FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BPS-17 UNDER THE FEDERAL GOVERNMENT, 2009

APPLIED MATH, PAPER-II

S.No.	
R.No.	

TIME ALLOWED: 3 HOURS	MAXIMUM MARKS:100

NOTE:	(i) Attempt **FIVE** question in all by selecting at least **TWO** questions from **SECTION–A, ONE** question from **SECTION–B** and **TWO** questions from **SECTION–C**. All questions carry **EQUAL** marks. **(ii) Use of Scientific Calculator is allowed.**

SECTION – A

Q.1. (a) Using method of variation of parameters, find the general solution of the differential equation.

$$y'' - 2y' + y = \frac{e^x}{x} \quad . \qquad \textbf{(10)}$$

(b) Find the recurrence formula for the power series solution around $x=0$ for the differential equation

$$y'' + xy = e^{x+1}. \qquad \textbf{(10)}$$

Q.2. (a) Find the solution of the problem **(10)**

$$u'' + 6u' + 9u = 0$$
$$u(0) = 2, \quad u'(0) = 0$$

(b) Find the integral curve of the equation

$$xz\frac{\partial z}{\partial x} + yz\frac{\partial z}{\partial y} = -(x^2 + y^2)\,. \qquad \textbf{(10)}$$

Q.3. (a) Using method of separation of variables, solve **(10)**

$$\frac{\partial^2 u}{\partial t^2} = 900\frac{\partial^2 u}{\partial x^2} \qquad \begin{cases} 0 < x < 2 \\ t > 0 \end{cases} \quad ,$$

subject to the conditions

$$u(0,t) = u(2,t) = 0$$
$$u(x,0) = 0 \qquad \frac{\partial u}{\partial t}\Big|_{t=0} = 30\sin 4\pi x.$$

(b) Find the solution of **(10)**

$$\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 4e^{3y} + \cos x\,.$$

SECTION – B

Q.4. (a) Define alternating symbol $\in_{ijk}$ and Kronecker delta δ_{ij}. Also prove that **(10)**

$$\in_{ijk}\in_{lmk} = \delta_{il}\,\delta_{jm} - \delta_{im}\,\delta_{jl}\ .$$

(b) Using the tensor notation, prove that

$$\nabla \times (\vec{A} \times \vec{B}) = \vec{A}(\nabla \bullet \vec{B}) - \vec{B}(\nabla \bullet \vec{A}) + (\vec{B} \bullet \nabla)\vec{A} - (\vec{A} \bullet \nabla)\vec{B} \qquad \textbf{(10)}$$

Q.5. (a) Show that the transformation matrix

$$\mathbf{T} = \begin{bmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

is orthogonal and right-handed. **(10)**

(b) Prove that **(10)**

$$l_{ik}\ l_{jk} = \delta_{ij}$$

where l_{ik} is the cosine of the angle between *ith-axis* of the system K' and *jth-axis* of the system *K*.

SECTION – C

Q.6. (a) Use Newton's method to find the solution accurate to within 10^{-4} for the equation **(10)**

$$x^3 - 2x^2 - 5 = 0, \qquad [1, 4].$$

(b) Solve the following system of equations, using Gauss-Siedal iteration method **(10)**

$4x_1 - x_2 + x_3 = 8,$
$2x_1 + 5x_2 + 2x_3 = 3,$
$x_1 + 2x_2 + 4x_3 = 11.$

Q.7. (a) Approximate the following integral, using Simpson's $\frac{1}{3}$ rules **(10)**

$$\int_0^1 x^2 e^{-x}\, dx.$$

(b) Approximate the following integral, using Trapezoidal rule **(10)**

$$\int_0^{\pi/4} e^{3x} \sin 2x\, dx.$$

Q.8. (a) The polynomial **(10)**

$f(x) = 230\,x^4 + 18x^3 + 9x^2 - 221x - 9$

has one real zero in [-1, 0]. Attempt approximate this zero to within 10^{-6}, using the Regula Falsi method.

(b) Using Lagrange interpolation, approximate. **(10)**

f(1.15), if f(1) = 1.684370, f(1.1) = 1.949477, f(1.2) = 2.199796, f(1.3) = 2.439189, f(1.4) = 2.670324

****