	(d)	There is a concept of an empty group.
	(e)	None of these.		
19	Given $\psi: G \to G'$, from G into G', is a group homomorphism. Then ψ is called epimorphism if:			
	(a)	$G' = G$	(b)	ψ is 1 -1
	(c)	ψ is onto G'	(d)	ψ is 1 – 1 and onto G', both
	(e)	None of these.		
20	A cyclic group of order n is generated by:			
	(a)	n elements	(b)	two elements
	(c)	One element	(d)	n – 1 elements.
	(e)	None of these.		

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FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN PBS-17, UNDER THE FEDERAL GOVERNMENT, 2002

PURE MATHEMATICS, PAPER-II

TIME ALLOWED: THREE HOURS **MAXIMUM MARKS: 100**

NOTE: Attempt **FIVE** questions in all, including **QUESTION NO. 8** which is **COMPULSORY**. Select at least **TWO** questions from each of the **SECTIONS I and II**. All questions carry **EQUAL** marks.

Q.No.		Question	Marks
		SECTION – I	
1	(a)	Prove that an open sphere in a metric space X is an open set.	07
	(b)	The intersection of any two open sets and hence of any number of open sets in X is open. (Prove for topological space (X, τ)).	07
	(c)	Define: (i) Interior point of A (ii) Exterior point of A. (iii) Boundary point of A. (iv) Closure of A; where A is a subset of a subset of a topological space X.	06
2	(a)	Let $X = \{x, y, z\}$, $\tau = \{\Phi, X, \{x\}, \{y,z\}\}$. Define $g: X \to X$ by $g(x) = y, g(y) = z, g(z) = x$. Verify whether g is continuous or not.	06
	(b)	Prove that $\beta(m,n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$	08
	(c)	Show that $\beta(m,n) = \beta(n,m)$; also evaluate $\Gamma(\frac{5}{2})$.	06
3	(a)	Evaluate $\lim\limits_{x \to \frac{\pi}{2}} \dfrac{\tan 3x}{\tan x}$.	06
	(b)	Find the volume of the tetrahedron bounded by the coordinate planes and the plane $\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{z}{c} = 1$, where a, b, c are positive constants.	08
	(c)	Calculate $\int_1^4 \dfrac{dx}{(x-2)^{\frac{2}{3}}}$.	06
4	(a)	Test the convergence of the series $\sum\limits_{n=1}^{\infty} \dfrac{n}{(n+1)(n+2)}$.	06
	(b)	Prove that the least perimeter of an isosceles triangle in which a circle of radius r can be inscribed is $6r\sqrt{3}$.	06
	(c)	State and prove Fundamental theorem of Integral calculus.	08
		SECTION – II	
5	(a)	Expand $\cos^5\theta \sin^3\theta$ in series of Sines of multiples of θ.	08
	(b)	Find the 6 Sixth roots of -1.	06
	(c)	Prove that $\cos h^{-1} z = \text{Log}(z + \sqrt{z^2 - 1})$.	06
6	(a)	Expand $f(x) = \sin x$ in a Fourier cosine series in the interval $0 \le x \le \pi$.	07

	(b)	Verify that $u = x^2 - y^2 - y$ is harmonic in C and find a conjugate harmonic function v of u.	07
	(c)	Evaluate $\oint_c \frac{dz}{z-i}$, c is the circle $\|z\| = 2$ (counter clockwise).	06
7	(a)	Find the center and radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{(z-2i)^n}{5^n}$.	06
	(b)	Define the following terms: (i) Pole (ii) Isolated essential singularity (iii) Zero of an analytic function (iv) Residue.	06
	(c)	Evaluate $\oint_c \frac{z}{z^2 - \frac{1}{4}} dz$, where c is the unit circle (counter clock wise).	08

COMPULSORY QUESTION

8. Write only the correct choice in the Answer Book. Don't reproduce the statement.

1	The function $f(x) = \frac{x^2 - 9}{x - 3}$ is discontinuous at:			
	(a)	$x = 0$	(b)	$x = 3$
	(c)	$x = 1$	(d)	None of these.
2	f(x) = Sin x has a minimum value at:			
	(a)	$x = 0$	(b)	$x = \frac{\pi}{2}$
	(c)	$x = \frac{3\pi}{2}$	(d)	None of these.
3	$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$ is:			
	(a)	0	(b)	1
	(c)	e	(d)	-e
4	Derivative of the function f(x) = ln x at x = 0 is:			
	(a)	1	(b)	0
	(c)	∞	(d)	None of these.
5	For a decreasing function g, let $x_1 < x_2$; then:			
	(a)	$g(x_1) > g(x_2)$	(b)	$g(x_1) < g(x_2)$
	(c)	$g(x_1) = g(x_2)$	(d)	None of these.

PURE MATHEMATICS, PAPER-II

6	Tangent to the parabola $y^2 = 5x$ at (5,5) is:			
	(a)	$y = x + 5$	(b)	$y = x - 5$
	(c)	$y = x$	(d)	None of these.
7	$(2i)(-3 - i)$ is equal to:			
	(a)	(2, -6)	(b)	(-2, 6)
	(c)	(2, 6)	(d)	(-2, -6)
8	Which of the following statements is not correct?			
	(a)	e^z is never zero	(b)	$5z > z$
	(c)	$e^z = 1$ iff z is an integral multiple of $2\pi i$	(d)	$\arg(z_1 z_2) = \arg z_1 + \arg z_2$.
9	$\int_0^{\frac{\pi}{4}} \sec^2 x \, dx$ is equal to:			
	(a)	1	(b)	Zero
	(c)	∞	(d)	None of these.
10	$\Gamma(\frac{1}{2})$ is equal to:			
	(a)	π	(b)	$\sqrt{\pi}$
	(c)	$\frac{1}{2}$	(d)	Zero.
11	The Jocobian of the rotation $x = u\cos\alpha - v\sin\alpha$, $y = u\sin\alpha + v\cos\alpha$ is:			
	(a)	Uv	(b)	α
	(c)	1	(d)	None of these.
12	$\int_0^1 \int_1^2 \int_2^3 dx\, dy\, dz$ is equal to:			
	(a)	1	(b)	2
	(c)	3	(d)	None of these.
13	$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$ is the expansion of:			
	(a)	$\cos x$	(b)	e^x
	(c)	$\frac{1}{1-x}$	(d)	None of these.

14	Which of the following statements is not correct?			
	(a)	An absolutely convergent series is convergent.	(b)	$\sum_{1}^{\infty}\frac{1}{n}$ is convergent
	(c)	$\sum_{1}^{\infty} n$ is divergent	(d)	$\left\{1+(-1)^{n+1} n\right\}$ oscillates infinitely.
15	The period of cos x sin x is:			
	(a)	$\frac{\pi}{2}$	(b)	2π
	(c)	π	(d)	Arbitrary.
16	Let the metric space be R and let $x_0 = 1$ and $r = \frac{1}{2}$. Then $S_{\frac{1}{2}}(1)$ is given by:			
	(a)	$]\frac{1}{2}, 1[$	(b)	$]0, \frac{3}{2}[$
	(c)	$]\frac{1}{2}, \frac{3}{2}[$	(d)	None of these.
17	Which of the following statements is not correct?			
	(a)	If gof is injective, then f is injective.	(b)	If gof is surjective, then g is surjective.
	(c)	If gof is surjective, and g is injective, then f is surjective.	(d)	If gof is injective, and f is surjective, then g is surjective.
	Note: $f: A \to B$ and $g: B \to C$ are functions.			
18	Select the correct statement;			
	(a)	Int (Int A) $\neq$ Int (A)	(b)	Int $(A \cup B)$=Int(A) $\cup$ Int (B)
	(c)	Int $(A \cap B)$=Int(A) $\cap$ Int(B)	(d)	Ext$(A \cup B) \neq$ Ext(A) $\cap$ Ext(B)
	where A and B are any two subsets of a topological space.			
19	$\int_c \cot z\,dz$ is equal to:			
	(a)	$2\pi i$	(b)	πi
	(c)	Zero	(d)	None of these.
	where c is the unit circle (Counter clockwise).			
20	The image of the region $1.5 \le \|z\| < 2.1$ under the mapping $w = z^2$ is:			
	(a)	$2.25 \le \|w\| < 4.41$	(b)	$1.5 \le \|w\| < 4.41$
	(c)	$2.25 \le \|w\| < 2.1$	(d)	None of these.

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