# FEDERAL PUBLIC SERVICE COMMIS 



COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT, 2013

APPLIED MATHEMATICS, PAPER-II

TIME ALLOWED: THREE HOURS
MAXIMUM MARKS: 100
NOTE: (i) Candidate must write Q. No. in the Answer Book in accordance with Q. No. in the Q. Paper.
(ii) Attempt FIVE questions in all by selecting TWO questions from SECTION-A and ONE question from SECTION-B and TWO questions from SECTION-C ALL questions carry EQUAL marks.
(iii) Extra attempt of any question or any part of the attempted question will not be considered.
(iv) Use of Calculator is allowed.

## SECTION-A

Q.No.1. Solve the following equations:
(a) $\frac{d^{3} y}{d x^{3}}+\frac{d y}{d x}=\operatorname{Sec}^{2} x$
(b) $\frac{2 d y}{d x}-\frac{x}{y}+x^{3} \operatorname{Cos} y=0$
Q.No.2. (a) Find the power series solution of the differential equation
$\left(1-x^{2}\right) y^{\prime \prime}-2 x y^{\prime}+2 y=0$, about the point $x=0$.
(b) Solve $Z(x+y) \frac{\partial Z}{\partial x}+Z(x-y) \frac{\partial Z}{\partial y}=\left(x^{2}+y^{2}\right)$.
Q.No.3. (a) Classify the following equations:
(i) $\frac{\partial^{2} Z}{\partial x^{2}}+x^{2} \frac{\partial^{2} Z}{\partial y^{2}}-\frac{1}{x} \frac{\partial Z}{\partial x}=0$
(ii)

$$
\begin{equation*}
x^{2} \frac{\partial^{2} Z}{\partial x^{2}}+2 x y \frac{\partial^{2} Z}{\partial x \partial y}+y^{2} \frac{\partial^{2} Z}{\partial y^{2}}=4 x^{2} \tag{15}
\end{equation*}
$$

(b) Solve: $\frac{\partial u}{\partial t}=\frac{\partial^{2} u}{\partial x^{2}},-1<x<1, t>0$

$$
\begin{aligned}
& u(-1, t)=u(1, t) ; \frac{\partial u}{\partial x}(-1, t)=\frac{\partial u}{\partial x}(1, t) \text { for } t>0 \\
& u(x, o)=x+1,-1<x<1 .
\end{aligned}
$$

## SECTION-B

Q.No.4. (a) Highlight the difference between a vector and a tensor. What happens if we permute the subscripts of a tensor?
(b) Transform $g^{a b}=\left(\begin{array}{cc}1 & 0 \\ 0 & 1 / r^{2}\end{array}\right)$ into Cartesian coordinates.
Q.No.5. (a) Workout the Christoffel symbols for the metric tensor $g_{a b}=\left(\begin{array}{cc}a^{2} & 0 \\ 0 & a^{2} \sin ^{2} \theta\end{array}\right)$
(b) Workout the two dimensional metric tensor for the coordinates $p$ and $q$ given by $p=(x y)^{\frac{1}{3}}, q=\left(x^{2} / y\right)^{1 / 3}$

## SECTION-C

Q.No.6. (a) Solve the following system of equations by Jacobi iteration method:

$$
\begin{aligned}
& 10 x+y-2 z=7.74 \\
& x+12 y+3 z=39.66 \\
& 3 x+4 y+15 z=54.8
\end{aligned}
$$

(b) Solve Sinx $=1+x^{3}$ Using Newton-Raphson method.
Q.No.7. (a) Find the root of $x e^{x}=3$ by regular falsi method correct to three decimal places.
(b) Evaluate $\int_{0}^{10} \frac{d x}{1+x^{2}}$ using
(i) Trapezoidal rule and
(ii) Simpson's rule.
Q.No.8. (a) Find the real root of the equation $\operatorname{Cos} x=3 x-1$ correct to seven decimal places by the iterative method.
(b) Use Lagrange's interpolation formula to find the value of $y$ when $x=10$, if the values of $x$ and $y$ are given below:

| X | 5 | 6 | 9 | 11 |
| :---: | :---: | :---: | :---: | :---: |
| Y | 12 | 13 | 14 | 16 |

