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FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BPS–17, UNDER THE FEDERAL GOVERNMENT, 2005

APPLIED MATHEMATICS, PAPER–I

TIME ALLOWED: THREE HOURS — MAXIMUM MARKS: 100

NOTE: Attempt **FIVE** questions in all, selecting **TWO** Questions from each of the Sections A and B. **QUESTION NO.8** is **COMPULSORY**. All questions carry **EQUAL** marks.

SECTION – A

1. (a) Find the volume of the tetrahedron having the vertices $-(\hat{j}+\hat{k}), 4\hat{i}+5\hat{j}+x\hat{k}$, $3\hat{i}+9\hat{j}+4\hat{k}$ and $4(-\hat{i}+\hat{j}+\hat{k})$. Also, find the value of x for which these four points are coplanar. (10)

   (b) (i) Prove that: (05)

   $$\hat{i}\times(\vec{a}\times\hat{i})+\hat{j}\times(\vec{a}\times\hat{j})+\hat{k}\times(\vec{a}\times\hat{k})=2\vec{a}.$$

   (ii) Show that the vector (05)

   $$(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})+(\vec{a}\times\vec{c})\times(\vec{d}\times\vec{b})+(\vec{a}\times\vec{d})\times(\vec{b}\times\vec{c})$$

   is parallel to the vector $\vec{a}$.

2. (a) The temperature at a point (x, y, z) in space is given by $T(x, y, z) = x^2 + y^2 - z$. A mosquito located at (1, 1, 2) desires to fly in such a direction that it will get warm as soon as possible. In what direction should it fly? (08)

   (b) A vector field $\vec{A}$ in space is defined by $\vec{A}=\vec{R}f(r)$, where $\vec{R}=x\hat{i}+y\hat{j}+z\hat{k}$ and $r=(x^2+y^2+z^2)^{1/2}$. Determine f(r) so that the field may be irrotational and solenoidal. (12)

3. (a) Two equal smooth spheres, each of weight W and redius r, are placed inside a hollow cylinder open at both ends which rests on a horizontal plane; if a (< 2r) be the readius of the cylinder, showthat the least weight it can have so as not to upset is $2W(1-\frac{r}{a})$. (10)

   (b) A rod AB of weight W is movable about a point at A and to B is attached a string whose other end is tied to a ring. The ring slides along a smooth horizontal wire passing through A. If the rod and the string make angles α and β with the horizon, find the horizontal force necessary to keep the ring at rest. (10)

SECTION – B

4. (a) A car of width b is moving with constant velocity V close to the edge of a straight road. If a pedestrian steps on the road at a point distant d in front of the car, what is the least uniform velocity at which he must be able to walk in order to cross the road in safety? If the car is at rest, but moves off with constant acceleration f at the same instant that the pedestrian starts to cross the road from a point on the edge of the road distant d in front of the car, show that the least uniform velocity at which he must be able to walk is $[f\{(d^2+b^2)^{1/2}-d\}]^{1/2}$ and that if he walks at this velocity his direction must be inclined to the edge of the road at an angle $\cot^{-1}[\{(d^2+b^2)^{1/2}-d\}/b]$, the distance between the car and the edge of the road being negligible and the position of the pedestrian be a point. (12)

   (b) Find the radial and transverse components of the acceleration of a particle moving along the circle $x^2+y^2=a^2$ with constant angular velocity c. (08)

5. (a) A particle moving along a straight line starts from rest and is accelerated uniformly till it attains a velocity v. The motion is then retarted and the particle comes to rest after traversing a total distance x. If the acceleration is f, find the retardation and the total time taken by the particle from rest to rest.

(b) An artificial satellite revolves round the earth in a circular orbit at a height h above earth's surface. Calculate the period of revolution of the satellite so that the astronaut in it may be in a state of weightlessness. (10)

6. (a) A gun of mass M fires a shell of mass m horizontally and the energy of the explosion is such that it would be sufficient to project the shell vertically to a height h. Show that the velocity of recoil of the gun has a magnitude. (10)

$$\sqrt{\frac{2m^2gh}{M(M+m)}}.$$

(b) An aeroplane is flying with constant speed v and at constant height h. Show that, if a gun is fired point blank at the aeroplane after it has passed directly over the gun when its angle of elevation as seen from the gun is α, the shell will hit the aeroplane provided that (10)

$$2v(u\cos\alpha - v) = gh\cot^2\alpha,$$

where u is the initial speed of the shot, the path being assumed parabolic.

7. (a) A planet is describing an ellipse about the sun as focus. Show that its velocity away from the sun is greatest when the radius vector to the planet is at right angles to the major axis of the path, and that it then is $\frac{2\pi ac}{T(1-e^2)}$, where the notations have their usual meanings. (12)

(b) A ball is dropped on the floor from a height h. If the coefficient of restitution is e, find the height of the ball at the top of the fifth rebound. (08)

COMPULSORY QUESTION

8. Write only the correct answer in the Answer Book.. Do not reproduce the question. Each part carries one mark.

(1) If the vectors $a\hat{i}+\hat{j}+\hat{k}$, $\hat{i}+b\hat{j}+\hat{k}$ and $\hat{i}+\hat{j}+c\hat{k}$ $(a \neq b \neq c \neq 1)$ are coplanar, then

$$\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}= \ldots.$$

(a) 1 (b) 0 (c) −1
(d) −4 (e) None of these

(2) The points with position vectors $\vec{a}, \vec{b}, \vec{c}$ are collinear if ...

(a) $[\vec{a}\ \vec{b}\ \vec{c}]=0$ (b) $(\vec{a}+\vec{b}).\vec{c}=0$

(c) $\vec{a}\times(\vec{b}\times\vec{c})=\vec{0}$ (d) $(\vec{a}\times\vec{b})+(\vec{b}\times\vec{c})+(\vec{c}\times\vec{a})=\vec{0}$

(3) For non-zero vectors $\vec{a}, \vec{b}$ and $\vec{c}$, $|(\vec{a}\times\vec{b}).\vec{c}| = |\vec{a}|\,|\vec{b}|\,|\vec{c}|$ holds if and only if ...

(a) $\vec{a}\cdot\vec{b}=0=\vec{b}\cdot\vec{c}$ (b) $\vec{b}\cdot\vec{c}=0=\vec{c}\cdot\vec{a}$

(c) $\vec{c}\cdot\vec{a}=0=\vec{a}\cdot\vec{b}$ (d) $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=0$ (e) None of these.

(11) If three forces acting on a rigid body are in equilibrium then they must be ...
(a) concurrent (b) parallel (c) non-coplanar
(d) concurrent or parallel and coplanar (e) None of these

(12) If three forces acting on a particle are in equilibrium, then they are ...
(a) not necessarily parallel (b) necessarily perpendicular
(c) necessarily parallel (d) not necessary coplanar
(e) None of these.

(13) The work done by a force $\vec{F} = 2\hat{i} - \hat{j} - \hat{k}$ through a displacement $\vec{r} = 3\hat{i} + 2\hat{j} - 5\hat{k}$ is...
(a) 3 (b) 6 (c) 9
(d) 12 (e) None of these

(14) A cricket ball is thrown with a velocity of 30 m/s, then the two directions in which the ball may be thrown so as to give a range of 45 m ($g=10m/s^2$), are ...
(a) 15° or 75° (b) 30° or 60° (c) 36° or 54°
(d) 45° or 54° (e) None of these

(15) If the radial and transverse velocities of a particle be non-zero constants, then the path described by the particle is ...
(a) an ellipse (b) a cardioid (c) a spiral
(d) a circle (e) None of these

(16) If a particle is thrown with a velocity $\sqrt{2gR}$ from the earth's surface, R being earth's radius, then the particle will ...
(a) never come back (b) come back after a time g^{R}
(c) come back after a time 2gR (d) come back after a time $\sqrt{gR}$
(e) None of these

(17) The acceleration of a freely falling body under gravity ...
(a) varies as the inverse of the distance traveled
(b) varies as the square of the distance travelled
(c) is uniform (d) is zero. (e) None of these

(18) If a particle moves on a cycloid, then its motion is ...
(a) linear (b) simple harmonic (c) parabolic
(d) elliptic (e) None of these

(19) The science which is concerned with the relations between the forces acting on rigid bodies and the resulting motion is called ...
(a) Kinematics (b) Kinetics (c) statics
(d) quantum mechanics (d) None of these

(20) If the collision is inelastic, then the coefficient of restitution e satisfies the condition...
(a) $e = 0$ (b) $e = 1$ (c) $0 \le e \le 1$
(d) $0 < e < 1$ (e) None of these

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(4) Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three non–zero vectors such that $\vec{c}$ is a unit vector perpendicular to both the vectors $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, then $\left|\vec{a}\cdot(\vec{b}\times\vec{c})\right|^2 = \ldots$

(a) 0

(b) $\frac{1}{4}(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)$

(c) $\frac{3}{4}(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)(c_1^2 + c_2^2 + c_3^2)$

(d) $\frac{1}{4}(a_1^2 + a_2^2 + a_3^2)(c_1^2 + c_2^2 + c_3^2)$

(e) None of these

(5) If $\vec{a} = (1,1,1)$, $\vec{c} = (0,1,-1)$ are given vectors, then vector b satisfying the equations $\vec{a}\times\vec{b} = \vec{c}$ and $\vec{a}\cdot\vec{b} = 3$ is:

(a) $\frac{1}{3}(5,1,1)$

(b) $\frac{1}{3}(5,2,2)$

(c) $\frac{5}{3}(1,1,1)$

(d) $\frac{1}{3}(5,1,2)$

(e) None of these

(6) If $\vec{a}, \vec{b}$ and $\vec{c}$ are any three coplanar unit vectors, then ...

(a) $\vec{a}\cdot(\vec{b}\times\vec{c}) = 1$

(b) $(\vec{a}\times\vec{b})\cdot\vec{c} = 0$

(c) $(\vec{a}\times\vec{b})\times\vec{c} = 1$

(d) $(\vec{a}\times\vec{b})\times\vec{c} = 0$

(e) None of these

(7) Let $\vec{a}, \vec{b}, \vec{c}$ be three non-coplanar vectors and $\vec{p}, \vec{q}, \vec{r}$ be three vectors defined by the relations $\vec{p} = \frac{\vec{b}\times\vec{c}}{[\vec{a}\ \vec{b}\ \vec{c}]}$, $\vec{q} = \frac{\vec{c}\times\vec{a}}{[\vec{b}\ \vec{a}\ \vec{c}]}$, $\vec{r} = \frac{\vec{a}\times\vec{b}}{[\vec{a}\ \vec{b}\ \vec{c}]}$. Then $(\vec{a}+\vec{b})\cdot\vec{p} + (\vec{b}+\vec{c})\cdot\vec{q} + (\vec{c}+\vec{a})\cdot\vec{r} = \ldots$

(a) 0

(b) 1

(c) 2

(d) 3

(e) None of these

(8) If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$ and vectors $\vec{p} = (1, a, a^2)$, $\vec{q} = (1, b, b^2)$, $\vec{r} = (1, c, c^2)$ are coplanar, then $[\vec{p}\ \vec{q}\ \vec{r}] = \ldots$

(a) -1

(b) -2

(c) 0

(d) -4

(e) None of these

(9) The resultant of any number of couples acting in the same plane on a rigid body is...

(a) a force

(b) a couple

(c) a force and a couple

(d) a force or a couple

(e) None of these

(10) If μ and λ are the coefficient and the angle of friction, then ...

(a) $\tan\lambda = \frac{1}{\mu}$

(b) $\tan\mu = \frac{1}{\lambda}$

(c) $\tan^{-1}\lambda = \mu$

(d) $\tan^{-1}\mu = \lambda$

(e) None of these

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APPLIED MATHEMATICS, PAPER–II

TIME ALLOWED: THREE HOURS MAXIMUM MARKS: 100

NOTE: Attempt **FIVE** questions in all, selecting **TWO** questions each from Sections **A** and **B**. **QUESTION NO.8**, is **COMPULSORY**. All questions carry **EQUAL** marks.

SECTION – A

1. (a) Solve: $4y'' - 4y' + y = x^{\frac{1}{2}} e^{\frac{x}{4}}$ (10)

   (b) Solve: $(1+x)^2 y'' + (1+x)y' + y = 4\cos\{\ln(1+x)\}$. (10)

2. (a) Solve the system of equations: (12)

   $x' - y = t^2, \quad y' + 4x = 2t.$

   (b) Solve the Lagrange's equation (8)

   $z(x+2y)p - z(y+2x)q = y^2 - x^2.$

3. (a) Use Monge's method to solve the following equation. (10)

   $q^2 r - 2pqs + p^2 t = p^2 qz.$

   (b) Obtain the solution of the wave equation $\frac{\partial^2 y}{\partial t^2} = c^2 \cdot \frac{\partial^2 y}{\partial x^2}$ using the method (10) of separation of variables.

4. (a) If λ_i and μ^i are the components of a covariant and contravariant vector, then show that the sum $\lambda_i \mu^i$ is an invariant. (10)

   (b) Prove that the transformations of covariant vectors form a group. (10)

5. (a) What are Christoffel symbols of the first and second kind? State all their properties. (10)

   (b) Find the real root of the equation: $x\log_{10} x - 1\cdot 2 = 0$, correct to five decimal places using the regula falsi method. (10)

6. (a) Compute the value of: $\int_{0.2}^{1.4} (\sin x - \ln x + e^x)dx$ (9)

   by (i) trapezoidal rule, (ii) Simpson's one-third rule and (iii) Simpson's three-eighth rule.

   (b) Solve by Gauss – Seidel method the following system of linear equations. (11)

   $$27x + 6y - z = 85,$$
   $$6x + 15y + 2z = 72,$$
   $$x + y + 54z = 110.$$

7. (a) Using Lagrange's interpolation formula, find the value of y corresponding to x = 10 from the following table: (10)

x:	5	6	9	11
y:	12	13	14	16

   (b) A factory produces two products chairs and tables. The factory makes a profit of Rs.40/= on each chair produced and Rs.50/= on each table. A chair requires the resources of 2 man hours, 3 hours of machine time and 1 unit of wood. A table requires 2 man hours, 1 hour machine time and 4 units of wood. The factory has 60 man hours, 75 hours machine time and 84 units of wood available each day for producing these two products. How should the resources be allocated between the two products in order to maximize the factory profit? (12)

COMPULSORY QUESTION

8. Write the correct answer in the Answer Book. Do not reproduce the question. Each part carry one mark.

(1) The solution of a differential equation subject to a condition satisfied at one particular point only is called ———.
- (a) a boundary value problem
- (b) a two-point boundary value problem
- (c) an initial value problem
- (d) a two-point initial value problem
- (e) None of these

(2) The order and degree of the differential equation $\left[\dfrac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}}\right]^{3/2} = b$ are respectively ———.
- (a) 1 and 3
- (b) 1 and 2
- (c) 3 and 2
- (d) 2 and 3
- (e) None of these

(3) The function obtained after solving a differential equation is called ———. Choose the odd one out.
- (a) a solution
- (b) an integral
- (c) a primitive
- (d) a root
- (e) None of these

(4) In the linear equation $F(D)y=f(x)$, the function $f(x)$ is called the ———. Choose the odd one out.
- (a) input function
- (b) forcing function
- (c) excitation
- (d) response function
- (e) None of these

(5) The singular solution of the differential equation $f(x,y,p)=0$, where $p=dy/dx$, is obtained by eliminating p from ———.
- (a) $f(x,y,p)=0$ and $\partial f/\partial p = 0$
- (b) $f(x,y,p)=0$ and $\partial f/\partial x = 0$
- (c) $f(x,y,p)=0$ and $\partial f/\partial y = 0$
- (d) $f(x,y)=0$ and $\dfrac{\partial f(x,y,p)}{\partial p} = 0$
- (e) None of these

(6) "Infinitely many differential equations have the same integrating factors." This statement is ———.
- (a) never true
- (b) may be true
- (c) semi-true
- (d) always true
- (e) may not be true

(7) If m is the degree of a given differential equation, then ———.
- (a) m can be zero
- (b) m is any non-negative integer
- (c) m is any integer
- (d) m is any natural number
- (e) None of these

(8) A general solution of an nth order differential equation contains ———.
- (a) n – 1 arbitrary constants
- (b) n arbitrary constants
- (c) n + 1 arbitrary constants
- (d) no constant
- (e) None of these

(9) An equation of the form $y = px+f(p)$, where $p = \dfrac{dy}{dx}$, is called ———.
- (a) Bernoulli's equation
- (b) Euler's equation
- (c) Clairaut's equation
- (d) Bessel's equation
- (e) None of these

(10) A homogeneous differential equation of the form

$$a_0 x^n \frac{d^n y}{dx^n} + a_1 x^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \ldots + a_n y = f(x),$$ where $a_0, a_1, \ldots, a_n$ are constants

and $f(x)$ is a function of x, is known as ———.

(a) Bernoulli's equation (b) Lagrange's equation
(c) Legendre's equation (d) Cauchy-Euler equation
(e) None of these

(11) If an index appears only once in a term of the tensor equation, then it is called a ———.

(a) bound index (b) free index (c) sliding index
(d) directional index (e) None of these

(12) The summation convention was introduced by ———.

(a) Ricci (b) Levi Civita (c) Einstein
(d) Christoffel (e) None of these

(13) A second rank tensor in an N – dimensional space has ———.

(a) N! components (b) N^2 components
(c) (n – 1)! Components (d) N components (e) None of these

(14) A second rank symmetric tensor in a four dimensional continuum will have only ——— independent components.

(a) 16 (b) 12 (c) 10
(d) 6 (e) None of these

(15) In a four dimensional space, an anti-symmetric second rank tensor can be represented by ——— independent components only.

(a) 6 (b) 12 (c) 4
(d) 8 (e) None of these

(16) If the indices are unequal and not in a cyclic order, then the alternating symbol ϵ_{ijk}=.....

(a) 0 (b) 1 (c) 2
(d) –1 (e) None of these

(17) Newton – Raphson method to solve an equation involves the formula ———.

(a) $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ (b) $x_{n+1} = x_n + \frac{f(x_n)}{f'(x_n)}$

(c) $x_n = x_{n+1} - \frac{f(x_{n+1})}{f'(x_n+1)}$ (d) $x_{n+1} = x_n - \frac{f'(x_n)}{f(x_n)}$

(e) None of these

(18) The root of the equation $x^3 - x - 9 = 0$ near $x = 2$ correct to three decimal places when solved by Newton – Raphson method is ———.

(a) 2.273 (b) 2.240 (c) 2.241
(d) 2.242 (e) None of these

(19) By means of the iterative process $x_{n+1} = \frac{1}{2}\left(x_n + \frac{N}{x_n}\right)$, the positive square root of 102 correct to four decimal places is ———.

(a) 10.0995 (b) 10.1995 (c) 10.2995
(d) 10.2225 (e) None of these

(20) Using the iterative process $x_{n+1} = \frac{1}{2}\left(x_n + \frac{N}{x_n}\right)$, the positive square root of 278 to five significant figures is ———.

(a) 16.670 (b) 16.671 (c) 16.672
(d) 16.673 (e) None of these

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