FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS
IN BPS-17 UNDER THE FEDERAL GOVERNMENT, 2001.

PURE MATHEMATICS
PAPER-I

TIME ALLOWED: THREE HOURS MAXIMUM MARKS: 100

NOTE: Attempt **FIVE** questions in all, including question No.8 which is **COMPULSORY**. At least select **TWO** questions from each section. All questions carry **EQUAL** marks.

SECTION-I

1\. (a) Prove that any group G can be embedded in a group of bijective mappings of a certain set. **(10)**

(b) Prove that the number of elements in a conjugacy class Ca of an element "a" in a group G is equal to the index of its normalizer. **(10)**

2\. (a) Let G be a group, prove that: **(12)**

(i) The derived subgroup G' is normal subgroup of G.

(ii) G/G' is abelian.

(iii) If K is a normal subgroup of G such that G/K is abelian then $k \supseteq G'$.

(b) Prove that a finite dimensional integral domain is a field. **(08)**

3\. (a) Prove that in a commutative ring with identity an ideal M of R is maximal ideal if and only if R/M is a field. **(07)**

(b) Find rank and nullity of $T: R^3 \to R^3$ defined by $T(x_1, x_2, x_3) = (x_1 - x_2, x_1 + x_3, x_2 + x_3)$ **(07)**

(c) Let V be a vector space of polynomials of degree ≤ 3, determin whether the vectors $x^3 - 3x^2 + 5x + 1$, $x^3 - x^2 + 8x + 2$ and $2x^3 - 4x^2 + 9x + 5$ of V are linearly independent. **(06)**

4\. (a) Find value of λ for which the following homogeneous system of linear equations has non-trivial solution. Find the solution **(07)**

$$(1-\lambda)x_1 + x_2 + x_3 = 0$$
$$x_1 - \lambda x_2 + x_3 = 0$$
$$x_1 - x_2 + (1-\lambda)x_3 = 0$$

(b) Find eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 2 & 3 \\ 4 & 3 \end{bmatrix}$. **(06)**

(c) Solve the following system of equations by reducing to reduced echlon form: **(07)**

$$2x_1 - x_2 + 3x_3 = 3$$
$$3x_1 + x_2 - 5x_3 = 0$$
$$4x_1 - x_2 + x_3 = 3$$

SECTION-II

5\. (a) Find equation of a sphere passing through the points (0,-2,-4), (2,-1,-1) and having the centre on the straight line 2x-3y=0=5y+2z **(08)**

(b) (i) Discuss the following surface and sketch it $9x^2 - 4y = 9z^2$ **(06)**

(ii) Find cylindrical and spherical polar coordinates of the point P with rectangular coordinates $(2\sqrt{3}, 2, -2)$. **(06)**

6. (a) Show that the lines:
L: $x=3+2t,\ y=2+t,\ z=-2-3t$
M: $x=-3+4s,\ y=5-4s,\ z=6-5s$
Intersect. Find an equation of the plane containing these lines.

(b) Show that the perpendicular distance D of a point $P(x_1, y_1, z_1)$ from the plane ax+by+cz+d=0 is given by $D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$ and hence find distance between the parallel planes 2x+2y-4z+3=0 and 3x+3y-6z+1=0. **(10)**

7. (a) Find length of one arch of the cycloid $x = a(\theta - \text{Sin}\theta), y = a(1 - \text{Cos}\theta)$. **(10)**

(b) Show that for the parabola $y = ax^2 + bx + c$, the curvature is minimum at its vertex. **(10)**

COMPULSORY QUESTION

8. Write only the correct answer in the answer book. Do not reproduce the questions.

(1) The set {i, -i, 1, -1} is:
(a) Semi group under addition (b) Group under addition
(c) Group under multiplication (d) None of these.

(2) Number of subgroups of order one of an infinite group G is:
(a) Zero (b) 1 (c) 2 (d) infinite (e) None of these.

(3) A cycli group of order n is generated by:
(a) n elements (b) (n-1)elements
(c) two elements (d) one element
(e) None of these.

(4) Let H be a subgroup of order m of a group of order n, the number of right cosets of H in G is:
(a) n (b) n - m (c) m – n
(d) $\frac{n}{m}$ (e) None of these.

(5) The dimension of a vector space V is the number of:
(a) Linearly independent vectors in V.
(b) Linearly dependent vectors in V.
(c) Linearly independent vectors spanning V.
(d) None of these.

(6) The characteristic of an integral domain is:
(a) zero (b) a prime (c) zero or a prime (d) None of these.

(7) The eigenvalue is related to the corresponding eigenvector (for a matrix A) as:
(a) $|A - \lambda I \underline{x}| = 0$ (b) $|A - \lambda I|\ \underline{x} = b$
(c) $A\underline{x} = \lambda\ \underline{x}$ (d) None of these.

(8) For two vectors $\vec{A}$ and $\vec{B}$, $\vec{A} . \vec{B}$ gives:
(a) Cos of angle between $\vec{A}$ and $\vec{B}$
(b) Area of parallelogram with $\vec{A}$ and $\vec{B}$ as its adjacent sides.
(c) Vector perpendicular to $\vec{A}$ and $\vec{B}$
(d) Vector parallel to the plane of $\vec{A}$ and $\vec{B}$
(e) None of these.

(9) If θ is angle between two vectors $\vec{A}$ and $\vec{B}$, then $\frac{\vec{A} \times \vec{B}}{|\vec{A}||\vec{B}|}$ gives:
(a) tan θ (b) cos θ
(c) sin θ (d) sec θ
(e) None of these.

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PURE MATHEMATICS
PAPER-II

TIME ALLOWED: THREE HOURS — MAXIMUM MARKS: 100

NOTE: Attempt **FIVE** questions in all, including question **No.8** which is **COMPULSORY**. At least select **TWO** questions from each section. All questions carry **EQUAL** marks.

SECTION-I

1. (a) (i) Find $\lim_{x\to a}\frac{x^p - a^p}{x-a}$. **(5 + 5)**

(ii) Find a and b such that $f(x)=\begin{cases} x^3, x<-1 \\ ax+b, -1\le x<1 \\ x^2+2, x>1\end{cases}$ is continuous for all x.

(b) (i) Find $\frac{dy}{dx}$ when Sin (ln xy) = $x + y^2$ **(5 + 5)**

(ii) Use Taylor's theorem to prove that ln sin (x+h) = ln sin x + h cot x - $\frac{1}{2}h^2\,\text{cosec}^2 x + \frac{1}{3}h^3 \cot x\,\text{cosec}^2 x$ + ---

2. (a) Evaluate $\int\frac{dx}{(1-2x^2)\sqrt{1-x^2}}$ **(08)**

(b) Evaluate $\int e^{3x}\sin 4x\,dx$ **(06)**

(c) Test the convergence or divergence of the series:
$\frac{2}{5}+\frac{2.4}{5.8}+\frac{2.4.6}{5.8.11}+\frac{2.4.6.8}{5.8.11.14}+\ldots$ **(06)**

3. (a) Find the asymptotes of the curve $x^2y^2(x^2-y^2)^2=(x^2+y^2)^2$. **(10)**

(b) Find maxima and minima of the radius vector of the curve:
$\frac{c^2}{r^2}=\frac{a^2}{\text{Sin}^2\theta}+\frac{b^2}{\text{Cos}^2\theta}$. **(10)**

4. (a) Trace the folium of Descartes $x^3+y^3=3axy$. **(08)**

(b) Define an open sphere in a metric space(x,d). Let (x,d_0) be the discrete metric space, write open balls centered at $x\in X$ with radius $1/2$ and $3/2$. **(06)**

(c) Let X = C [a,b] be the set of all real valued continuous defined on [a,b]. Define a function $d: X\times X\to R$ as follows: **(06)**
For f, g ∈ X, $d(f,g)=\int_a^b |f(x)-g(x)|\,dx$. Prove that (x,d) is a metric space.

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SECTION-II

5. (a) Separate into real and imaginary parts $\tan^{-1}(x+iy)$. **(07)**

   (b) Show that $\log(1+\cos\theta + i\sin\theta) = \ln(2\cos\frac{\theta}{2}) + i\frac{\theta}{2}$ **(06)**

   (c) Sum the series: **(07)**

   $1 + c\cos\theta + \frac{c^2}{2!}\cos 2\theta + \frac{c^3}{3!}\cos 3\theta + \ldots..$

6. (a) Define an analytic function. Prove that the necessary and sufficient condition for a function $W=f(z)=U(x,y)+iV(x,y)$ to be analytic is that U_x, V_x, U_y, V_y exist and are continuous such that $U_x = V_y$, $U_y = -V_x$. **(10)**

   (b) Using cauchy's integral formula evaluate $\int_c \frac{dz}{1+z^2}$ where C is part of the parabola $y=4-x^2$ from A(2,0) to B(-2,0). **(10)**

7. (a) Expand $f(z) = \frac{1}{z^2}$ about $z = 2$ using Taylor's series expansion. **(10)**

   (b) Consider the transformation $W = e^{i\frac{\pi}{4}}.Z$ and determine the region in w-plane corresponding to the triangular region bounded by the lines $x = 0$, $y = 0$ and $x + y = 1$ in the z-plane. **(10)**

COMPULSORY QUESTION

8. Write only the correct answer in the answer book. Do not reproduce the questions.

(1) The function $f(x)=\frac{x^2-a^2}{x-a}$ is discontinuous at:

(a) $x = 1$ (b) $x = a$
(c) $x = 0$ (d) $x = \sqrt{a}$ (e) None of these.

(2) $f(x) = \cos x$ has a maximum value at:

(a) $x = 0$ (b) $x = 1$
(c) $x = \frac{\pi}{2}$ (d) $x = \frac{3\pi}{2}$ (e) None of these.

(3) $\lim_{x\to 0} \frac{\sin x}{x}$ is :

(a) zero (b) 1
(c) undefined (d) -1 (e) None of these.

(4) Derivative of the function $f(x) = \tan x$ at $x = \frac{\pi}{4}$ is:

(a) 2 (b) $\frac{1}{2}$
(c) 1 (d) Zero (e) None of these.

(5) For an increasing function f, let $x_1 < x_2$ then:

(a) $f(x_1) > f(x_2)$ (b) $f(x_1) < f(x_2)$
(c) $f(x_1) = f(x_2)$ (d) None of these..

(6) Area under the curve $f(x) = e^x + 2$ bounded by x=0, x = 2 and x-axis is given by:

(a) 3 (b) e^3+2

(c) e^2+1 (d) e^2+3 (e) None of these..

(7) Normal to the parabola $y^2 = 12x$ at (3,-6) is:

(a) $y = x + 3$ (b) $y = x - 9$

(c) $y + x + 3 = 0$ (d) None of these.

(8) Equation of tangent to the circle $x^2 + y^2 = a^2$ at (x_1, y_1) is given by:

(a) $x_1^2 + y_1^2 + 2gx + 2fy + c = 0$

(b) $x^2 + y^2 + 2gx_1 + 2fy_1 + c = 0$

(c) $xx_1 + yy_1 + 2gx_1 + 2fy_1 + c = 0$

(d) $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$

(e) None of these.

(9) In a complete metric space:

(a) Every sequence is bounded

(b) Every sequence converges

(c) Every cauchy sequence converges

(d) There is no convergent sequence.

(e) None of these.

(10) The open ball of radius 1 and center at zero in R is given by:

(a) (0,1) (b) [0,1]

(c) (-1,1) (d) {0} (e) None of these.

(11) For the two positive term series $\sum_1^\infty a_n$ and $\sum_1^\infty b_n$ if

$a_n \le b_n \forall_n = 1,2....$ if $\sum_1^\infty b_n$ is convergent, then:

(a) $\sum_1^\infty a_n$ diverges (b) $\sum_1^\infty a_n$ converges

(c) $\sum_1^\infty a_n$ converges absolutely (d) None of these.

(12) Polar form of the complex number $z = 3 - 4i$ is:

(a) $5e^{i\theta}$ (b) $5e^{-i\theta}$

(c) $5e^{2i\theta}$ (d) $e^{i\theta}$. (e) None of these.

(13) Log (x + iy) is given by ($|z| = \sqrt{x^2 + y^2}$, $\theta = \tan^{-1}\frac{y}{x}$):

(a) $\log|z| + i\theta$ (b) $\log|z| + i\lambda\theta$

(c) $\log(|z| + i\lambda\theta)$ (d) $\log(|z| + i\theta)$

(e) None of these..

(14) A curve Z = f(t) is smooth if for $t \in [a,b]$:

(a) $f'(t) = 1$ (b) $f'(t) = 0$

(c) $f'(t) \neq 0$ (d) $f(a) = f(b)$

(e) None of these..

(15) On a Simply connected domain D and any closed con... C in D, for an analytic function f(z), $\int_c f(z)dz$ is:

(a) Zero (b) non - zero
(c) 1 (d) $\frac{1}{2}$ (e) None of these.

(16) $\lim_{n\to\infty}(1+\frac{1}{n})^n$ is:

(a) 1 (b) zero
(c) e (d) e^n (e) None of these.

(17) The set of integers together with the operation of multiplication forms:

(a) a semi-group (b) group
(c) Integral domain (d) field (e) None of these.

(18) $\int \tan x dx$ is:

(a) sec x tan x (b) $\sec^2 x$
(c) ln sec x (d) sec x (e) None of these.

(19) $\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}}(2+x)dxdy$ is:

(a) $\frac{\pi}{2}$ (b) 1
(c) 2π (d) Zero (e) None of these.

(20) $\left|\int\frac{dz}{z^2}\right|$ is:

(a) ≤ 2 (b) ≤ 1
(c) 2 (d) 1 (e) None of these.

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