FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN PBS-17, UNDER THE FEDERAL GOVERNMENT, 2002

APPLIED MATHEMATICS, PAPER-I

TIME ALLOWED: THREE HOURS **MAXIMUM MARKS: 100**

NOTE: Attempt **FIVE** questions in all, including **QUESTION NO. 8** which is **COMPULSORY**. Select at least TWO questions from each of the **SECTIONS I and II**. All questions carry **EQUAL** marks.

SECTION – I

1 (a) Find an equation for the plane passing through the points $P_1(2,-1,1)$, $P_2(3,2,-1)$ and $P_3(-1,3,2)$. 10

(b Prove (i) $\nabla \times (\nabla\phi) = 0$ (ii) $\nabla.(\nabla \times \vec{A}) = 0$ 10

2 (a) If $\vec{F} = y\hat{i} + (x - 2xz)\hat{j} - xy\hat{k}$, Evaluate $\iint_S (\nabla \times \vec{F}).\hat{n}dS$ where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$, above the xy plane. 10

(b Verify the divergence theorem for $\vec{A} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ taken over the region bounded by $x^2 + y^2 = 4$, $z = 0$ and $z = 3$. 10

3

(a) Forces of magnitude P, 2P, 3P, 4P act respectively along the sides AB, BC, CD, DA of a square ABCD of side a and forces each of magnitude $(8\sqrt{2})P$ act along the diagonals BD, AC. Find the magnitude of the resultant force and the distance of its line of action from A. 10

(b Find the Centriod of the arc of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ lying in the first quadrant. 10

SECTION – II

4 (a) Find the radial and transverse components of the velocity of a particle moving along the curve $ax^2 + by^2 = 1$ at any time t if the polar angle $\theta = ct^2$. 10

(b A particle is projected vertically upwards with a velocity $\sqrt{2gh}$ and another let fall from a height h at the same time. Find the height of the point where they meet each other. 10

5 (a) A particle of mass m' is attached by a light and inextensible string of length ℓ to a ring of mass m, free to slide on a smooth horizontal rod. Initially the two masses are held with a string taut along the rod and then they are set free. Prove that the greatest angular velocity of the string has magnitude $\sqrt{\frac{2g(m+m')}{\ell m}}$. 10

(b Prove that the speed required to project a particle from a height h to fall a horizontal distance a from the point of projection is at least $\sqrt{g\left(\sqrt{a^2+h^2}-h\right)}$. 10

6 (a) Discuss the motion of a particle on a circle. 10

(b Show that the law of force towards the pole, of a particle describing the curve $r^n = a^n \cos n\theta$ is given by $f = \frac{(n+1)h^2a^{2n}}{r^{2n+3}}$. 10

7 (a) Find the moment inertia of the circle $x^2 + y^2 = a^2$ about the line $y = a$. 10

(b AB, BC are two equal rods freely-hinged at B and lie in a straight line on a smooth table. The end A is struck by a blow P ⊥ to AB. Show that the resulting velocity of A is $3\frac{1}{2}$ times that of B. 10

COMPULSORY QUESTION

8. Write only the correct choice in the Answer Book. Don't reproduce the statement.

1	If $\vec{A}\cdot(\vec{B}\times\vec{C})=0$, then vectors $\vec{A}, \vec{B}, \vec{C}$ are:			
	(a)	Collinear	(b)	Coplanar
	(c)	Parallel	(d)	None of these.
2	If $\vec{A}\cdot\vec{B}=0$, then the vectors are:			
	(a)	Perpendicular	(b)	Parallel
	(c)	Collinear	(d)	None of these.
3	The directional derivative of ϕ in the direction of $\nabla\phi$ is:			
	(a)	Maximum	(b)	Minimum
	(c)	Constant	(d)	None of these.
4	If $\nabla\cdot\vec{V}=0$, the motion of the fluid is:			
	(a)	Continuous	(b)	Discontinuous
	(c)	Irrolational	(d)	None of these.

5	The surface integral is:			
	(a)	Single integral	(b)	Double integral
	(c)	Triple integral	(d)	None of these.
6	The force field $\vec{F}$ is conservative if:			
	(a)	$\nabla \cdot \vec{F} = 0$	(b)	$\nabla \times \vec{F} = 0$
	(c)	$\nabla \vec{F} = 0$	(d)	$\nabla \cdot \nabla \times \vec{F} = 0$
7	Which one is correct:			
	(a)	$\nabla \cdot \nabla \times \phi$	(b)	$\nabla \times \nabla \cdot \vec{A}$
	(c)	$\nabla \times \nabla \times \vec{A}$	(d)	$\nabla \times \nabla \cdot \phi$
8	The minimum number of forces required for equilibrium are:			
	(a)	1	(b)	2
	(c)	3	(d)	4
9	The friction is maximum if it is:			
	(a)	Static	(b)	Limiting
	(c)	Dynamic	(d)	None of these.
10	The resultant of a system of forces acting on a rigid body is always:			
	(a)	A force	(b)	A couple
	(c)	A force and a couple	(d)	None of these.
11	For a particle moving in a central force the angular momentum is:			
	(a)	Conserved	(b)	Zero
	(c)	Variable	(d)	None of these.
12	The center of mass of a semi-circular lamina $x^2 + y^2 = a^2$ in the upper half lies on:			
	(a)	The origin.	(b)	x-axis
	(c)	y-axis.	(d)	None of these.
13	If the amplitude of oscillation of a particle performing simple harmonic is doubled, then its time period is:			
	(a)	Doubled	(b)	Halved
	(c)	Unchanged	(d)	None of these.

14	The ratio of coefficient of static friction and coefficient of kinetic friction is:				
	(a)	Greater than 1	(b)	Less than 1	
	(c)	Equal to 1	(d)	None of these.	
15	The simple harmonic motion, the acceleration at distance x is:				
	(a)	$-\lambda x$	(b)	λx	
	(c)	λx^2	(d)	$-\lambda x^2$	
16	The range of the projectile is maximum, if the angle of projection is:				
	(a)	$\frac{\pi}{3}$	(b)	$\frac{\pi}{4}$	
	(c)	$\frac{\pi}{6}$	(d)	$\frac{2\pi}{3}$	
17	The transverse component of acceleration is:				
	(a)	$\dot{r}$	(b)	$r\dot{\theta}$	
	(c)	$2\dot{r}\dot{\theta} + r\ddot{\theta}$	(d)	$\ddot{r} - r(\dot{\theta})^2$	
18	The arbit of a planet is:				
	(a)	Circle	(b)	Eclipse	
	(c)	Hyperbola	(d)	Parabola	
19	The angular speed of the earth about its axis is:				
	(a)	7.29×10^{-5} rad / sec	(b)	7.5×10^{-5} rad / sec	
	(c)	6.89 rad / sec	(d)	None of these.	
20	The moment of inertia of a hollow sphere of radius a and mass M about a diameter is				
	(a)	$\frac{1}{2}Ma^2$	(b)	$\frac{1}{3}Ma^2$	
	(c)	Ma^2	(d)	$2Ma^2$	

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Marks

SECTION – I

1 (a) Solve $(2y+3x+1)dx-(3x+2y-1)dy=0$ 10

(b) Solve $y''-4y'+5y=2x^2e^{2x}+4e^{2x}\operatorname{Sin}x$ 10

2 (a) Using the variation of parameters Method, solve the differential equation $\frac{d^2y}{dx^2}+a^2y=b\sec^2 ax$. 10

(b) Solve $y''+(x-1)y'+y=0$ in powers of $(x-1)$. 10

3 (a) Using Monge's method solve $x(r+2xr+x^2t)=p+2x^3$. 10

(b) Solve $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$ with boundary conditions $U(x,0)=3\operatorname{Sin} n\pi x$, $u(0,t)=0$, $u(\ell,t)=0$ where $0<x<\ell$ and $t>0$. 10

SECTION – II

4 (a) Let A^{pq}_{rst} be a tensor. Show that A^{pq}_{rsp} is also a tensor. Find its rank. 10

(b) Evaluate the Christoffel symbols of: 10
(i) the first-kind (ii) the second kind, for spaces where $g_{pq}=0$ $p\neq q$.

5 (a) Find the real roots of the equation $2x-3\operatorname{Sin}x-5=0$ using Newton – Raphson method. 10

(b) Find a 4th degree polynomial which passes through the following points: 10

x	1.0	2.0	4.0	7.0	8.0
f(x)	-9	-41	-189	9	523

6 (a) Find the solution of the system: 10

$2x_1 + x_2 + 3x_3 = 11$

$4x_1 + 3x_2 + 10x_3 = 28$

$2x_1 + 4x_2 + 17x_3 = 31$ by Gauss's & elimination method.

(b Solve the system: 10

$10x_1 + x_2 + 2x_3 = 44$

$x_1 + 2x_2 + 10x_3 = 61$

$2x_1 + 10x_2 + x_3 = 51$ by Gauss – Seidel – iterative method.

7 (a) Find the first and second derivatives of the function: 10

x	0	1	2	3	4	5	6
f(x)	2	3	10	29	66	127	128

at x = 2.31 and x = 2.8.

(b Derive Simpson's 1/3 rule with error. 10

COMPULSORY QUESTION

8. Write only the correct choice in the Answer Book. Don't reproduce the statement.

1	The differential equation $\left[1+(y'')^2\right]^{1/3} = y'''$ has the degree and order respectively.			
	(a)	2,1	(b)	1,1
	(c)	1,3	(d)	3,1
2	The differential equation $\frac{dy}{dx} + 2xy + xy^4 = 0$ is linear equation of:			
	(a)	Cauchy	(b)	Bessel
	(c)	Bernoulli's	(d)	None of these.
3	The differential equation $\frac{dy}{dx} = \frac{x^2 + xy + 2y^2}{2x^2 + y^2}$ is:			
	(a)	Exact	(b)	Homogeneous
	(c)	Cauchy	(d)	None of these.
4	The differential equation $(1-x^2)y'' - 2xy' + n(n+1)y = 0$			
	(a)	The Guass equation	(b)	The Legendr equation
	(c)	The Bessel equation	(d)	None of these.

5	The differential equation $\frac{d^3y}{dx^3} + y^2 = x^4$ is:			
	(a)	Non-homogeneous and linear	(b)	Homogeneous and non-linear
	(c)	Non-homogeneous and non linear	(d)	None of these.
6	The differential equation $\frac{d^2y}{dx^2} = 0$ has the primitive:			
	(a)	$Y = Ax^3 + Bx^2 + C$	(b)	$Y = Ax^2 + Bx^2 + C$
	(c)	$Y = Ax^3 + Bx + C$	(d)	None of these.
7	The differential equation $(x - x^2)y'' + \int y - (\alpha + \beta + Z)xy - \alpha\beta y = 0$ is:			
	(a)	The legendre equation	(b)	The Bessle equation
	(c)	The Gauss equation.	(d)	None of these.
8	The differential equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ is			
	(a)	One-dimensional wave equation	(b)	One dimensional heat equation
	(c)	None of these.		
9	The linear partial differential equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ is			
	(a)	2-dimensional poisson equation	(b)	2-dimensional Laplace equation
	(c)	Three dimensional Laplace equation	(d)	None of these.
10	The linear partial differential equation $AU_{xx} + 2BU_{xy} + CU_{yy} = F(x, y, u, u_x, u_y)$ is elliptic if:			
	(a)	$AB - C^2 > 0$	(b)	$AC - B^2 > 0$
	(c)	$AC - B^2 < 0$	(d)	$AB - C^2 = 0$
11	The heat equation $u_t = c^2 u_{xx}$ is:			
	(a)	Elliptic	(b)	Parabolic
	(c)	Hyperbolic	(d)	None of these.
12	The wave equation $u_{tt} = c^2 u_{xx}$ is:			
	(a)	Ecliptic	(b)	Parabolic
	(c)	Hyperbolic	(d)	Mixed type.

13	The velocity of a fluid at any point is:			
	(a)	Covariant	(b)	Contra-variant
	(c)	Mixed tensor	(d)	None of these.
14	The inner product of tensors A_r^p and B_t^{qs} is a tensor of rank:			
	(a)	2	(b)	3
	(c)	4	(d)	1
15	Relative error is equal to:			
	(a)	Approximate value	(b)	Error/true value.
	(c)	Truncation error	(d)	None of these.
16	E is called -----			
	(a)	Shifting	(b)	Forward difference
	(c)	Central difference	(d)	Backward difference,
	operator where E f(x) = f(x +h).			
17	If $\mu f(x) = \frac{1}{2}\left[f(x+\frac{h}{2}) + f(x-\frac{h}{2})\right]$, then μ is called:			
	(a)	Mean value	(b)	Shifting
	(c)	Forward difference	(d)	None of these.
	operator.			
18	$y_n + 3y_{n-1} + 2y_{n-2} = 3$ is a:			
	(a)	Differential equation	(b)	Homogeneous differential equation
	(c)	Non-Homogeneous differential equation.	(d)	None of these.
19	In Simpson's rule, if the interval is reduced by 1/3rd then the truncation error is reduced to:			
	(a)	1/3	(b)	1/9
	(c)	1/27	(d)	1/81
20	Lagrange interpolating polynomial is for:			
	(a)	Equi-spaced intervals	(b)	Un-equal intervals
	(c)	Half-intervals	(d)	None of these.

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