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FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN PBS-17, UNDER THE FEDERAL GOVERNMENT, 2003

APPLIED MATHEMATICS, PAPER-I

TIME ALLOWED: THREE HOURS **MAXIMUM MARKS: 100**

NOTE: Attempt **FIVE** questions in all, including question **NUMBER- 8** which is **COMPULSORY**. Select at least **TWO** questions from each of the **SECTIONS I** and **II**. All questions carry **EQUAL MARKS**.

SECTION -I

1. (a) Prove that $(\vec{b}\times\vec{c}).(\vec{a}\times\vec{d})+(\vec{c}\times\vec{a}).(\vec{b}\times\vec{d})+(\vec{a}\times\vec{b}).(\vec{c}\times\vec{d})=0$ and use it to show that

$$\sin(A+B).\sin(A-B)=(\sin^2A-\sin^2B)=\frac{1}{2}(\cos 2B-\cos 2A).$$

(b) Prove that $\nabla\times(\nabla\times\vec{a})=\nabla(\nabla.\vec{a})-\nabla^2\vec{a}$.

2. (a) Find an expression for the square of the arc length in curvilinear coordinates.

(b) Verify Green's theorem in the plane for $\int_c(x^2ydx-y^3dy)$, where C is the closed curve formed by y=x and $y^3=x^2$ from (0, 0) to (1, 1).

3. (a) A uniform rod of length a rests against a smooth verticle wall and is supported by a string of length l attached to one end of the rod and to a point in the wall. If in the position of equilibrium the rod is inclined at an angle of measure θ to the vertical, show that $3a^2\cos^2\theta=l^2-a^2$.

(b) A heavy uniform rod of length 2a lies over a rough peg with one end leaning against a rough vertical wall. If c be the distance of the peg from the wall and the coefficient of friction both at the peg and the wall be μ, show that when the point of contact of the rod with the wall is above the peg, then the rod is on the point of sliding downwards, when $c=a(1+\mu^2)\sin^3\theta$.

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SECTION -II

4. (a) A cat is running along a straight edge of a garden. A dog sitting in the garden at a distance b from the edge, sees the cat when it is at its nearest point. The dog immediately chases the cat with twice the cat's speed in such a way that it is always running towards the cat. Find the time that elapses before the cat is caught and show the cat runs a distance $\frac{2b}{3}$ before being caught.

(b) A ship is approaching a cliff of height 105 metres above sea level. A gun fitted on the ship can fire shots with a speed of 110 metres per sec. Find the maximum distance from the foot of the cliff from where the gun can hit an object on the top of the cliff.

5. (a) A particle rest in equilibrium under the attraction of two centers of forces which attract directly as the distance, their intensities being λ and μ; the particle is slightly displaced towards one of them; show that the time of a small oscillation is $\frac{2\pi}{\sqrt{\lambda+\mu}}$.

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(b) A gun of mass M fires a shell of mass m horizontally and the energy of explosion is such as would be sufficient to project the shell vertically to a height h. Show that the velocity of recoil is $\sqrt{\dfrac{2m^2gh}{M(M+m)}}$.

6. (a) The angular velocity of a particle about a point in its plane of motion is constant. Prove that the transverse component of its acceleration is proportional to the radial component of its velocity.

(b) A planet is describing an ellipse about the sun as focus. Show that its velocity away from the sun is greatest when the radius vector to the planet is at right angles to the major axis of the path, and that it then is $\dfrac{2\pi ae}{T(1-e^2)}$, where 2a is the major axis, e the eccentricity and T the time period.

7. (a) A small insect moves along a uniform bar, of mass equal to itself and of length 2a, the ends of which are constrained to remain on the circumference of a fixed circle whose radius is $\dfrac{2a}{\sqrt{3}}$. If the insect starts from the middle point of the bar and moves along the bar with relative velocity V, show that the bar in time t will turn through an angle $\dfrac{1}{\sqrt{3}}$ are $\tan\dfrac{Vt}{a}$.

(b) Write short notes on any **TWO** of the following:
(i) Movements and Products of Inertia.
(ii) Impulsive Motion.
(iii) Coefficient of Restitution.

COMPULSORY QUESTION

8. Write only the correct choice in the Answer Book. Do not reproduce the question.

(1) If $\vec{a}$, $\vec{b}$, $\vec{c}$ be three collinear vectors, then there exists three non-zero scalars r, s, t such that $r\vec{a}$, $s\vec{b}$, $t\vec{c}$ with :

(a)	$r = s = t$	(b)	$r + s = t$
(c)	$r + s + t = 0$	(d)	$r = s + t$
(e)	None of these.		

(2) The vector equation of a line through a point with position vector $\vec{a}$ and parallel to a vector $\vec{b}$ is (t being a scalar)

(a)	$\vec{r} = t\vec{a} + \vec{b}$	(b)	$\vec{r} = \vec{a} + t\vec{b}$
(c)	$\vec{r} = (1-t)\vec{a} + t\vec{b}$	(d)	None of these.

(3) $\nabla \times (\phi \vec{f}) = ?$

(a)	$\nabla \times \vec{f} - (\nabla \times \vec{f})\phi$	(b)	$\nabla \times \vec{f} + (\nabla \phi) \times \vec{f}$

(c) $(\nabla \phi) \times \vec{f} + \phi (\nabla \times \vec{f})$ (d) None of these.

(4) $\nabla . (\nabla \times \vec{f}) = ?$

(a) div grad $\vec{f}$ (b) div curl $\vec{f}$

(c) curl div $\vec{f}$ (d) None of these.

(5) The work done be a force $\vec{F} = \vec{i} + \vec{j} + \vec{k}$ acting on a particle, if the particle is displaced from the point A(3,3,3) to the point B(4,4,4) is :

(a) 2 units (b) 3 units

(c) 4 units (d) None of these.

(6) If $\nabla \times \vec{f} = 0$, then $\vec{f}$ is said to be ....

(a) solenoidal (b) irrotational

(c) constant (d) None of these.

(7) If $\vec{a} = 3\vec{i} - 4\vec{j} - \vec{k}$ and $\vec{b} = -6\vec{i} + 8\vec{j} + 2\vec{k}$, then $\vec{a}$ and $\vec{b}$ are

(a) orthogonal (b) parallel

(c) non-coplanar (d) None of these.

(8) If $\vec{x} . \vec{a} = \vec{x} . \vec{b} = \vec{x} . \vec{c} = 0$ for some non-zero vector $\vec{x}$, then $[\vec{a}\ \vec{b}\ \vec{c}] = ?$

(a) 0 (b) 1

(c) 4 (d) None of these.

(9) The vector relation $l\vec{a} + m\vec{b} + n\vec{c} = 0$ is independent of the origin of vectors, where l, m, n are scalars, if

(a) $l + m + n = 0$ (b) $l = m = n = 0$

(c) $l = \frac{m\vec{b} + n\vec{c}}{\vec{a}}$ (d) None of these.

(10) If $\vec{a}, \vec{b}, \vec{c}$ are position vectors of the vertices of a triangle, then:

(a) $\frac{1}{2}[\vec{a}\ \vec{b}\ \vec{c}]$ (b) $\frac{1}{2}[\vec{b} \times \vec{c} + \vec{c} \times \vec{a} + \vec{a} \times \vec{b}]$

(c) $\frac{1}{2} \times \vec{a}(\vec{b} \times \vec{c})$ (d) None of these.

(11) Two balls are projected, respectively, from the same point in directions inclined at 60°C and 30°C to the horizontal. If they attained the same height, the ratio of their velocities of projection is:

(a) $\sqrt{3} : 1$ (b) $1 : \sqrt{3}$

(c) 1 : 2 (d) None of these.

(12) Forces of 7,5 and 3 units, acting on a particle, are in equilibrium. The angle between the last pair of forces is:

(a) 120° (b) 90°

(c) 60° (d) None of these.

(13) ABCD is square. Equal forces $\vec{P}$ are acting along AB, CB, AD and BC. Their resultant is $2\vec{P}$ acting along:

(a) DC (b) AB
(c) AC (d) None of these.

(14) If μ and λ be the co efficient and angle of friction, respectively then μ=?

(a) $\sin\lambda$ (b) $\tan\lambda$
(c) $\cos\lambda$ (d) None of these.

(15) If the radial and transverse velocities of a particle be non-zero constants, then the path described by the particle is:

(a) an ellipse (b) a spiral
(c) a circle (d) None of these.

(16) If m be the mass $\vec{r}$ the position vector and $\vec{v}$ be the velocity of a particle then its angular momentum is,

(a) $m(\vec{r}\cdot\vec{v})$ (b) $m(\vec{r}+\vec{v})$
(c) $m(\vec{r}\times\vec{v})$ (d) None of these.

(17) The path described by a particle moving with zero velocity and acceleration is a:

(a) circle (b) straight line
(c) point (d) None of these.

(18) If three forces acting on a body are in equilibrium, then they:

(a) are parallel (b) Meet in a point
(c) are-coplanar (d) None of these.

(19) The moment of inertia of a uniform solid sphere of mass m and radius r is:

(a) $\frac{4}{3}mr^3$ (b) $\frac{2}{5}mr^2$
(c) $\frac{3}{4}mr^2$ (d) None of these.

(20) An alternate to Newton's law of restitution is called:

(a) Laplace's hypothesis (b) Cauchy's hypothesis
(c) Poisson's hypothesis (d) None of these.

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NOTE: Attempt **FIVE** questions in all, including question **NUMBER- 8** which is **COMPULSORY**. Select at least **TWO** questions from each of the **SECTIONS I** and **II**. All questions carry **EQUAL MARKS**.

SECTION -I

1. Solve any **TWO** of the following differential equations:
   (a) $(D^2 - 4D + 4)y = 8x^3 e^{2x} \sin 2x$.
   (b) $\{(x+1)^2 D^2 + (x+1)D - 1\}y = x - 1 + \ln(x+1)^2$.
   (c) $x^2 \dfrac{d^2y}{dx^2} - 2x(1+x)\dfrac{dy}{dx} + 2(1+x)y = x^3$.

2. (a) Solve: $(y^2 + yz)dx + (z^2 + zx)dy + (y^2 - xy)dz = 0$.
   (b) Find a series solution of the differential equation

$$(2 + x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (1+x)y = 0$$

3. (a) $(D^2 + 3DD' + 2D'^2)z = x + y$
   (b) Solve: $(y^3x - 2x^4)p + (2y^4 - x^3y)q = 9z(x^3 - y^3)$

SECTION -II

4. (a) Write short notes on KRONECKER'S DELTA and LEVI-CIVITA SYMBOL.
   (b) Define CHARISTOFFEL SYMBOLS [ij, k] and $\left\{ {k \atop ij} \right\}$, respectively, of the first and second kind. Prove that:

$$[ij, k] = [ji, k] \text{ and } \left\{ {k \atop ij} \right\} = \left\{ {k \atop ji} \right\}.$$

5. (a) Use the method of Regula Falsi to find the real root to three decimal places of $x^5 - x^4 - x^3 - 1 = 0$ which lies between 1 and 2.
   (b) Solve $3x - \cos x - 1 = 0$ by Newton –Raphson method.

6. (a) Apply Gauss – Seidal iteration method to solve the system of equations:

$$27x + 6y - z = 85,$$
$$6x + 15y + 2z = 72,$$
$$x + y + 54z = 110.$$

   (b) Using Lagrange's interpolation formula, find the value of y corresponding to x = 10 from the following table:

x	5	6	9	11
y	12	13	14	16

7. (a) Compute the value of $\int_{0.2}^{1.4} (\sin x - \ln x + \exp x)dx$ by Simpson's one-third and three-eighth rules.
   (b) In a given factory there are three machines M_1, M_2 and M_3 used in making two products P_1 and P_2. One unit of P_1 occupies M_1 for 5 minutes, M_2 for 3 minutes and M_3 for 4 minutes. The corresponding figures for one

unit of P_2 are: M_1 for 1 minute, M_2 for 4 minutes and M_3 for 3 minutes. The net profit per unit of P_1 produced is Rs.300 and for P_2 is Rs.200. What production plan will give the most profit?

COMPULSORY QUESTION

8. Write only the correct choice in the Answer Book. Do not reproduce the question.

1. An invariant is a tensor of rank:
   (a) 0 (b) 1
   (c) 2 (d) None of these.

2. The tensor A^{ij} is a ---- tensor:
   (a) covariant (b) contravariant
   (c) mixed (d) None of these.

3. A^{ijk}_{pqrs} is a tensor of rank ----:
   (a) 3 (b) 4
   (c) 7 (d) None of these.

4. The product of two tensors of ranks 2 and 3 is a tensor of rank ----:
   (a) 2 (b) 3
   (c) 5 (d) 6
   (e) None of these.

5. $\delta^p_q \delta^q_r = ?$
   (a) δ^p_{qr} (b) δ^{pq}_r
   (c) δ^p_r (d) None of these.

6. The ORDER and DEGREE of differential equation $\frac{d^2y}{dx^2} = [y + (\frac{dy}{dx})^2]^{\frac{3}{4}}$ are:
   (a) 2 and 3 (b) 4 and 2
   (c) 2 and 4 (d) None of these.

7. The differential equation for the primitive $y = A \cos ax + B \sin ax$, A and B being arbitrary constants, is:
   (a) $(D^2 - a^2)y = 0$ (b) $(D^2 + a^2)y = 0$
   (c) $(D^2 + a)y = 0$ (d) None of these.

8. An INTEGRATING FACTOR for $\cos x \frac{dy}{dx} + y \sin x = 1$ is:
   (a) cos x (b) tan x
   (c) sec x (d) None of these.

9. The function $x = At^2 + Bt + C$ gives a differential equation of order ----
   (a) 1 (b) 2
   (c) 3 (d) None of these.

10. "Infinitely many differential equations may have the same integrating factors". This statement is:
    (a) never true (b) may be true
    (c) semi true (d) always true

11. If m is the degree of a given differential equation, then:
    (a) m can be zero
    (b) m is any non-negative integer
    (c) m is any integer
    (d) m is any natural number.

12. Which one of the following differential equations is not-linear?
    (a) $(x^2 D^2 + D + 1)y = 0$
    (b) $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + x^2y^2 = 0$
    (c) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + x^2y = 0$
    (d) None of these.

13. The number of arbitrary constants appearing in the particular solution of an nth order differential equation is:
    (a) n
    (b) n - 1
    (c) n + 1
    (d) None of these.

14. The particular integral of the differential equation $(D^2 + a^2)y = \cos ax$ is:
    (a) $-\frac{x}{2a}\cos ax$
    (b) $\frac{x}{2a}\sin ax$
    (c) $-\frac{x}{2a}\sin ax$
    (d) None of these.

15. $\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$ is .....
    (a) Heat equation
    (b) Wave equation
    (c) Equation of a vibrating string
    (d) None of these.

16. Use numeric magnification method. Find the positive real root of the equation $1.5\,x - x^2 = 0$ for x near 13. Give answer correct to two decimal places:
    (a) 1.30
    (b) 1.31
    (c) 1.33
    (d) None of these.

17. Solve the equation $x^2 - 2x - 4 = 0$ for x near 3 by iterative process. Give answer correct to three decimal places:
    (a) 3.136
    (b) 3.139
    (c) 3.236
    (d) None of these.

18. Use Newton's method to solve $x^3 - x - 9 = 0$ for x near 2 . Give the answer correct to three decimal places.
    (a) 2.273
    (b) 2.241
    (c) 2.242
    (d) None of these.

19. An interpolation formula used for unequal intervals is.....
    (a) Jacobi's formula
    (b) Stirling's formula
    (c) Lagrange's formula
    (d) None of these.

20. Numerical solutions of linear algebraic equations can be obtained by ....
    (a) Euler's method
    (b) Runge – Kutta method
    (c) Euler's modified method
    (d) None of these.

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