FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT, 2013

Roll Number

APPLIED MATHEMATICS, PAPER-I

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS: 100

NOTE:
- **(i)** Candidate must write **Q.No.** in the **Answer Book** in accordance with **Q.No.** in the **Q.Paper**.
- **(ii)** Attempt **FIVE** questions in all by selecting **THREE** questions from **SECTION-A** and **TWO** questions from **SECTION-B.** All questions carry **EQUAL** marks.
- **(iii)** **Use of Calculator is allowed.**
- **(iv)** Extra attempt of any question or any part of the attempted question will not be considered.

SECTION-A

Q.1. **(a)** Find a function φ such that $\nabla\varphi = \overset{\leftrightarrow}{f}$ **(10)**

$$\overset{\leftrightarrow}{f} = x\hat{i} + 2y\hat{j} + 2\hat{k}$$

(b) Prove that **(10)**

$$\nabla\varphi^n = n\varphi^{n-1}\nabla\varphi$$

Q.2. **(a)** Show that for any vectors $\vec{a}$ and $\vec{b}$ **(10)**

$$\left|\vec{a}+\vec{b}\right|^2 + \left|\vec{a}-\vec{b}\right|^2 = 2\left(\left|\vec{a}\right|^2 + \left|\vec{b}\right|^2\right)$$

(b) Prove that **(10)**

$$\left(\vec{a}\times\vec{b}\right)\bullet\left(\vec{b}\times\vec{c}\right)\times\left(\vec{c}\times\vec{a}\right) = \left(\vec{a}\bullet\vec{b}\times\vec{c}\right)^2$$

Q.3. **(a)** The greatest result that two forces can have is of magnitude P and the least is of magnitude Q. Show That when they act an angle α their resultant is of magnitude **(10)**

$$\sqrt{P^2\cos^2\alpha/2 + Q^2\sin^2\alpha/2}$$

(b) A uniform rod of length $2a$ rests in equilibrium against a smooth vertical wall and upon a smooth peg at a distance b from the wall. Show that in the position of equilibrium the rod is inclined to the wall at an angle $\sin^{-1}\left(\frac{b}{a}\right)^{\frac{1}{3}}$ **(10)**

Q.4. **(a)** Three forces P, Q and R act along the BC, CA and AB respectively of triangle ABC. Prove that if $P\cos A + Q\cos B + R\cos C = 0$, then the line of action of the resultant passes through the circum center of the triangle. **(10)**

(b) A sphere of weight W and radius a is suspended by a string of length l from a point P and a weight w is also suspended from P by a string sufficiently long for the weight to hang below the sphere. Show that the inclination of the first string to the vertical is **(10)**

$$\sin^{-1}\left(\frac{wa}{(W+w)(a+l)}\right)$$

Q.5. **(a)** Find the volume $\iint_R \left(x^3 + 4y\right)\, dA$ where R is the region bounded by the parabola $y = x^2$ and the line $y = 2x$.

(b) Evaluate the following line integral

$$\int_c x^2\, dy$$

bonded by the triangle having the vertices (-1,0) to (2,0), and (1,1)

SECTION-B

Q.6. **(a)** The position of a particle moving along an ellipse is given by $\overleftrightarrow{r} = a\cos t\,\hat{i} + b\sin t\hat{j}$. If $a > b$, find the position of the particle where its velocity has maximum or minimum magnitude. **(10)**

(b) Prove that the speed at any point of a central orbit is given by: **(10)**

$$vp = h,$$

When h is the areal speed and p is the perpendicular distance from the centre of force, of the tangent at the point, Find the expression for v when a particle subject to the inverse square law of force describes an ellipse, a parabolic and hyperbolic orbit.

Q.7. **(a)** A particle is moving with the uniform speed v along the curve **(10)**

$$x^2 y = a\left(x^2 + \frac{a^2}{\sqrt{5}}\right)$$

Show that its acceleration has the maximum value at $\frac{10v^2}{9a}$

(b) An aeroplane is flying with uniform speed v_0 in an arc of a vertical circle of radius a, whose centre is a height h vertically above a point O of the ground. If a bomb is dropped from the aeroplane when at a height Y and strikes the ground at O, show that Y satisfies the equations **(10)**

$$KY^2 + Y\left(a^2 - 2hK\right) + K\left(h^2 - a^2\right) = 0,$$

where $K = h + \frac{ga^2}{2v^2_0}$

Q.8. **(a)** Find the tangential and normal components of the acceleration of a particle describing the ellipse **(10)**

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

With uniform speed v when the particle is a $a > b$

(b) Find the velocity acquired by a block of wood of mass M lb., which is free to recoil when it is struck by a bullet of mass m lb. moving with velocity v in a direction passing through the centre of gravity. If the bullet is embedded a ft., show that the resistance of the wood to the bullet, supposed uniform, is $\frac{Mm^2}{2(M+m)ga}$ lb.wt. and that the time of penetration is $\frac{2a}{v}$ sec., during which time the block will move $\frac{ma}{m+M}$ ft. **(10)**

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