FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS
IN B.P.S.-17 UNDER THE FEDERAL GOVERNMENT, 2001

APPLIED MATHEMATICS, PAPER-I

TIME ALLOWED: 3 HOURS **MAXIMUM MARKS: 100**

Note: Attempt **FIVE** questions in all, including **QUESTION # 8** which is **COMPULSORY**. Select at least **TWO** questions from **EACH SECTION**. All questions carry equal marks.

SECTION-I

1. (a): In a temperature field, heat flows in the direction of maximum decrease of temperature T. Find this direction at $P(2,1)$ when $T = x^3 - 3xy^2$. Also, show that $\nabla^2 T = 0$.

   (b): Show that $div(f\nabla g) = f\nabla^2 g - \nabla f \cdot \nabla g$.

2. (a)(i): Find the equation for the tangent plane to the surface $3x^2 + 2y^2 + z^2 = 20$ at the point $P(1,2,3)$.

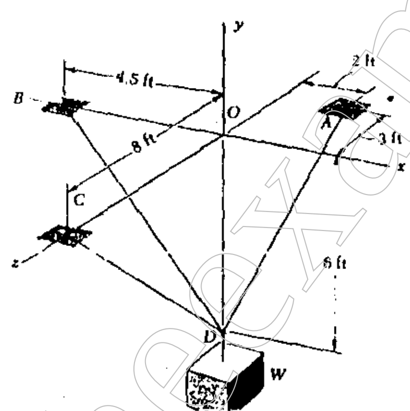
   (ii): Verify Stokes' theorem,

$$\iint_S (curl\mathbf{F})\cdot \hat{\mathbf{n}}dA = \oint_C \mathbf{F}\cdot \mathbf{r}(s)ds$$

   when $\mathbf{F} = y\hat{\mathbf{i}} + z\hat{\mathbf{j}} + x\hat{\mathbf{k}}$ and S is a paraboloid $z = f(x,y) = 1 - (x^2 - y^2)$, $z \geq 0$.

   (b): Show that the circular helix has constant curvature and torsion.

3. (a): A load W of magnitude 262 lb is supported by three cables as shown below. Determine the tension in each cable.



   (b): A lamina has the shape of the region R in the xy-plane bounded by the parabola $x = y^2$ and the line $x = 4$. The area mass density at the point $P(x,y)$ is directly proportional to the distance from the $y-axis$ to P. Find the centre of mass of the lamina.

SECTION-II

4. (a): Find a minimum speed with which a particle must be projected so that it passes through two points P and Q at height p and q respectively.

   (b): A gun fires two shots, the muzzle velocity in each case being v_0. In the first case the shot is fired at an angle of elevation α, and in the second case the shot is fired at a smaller angle of elevation β. Find the time interval between the two firings such that two shots will collide in the mid-air.

5. (a)(i): State Kepler's laws.

(ii): Deduce the law of force from Kepler's laws.

(b): A particle of mass m describes an orbit about a centre of force. The law of force is,

$$-km[n^2 + 1 - (2n^2a^2/r^2)]/r^3,$$

where k, n, a are positive numbers. At $t = 0$ the particle is at an apsis at a distance a from the force centre and has an initial velocity equal in magnitude to that it would have acquired if it had dropped from infinitely to that point. Find the equation of the orbit.

6. (a)(i): Define the followings:

1. Moment of Inertia.
2. Rotational Kinetic Energy.
3. Radius of Gyration.

(ii): Find the moment of inertia of a homogeneous circular disk, of radius b and mass m about an axis perpendicular to sheet, passing through the point at the edge of the disk.

(b) Find the velocity acquired by a block of wood, of mass M lb, which is free to recoil when it is struck by a bullet of mass m lb moving with velocity v, in the direction passing through the centre of gravity. If the bullet is embedded a $ft.$, show that the resistance of the wood to the bullet, supposed uniform, is $\frac{Mmv^2}{2(M+m)ga}$ lb and that the time of penetration is $\frac{2a}{v}$ sec., during which time the block will move $\frac{ma}{M+m}$ $ft.$

7. (a): A point moving with simple harmonic motion is observed to have velocities 3 $ft./\text{sec}$. and 4 $ft./\text{sec}$. when at distances of 4 $ft.$ and 3 $ft.$ respectively from its equilibrium position. Find the amplitude and period of the motion.

(b): A particle of mass m lies at the middle, A, of a hollow tube of length $2b$ and mass M. The tube, which is closed at both ends, lies on a smooth table. The coefficient of restitution between m and M is e. Let m be given initial velocity v_0 along the tube.

(i): Find the velocities of m and M after the first impact.

(ii): Find the loss in energy during the first impact.

(iii): Find the time required for m to arrive back at A travelling in the original direction.

8. (COMPULSORY QUESTION)

(1): An equation of a plane determined by the points $P_1(2,-1,1)$, $P_2(3,2,-1)$ and $P_3(-1,3-2)$ is:

(a): $11x + 5y + 13z = 30$.

(b): $2x + y - z = 15$.

(c): $x - y + z = 10$.

(d): none of these.

(2): The projection p of a vector $\overrightarrow{a}$ in the direction of a vector $\overrightarrow{b}$ is:

(a): $\overrightarrow{a} \cdot \overrightarrow{b}$ (b): $\frac{\overrightarrow{a} \cdot \overrightarrow{b}}{|\overrightarrow{b}|}$ (c): $\frac{\overrightarrow{a} \cdot \overrightarrow{b}}{|\overrightarrow{a}|}$ (d): none of these.

(3): Let $\varphi(x, y, z)$ be a differentiable function at a point $P(x, y, z)$ in a certain region. The directional derivative of φ in the direction of $\overrightarrow{a}$ is:

(a): $\nabla\varphi \cdot \overrightarrow{a}$ (b): $\frac{\nabla\varphi \cdot \overrightarrow{a}}{|\overrightarrow{a}|}$ (c): $\frac{\nabla\varphi \cdot \overrightarrow{a}}{|\nabla\varphi|}$ (d): none of these.

MATHEMATICS, PAPER-I:

Let $\vec{V}$ be a vector field, in a region R, whose partial derivatives are continuous at points of If $curl\,\vec{V} = 0$ at all points of R, then:

(a): $\vec{V}$ is said to be rotational in R.

(b): $\vec{V}$ is said to be irrotational in R.

(c): $\vec{V}$ is said to be solenoidal in R.

(d): none of these.

(5): Let f be a differentiable function that represents a surface S. Then the gradient of f (none zero vector) at a point P of S is:

(a): a normal vector on S at P.

(b): a vector in the tangent plane at P.

(c): an oblique vector on S at P.

(d): none of these.

(6): A potential function is a scalar function which satisfies:

(a): Heat equation.

(b): Laplace equation.

(c): Wave equation.

(d): none of these.

(7): The spherical coordinate system is:

(a): An orthogonal system.

(b): Not an orthogonal system.

(c): equivalent to Cartesian coordinate system.

(d): none of these.

(8): The range of a projectile is maximum if the angle of projection is:

(a): $\frac{\pi}{3}$ (b): $\frac{\pi}{6}$ (c): $\frac{\pi}{4}$ (d): none of these.

(9): The centre of mass of a set of particles is the point with respect to which:

(a): Linear momentum of the set of particles is zero.

(b): Sum of the vectors joining the set of particles is zero.

(c): the mass is zero.

(d): none of these.

(10): The centre of mass of a right circular solid cone of height h is:

(a): $\frac{2}{3}h$ (b): $\frac{3}{4}h$ (c): $\frac{1}{3}h$ (d): none of these.

(11): Let $\vec{A}$ be a unit vector. The $\vec{A}$ and $\frac{d\vec{A}}{dt}$ will be:

(a): parallel to each other.

(b): anti-parallel to each other.

(c): perpendicular to each other.

(d): none of these.

(12): The components of the force at any point, in a conservative field, are the negative of the components of the gradient of:

(a): the potential energy at that point.

(b): the kinetic energy at that point.

(c): the Lagrangian at that point.

(d): none of these.

(13): In a motion of a particle which is acted upon only by a conservative forces, kinetic and potential energies remains:

(a): zero. (b): constant (c): variable (d): none of these.

(14): The time rate of change of momentum of a particle remains constant in magnitude direction if:

(a): force is acting on the particle.

(b): no force is acting on the particle..

(c): velocity is zero.

(d): none of these.

(15): The Lagrangian of a system of particles is equal to:

(a): the difference of kinetic and potential energies.

(b): the sum of kinetic and potential energies.

(c): half of the kinetic energy.

(d): none of these.

(16): The transverse component of a velocity is:

(a): r. (b): $r\theta$ (c): $r^2\theta$ (d): none of these.

(17): The slope of the velocity time curve of a particle moving in a straight line gives:

(a): its distance travelled by the particle.

(b): its acceleration.

(c): its constant value.

(d): none of these.

(18): Dimensions of force is:

(a): MLT^{-1}. (b): MLT^{-2} (c): ML^2T^{-2} (d): none of these.

(19): Time of flight of a projectile, with velocity v_0 and angle α, is:

(a): $\frac{v_0}{g}$. (b): $\frac{v_0 \sin\alpha}{g}$ (c): $\frac{2v_0 \sin\alpha}{g}$ (d): none of these.

(20): Centre of mass of a hollow right circular cone, of semi-vertical angle α and height h, is:

(a): $\frac{2}{3}h$ (b): $\frac{1}{3}h$ (c): $\frac{3}{4}h$ (d): none of these.

FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION FOR RECRUITMENT TO POSTS IN BPS-17 UNDER THE FEDERAL GOVERNMENT, 2001.

APPLIED MATHEMATICS PAPER-II

TIME ALLOWED: THREE HOURS MAXIMUM MARKS: 100

NOTE: Attempt **FIVE** questions in all, including question **No.8** which is **COMPULSORY**. At least select **TWO** questions from each section. All questions carry **EQUAL** marks.

SECTION-I

1. (a) Solve: $\frac{dy}{dx} = \frac{2x-5y+3}{2x+4y-6}$ **(10)**

   (b) Solve: $y'' - 4y' + 3y = 2xe^{3x} + 3e^{x}\cos 2x$ **(10)**

2. (a) Solve: $x^3y''' + 2x^2y'' = x + \sin(\ln x)$ **(10)**

   (b) Solve the system:
   $(D-1)x + (D+3)y = e^{-t} - 1$
   $(D+2)x + (D+1)y = e^{2t} + t$, where $d = d/dt$ **(10)**

3. (a) Obtain partial differential equations by eliminating the arbitrary function f and g from $x = f(z) + g(y)$. **(08)**

   (b) Solve the boundary value problem by the method of separation of variable: **(12)**
   $\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}$, $\partial(0,t) = 0$, $\partial(\pi,t) = 0$ and $\partial(x,0) = 2\sin 3x - 4\sin 5x$.

SECTION-II

4. (a) If A_r^{pq} and B_r^{pq} are tensors, prove that their sum and difference are also tensors. **(10)**

   (b) If A_λ is a covariant tensor of rank one, show that $\frac{\partial A_\lambda}{\partial x_\mu}$ is not a tensor. **(10)**

5. (a) Use Newton's Raphson method to solve the equation
   $3x^3 + 4x^2 - 8x + 1 = 0$. **(10)**

   (b) Solve the following system of equation by Gauss – Seidal method: **(10)**
   $3x - y + 5z = 62$
   $2x + 5y + z = 36$
   $7x - 3y + z = 14$.

6. (a) Use Lagranges formula to produce a 4th degree polynomial which includes the following x_k, y_k number pairs: **(10)**

x_k	0	1	2	4	5
y_k	0	16	48	88	0

   (b) Find the first and second derivatives of the function f(x) at the point x = 1.1: **(10)**

x	1	1.2	1.4	1.6	1.8	2.0
f(x)	0	.1280	.5440	1.2960	1.4320	4.0

7. Write notes on the following : (5 x 4)
   (i) Lagranges interpolation.
   (ii) Classification of partial differential Equation.
   (iii) Orthogonal Trajectories.
   (iv) Method of variation of parameters.

COMPULSORY QUESTION

8. Write only the correct answers in the answer book. Do not reproduce the questions:

(1) The differential equation $\left[1+(y')^2\right]^{\frac{1}{2}} = y''$ has the order and degree respectively.
(a) 2,1 (b) 2,2
(c) 1,2 (d) None of these.

(2) Elimination of constants a, b from the equation $y = a\,e^{3x} + be^{x}$ gives a differential equation of order:
(a) 1 (b) 2
(c) 3 (d) None of these.

(3) The differential equation $y' + y = xy^3$ may be called as linear equation of :
(a) Cauchy (b) Bessel
(c) Bernoulli's (d) None of these.

(4) The differential equation $x^2y'' + xy' + (x^2 - k^2)y = 0$ is:
(a) Gauss equation (b) Bassel equation
(c) Legendre equation (d) None of these.

(5) The differential equation $(1-x^2)y'' - 2xy' + n(n+1)y = 0$ is:
(a) Gauss equation (b) Bassel equation
(c) Legendre equation (d) None of these.

(6) The equation $y'' + 4y' + y = 0, y(0) = 1, y'(0) = 2$, define:
(a) Initial value problem (b) Boundary value problem
(c) None of these.

(7) The differential equation $(x + y - 10)dx - (y - x + 2)dy = 0$ is:
(a) Linear (b) Exact
(c) Homogeneous (d) None of these.

(8) Particular integral of $(D^2 + 4)y = \text{Sin}\,3x$ is:
(a) $-\frac{1}{5}\text{Sin}\,3x$ (b) $-\frac{1}{5}\text{Cos}\,3x$
(c) $\frac{1}{5}\text{Sin}\,3x$ (d) None of these.

(9) The equation $\frac{\partial^2 u}{\partial x^2} + \frac{5\partial^2 u}{\partial y^2} = \frac{\partial u}{\partial z}$, where x, y, z are variables, is a partial differential equation of order and degree:
(a) 2,1 (b) 2,2
(c) 1,2 (d) None of these.

(10) The partial differential equation $\ln p + q = z^2$ is:
(a) of order 1 and is linear (b) of order 1 and is not linear
(c) not of order 1 and is linear (d) None of these.

(11) A partial differential equation $A\frac{\partial^2 u}{\partial x^2}+B\frac{\partial^2 u}{\partial y^2}+C\frac{\partial^2 u}{\partial x\partial y}=\text{Sin}\,x+2y$, where A, B, C are real constants, is:

(a) Linear (b) Homogeneous
(c) Non-Homogeneous (d) None of these.

(12) The Solution of $xp + yq = z$ is:

(a) $f(x,y)=0$ (b) $f(\frac{x}{y},\frac{y}{z})=0$
(c) $f(xy, yz)=0$ (d) $f(x^2,y^2)=0$
(e) None of these.

(13) If E is the "Shift operator" and Δ the "forward difference operator" then E - Δ=:

(a) -1 (b) 1
(c) 0 (d) None of these.

(14) An equation of the form $AU_{xx}+2BU_{xy}+CU_{yy}=F(x,y,u,u_x,u_y)$ is said to be elliptic if:

(a) $AC-B^2>0$ (b) $AC-B^2=0$
(c) $AC-B^2<0$ (d) None of these.

(15) Heat equation $U_t=C^2U_{xx}$ is:

(a) Parabolic (b) elliptic
(c) Hyperbolic (d) None of these.

(16) If a is given as first approximation to a root of the equation f(x)=0 and f'(a) is very small, then the Newton Raphson method is:

(a) applicable (b) Not-applicable

(17) Lagranges interpolation formula is used for:

(a) equal interval (b) Un-equal interval
(c) Half-interval (d) None of these.

(18) If A^λ, B_μ are components of a contravariant and covariant tensor of rank one, then $C^\lambda_\mu=A^\lambda B_\mu$ are the components of a mixed tensor of rank:

(a) One (b) Two
(c) Three (d) None of these.

(19) $\delta_{ik}\,\epsilon_{ikm}$ has the value:

(a) 0 (b) -1
(c) 3 (d) None of these.

(20) The smallest +ve root of $x^3-5x+3=0$ lies between:

(a) 1 and 2 (b) 0 and 1
(c) 2 and 3 (d) None of these.

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