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Part III — STATISTICS

(English Version)

Time Allowed : 3 Hours]   [Maximum Marks : 150

PART – I

Note : Answer *all* the questions.

Choose the correct answer : $50 \times 1 = 50$

1. The probability of an impossible event is

   a) 0   b) 1

   c) – 1   d) ∞ .

2. The probability that a leap year selected at random will contain 53 Sundays is

   a) $\frac{1}{2}$ b) $\frac{2}{7}$

   c) $\frac{3}{7}$ d) $\frac{7}{2}$.

3. Three dice are thrown simultaneously. The probability that sum being 3 is

   a) 0   b) $\frac{1}{216}$

   c) $\frac{2}{216}$ d) $\frac{3}{216}$.

[Turn over

4. Probability of drawing a spade queen from a well shuffled of cards is

    a) $\dfrac{1}{13}$ b) $\dfrac{1}{52}$

    c) $\dfrac{4}{13}$ d) 1.

5. An integer is chosen from 20 to 30. The probability that it is a prime number is

    a) $\dfrac{2}{10}$ b) $\dfrac{2}{11}$

    c) $\dfrac{3}{11}$ d) $\dfrac{4}{11}$.

6. When a coin and a die are thrown simultaneously, the number of all possible cases is

    a) 7   b) 8

    c) 12   d) 0.

7. When a single die is thrown, the events of getting odd number or even number are

    a) mutually exclusive events   b) not mutually exclusive events

    c) independent events   d) dependent events.

8. From the given random variable table,

$X = x$:	0	1	2
P_i :	a	$2a$	a

the value of a is

    a) 1   b) $\dfrac{1}{2}$

    c) 4   d) $\dfrac{1}{4}$.

9. Variance of a constant c is

   a)   0   b)   1

   c) c d) c^2.

10. If $f(x)$ is the *p.d.f.* of the continuous random variable X, then $E\left(X^2\right)$ is

   a) $\displaystyle\int_{-\infty}^{\infty} f(x)\, dx$ b) $\displaystyle\int_{-\infty}^{\infty} x \cdot f(x)\, dx$

   c) $\displaystyle\int_{-\infty}^{\infty} x^2 \cdot f(x)\, dx$ d) $\displaystyle\int_{-\infty}^{\infty} f(x^2)\, dx$.

11. If X is a random variable with its mean $\overline{X}$, then expression $E\left(X - \overline{X}\right)^2$ is

   a)   the variance of X b)   second central moment

   c)   both (a) and (b)   d)   none of (a) and (b).

12. $E(2X + 3)$ is

   a) $E(2X)$ b) $2E(X) + 3$

   c) $E(3)$ d) $2X + 3$.

13. If $f(x)$ is the probability function of a continuous random variable X with *p.d.f.* $f(x)$, then $\displaystyle\int_{-\infty}^{\infty} f(x)\, dx =$

   a)   1   b)   0

   c)   0·5   d)   2.

[Turn over

14. Variance of the random variable X is

    a) $E(X^2) - [E(X)]^2$ b) $[E(X)]^2 - E(X^2)$

    c) $E(X^2)$ d) $[E(X)]^2$.

15. The mean and variance of a Binomial distribution are 8 and 4 respectively. Then

    $P(X = 1)$ is equal to

    a) $\dfrac{1}{2^{12}}$ b) $\dfrac{1}{2^4}$

    c) $\dfrac{1}{2^6}$ d) $\dfrac{1}{2^8}$.

16. In a Binomial distribution if the numbers of independent trial is n, then the

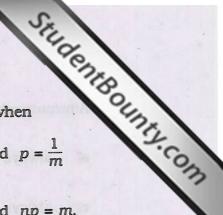   probability of n success is

    a) $nC_x \cdot p^x \cdot q^{n-x}$ b) 1

    c) p^n d) q^n.

17. $\left(\dfrac{2}{3} + \dfrac{1}{3}\right)^9$ refers the Binomial distribution and its standard deviation is

    a) 2 b) 3

    c) $\sqrt{2}$ d) 6.

18. Poisson distribution corresponds to

    a) rare events b) certain event

    c) impossible event d) almost sure event.

19. Poisson distribution is a limiting case of Binomial distribution when

a) $n \to \infty$, $p \to 0$ and $np = \sqrt{m}$ b) $n \to 0$, $p \to \infty$ and $p = \dfrac{1}{m}$

c) $n \to \infty$, $p \to \infty$ and $np = m$ d) $n \to \infty$, $p \to 0$ and $np = m$.

20. In a normal distribution, skewness is

a) one   b) zero

c) greater than one   d) less than one.

21. The standard normal distribution is represented by

a) $N(0, 0)$ b) $N(1, 1)$

c) $N(1, 0)$ d) $N(0, 1)$.

22. $P(-3 < Z < 3)$ takes the value

a) 0·9544   b) 0·9973

c) 0·0027   d) 1.

23. The normal distribution is a limiting form of Binomial distribution if

a) $n \to \infty$, $p \to 0$ b) $n \to 0$, $p \to q$

c) $n \to \infty$, $p \to n$ d) $n \to \infty$ and neither p nor q is small.

24. A measure characterizing a sample such as $\overline{X}$ or s is called

a) Population   b) Statistic

c) Universe   d) Mean.

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25. Alternative hypothesis is

   a)   always left tailed   b)   always right tailed

   c)   always one tailed   d)   one tailed or two tailed.

26. When the hypothesis is false and the test accepts it this is called

   a)   Type I error   b)   Type II error

   c)   Simple error   d)   Sample error.

27. Area of the critical region depends on

   a)   size of type I error   b)   size of type II error

   c)   value of the statistics   d)   number of observations.

28. Standard error of number of success is given by

   a) $\sqrt{\dfrac{pq}{n}}$ b) $\sqrt{npq}$

   c) npq d) $\sqrt{\dfrac{np}{q}}$.

29. Large sample theory is applicable when

   a) $n > 30$ b) $n < 30$

   c) $n < 100$ d) $n < 1000$.

30. The variance of observed sample proportion p is

   a) $\dfrac{P(1-Q)}{n}$ b) $\sqrt{\dfrac{PQ}{n}}$

   c) $\dfrac{PQ}{n}$ d) npq.

31. Statistic $Z = \dfrac{\overline{X} - \overline{Y}}{\sigma\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$ is used to test the null hypothesis

   a) $H_0 : \mu_1 + \mu_2 = 0$
   b) $H_0 : \mu_1 - \mu_2 = 0$

   c) $H_0 : \mu = \mu_0$ (a constant)
   d) none of these.

32. t-distribution ranges from

   a) $-\infty$ to 0
   b) 0 to ∞

   c) $-\infty$ to ∞
   d) 0 to 1.

33. The mean difference between 9 paired observations is 15·0 and the standard deviation of difference is 5·0. The value of test statistic t is

   a) 27
   b) 9

   c) 3
   d) zero.

34. The calculated value of χ^2 is

   a) always positive

   b) always negative

   c) can be either positve or negative

   d) none of these.

35. The χ^2-test was derived by

   a) R. A. Fisher
   b) Gauss

   c) Karl Pearson
   d) Laplace.

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36. Standard error of the sample mean in testing the difference between population mean and sample mean under t-statistic is

a) $\dfrac{\sigma}{\sqrt{n}}$

b) $\dfrac{S}{\sqrt{n}}$

c) $\dfrac{S}{n}$

d) $\dfrac{\sigma^2}{n}$.

37. The test statistic $F = \dfrac{S_1^2}{S_2^2}$ is used for testing

a) $H_0 : \mu_1 = \mu_2$

b) $H_0 : \sigma_1^2 = \sigma_2^2$

c) $H_0 : \sigma_1 = \sigma_2$

d) $H_0 : \sigma^2 = \sigma_0^2$.

38. In χ^2-test no theoretical cell-frequency should be less than

a) 1

b) 10

c) 0

d) 5.

39. Analysis of variance technique originated in the field of

a) Agriculture

b) Industry

c) Biology

d) Genetics.

40. In the case of one way classification with t treatments the mean sum of squares for treatment is

a) $\dfrac{SST}{N-1}$

b) $\dfrac{SST}{t-1}$

c) $\dfrac{SST}{N-t}$

d) $\dfrac{SST}{t}$.

41. With 90, 35, 25 as *TSS*, *SSR* and *SSC* respectively in case of two

   classification, *SSE* is

   a) 50

   b) 40

   c) 30

   d) 20.

42. Simple average method is used to calculate

   a) trend values

   b) cyclic variations

   c) seasonal Indices

   d) none of these.

43. The sales of a departmental store on Diwali are associated with the component

   of time series

   a) Secular trend

   b) Seasonal variation

   c) Irregular variation

   d) all of these.

44. The economic rhythm theory comes under the category of

   a) Analytical method

   b) Naive method

   c) Barometric method

   d) none of these.

45. Measure of association usually deals with

   a) attribute

   b) quantitative factors

   c) variables

   d) numbers.

[Turn over

46. The frequency of the class can always be expressed as a sum of frequencies of

    a)   lower order classes   b)   higher order classes

    c)   zero order classes   d)   none of these.

47. In case of two attributes A and B the class frequency $(AB) = 0$, the value of Q is

    a)   1   b) -1

    c)   0   d) $-1 \leq Q \leq 1$.

48. Which of the following criteria does not apply to decision making under uncertainty ?

    a)   Maximin return   b)   Maximax return

    c)   Minimax return   d)   Maximize expected return.

49. Maximin return, maximax return and minimax regret are criteria that

    a)   lead to the same optimal decision

    b)   cannot be used with probabilities

    c)   both (a) and (b)

    d)   none of these.

50. The criterion which selects the action for which maximum pay-off is lowest is known as

    a)   maximin criterion   b)   minimax criterion

    c)   maximax criterion   d)   none of these.

PART – II

Note : Answer any *fifteen* questions. $15 \times 2 = $

51. State Bayes' Rule.

52. If two events *A* and *B* are not mutually exclusive and are not connected with one random experiment $P(A) = \frac{1}{4}$, $P(B) = \frac{2}{5}$ and $P(A \cup B) = \frac{1}{2}$ then find the value of $P(B/A)$.

53. Write any two properties of distribution function.

54. A random variable *X* has the following probability distribution :

$X = x$:	5	2	1
$P(X = x)$:	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

Find the expected value of *X*.

55. Examine whether $f(x) = 5x^4$, $0 < x < 1$ can be a *p.d.f.* of a continuous random variable *X*.

56. Explain the fitting of a binomial distribution.

57. Write any two examples of Poisson distribution.

58. Find the area to the right of $Z = 0.25$.

59. What do you mean by testing of hypothesis ?

60. Define standard error.

61. In a test if $Z_0 > Z_e$, what is your conclusion about the null hypothesis ?

62. State any two properties of *t*-distribution.

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63. Explain the term 'Degrees of freedom'.

64. Define non-parametric test.

65. Write briefly about seasonal variation.

66. Discuss irregular variation in the context of time series.

67. What do you understand by consistency of data ?

68. For two attributes A and B, we have $(AB) = 35$, $(A) = 55$, $N = 100$ and $(B) = 65$. Calculate the missing values.

69. Explain the meaning of 'Statistical decision theory'.

70. Write a short note on decision tree.

PART – III

Note : Answer any *six* questions. $6 \times 5 = 30$

71. Two persons A and B appeared for an interview for a job. The probability of selecting A is $\frac{1}{3}$ and that of B is $\frac{1}{2}$. Find the probability that only one of them will be selected.

72. A random variable X has the density function $f(x) = Ax^3$, $0 < x < 1$. Find A and the expected value of X.

73. With the usual notation find p for binomial random variable X if $n = 6$ and $9P(X = 4) = P(X = 2)$.

74. If a random variable X follows Poisson distribution such that

$P(X = 1) = P(X = 2)$, find the mean of the distribution and $P(X = 0)$.

75. A sample of size 400 was drawn and the sample mean was found to be [...] whether this sample could have come from a normal population with mean [...] and variance 64 at 5% level of significance.

76. A random sample of size 20 from a population gives the sample standard deviation of 6. Test the hypothesis that the population standard deviation is 9.

77. Calculate the trend value to the following data by the method of semi-averages :

Year	1995	1996	1997	1998	1999	2000	2001
Expenditure (Rs. in lakhs)	1·5	1·8	2·0	2·3	2·4	2·6	3·0

78. 1,500 candidates appear for competitive examination. 425 were successful. 250 had attended a coaching class and of them 150 came out successful. Estimate the utility of the coaching class.

79. Consider the following pay-off (profit) matrix :

Action	States of Nature			
	S_1	S_2	S_3	S_4
A_1	5	10	18	25
A_2	8	7	8	23
A_3	21	18	12	21
A_4	30	22	19	15

No probabilities are known for the occurrence of the nature states. Compare the solutions obtained by each of the following criteria :

i) Maximin

ii) Laplace

iii) Hurwicz

(Assume that $\alpha = 0.5$)

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PART – IV

Note : Answer any *four* questions. 4 × 10 = 40

80. A bag contains 6 red and 8 black balls. Another bag contains 7 red and 10 black balls. A bag is selected and a ball is drawn. Find the probability that it is a red ball.

81. Students of a class were given an aptitude test. Their marks were found to be normally distributed with mean 60 and standard deviation 5. What percentage of students scored

    i) more than 60 marks ?

    ii) less than 56 marks ?

    iii) between 45 and 65 marks ?

82. In a certain city 125 men in a sample of 500 are found to be self-employed. In another city, the number of self-employed is 375 in a random sample of 1000. Does this indicate that there is a greater population of self-employed in the second city than in the first ?

83. Two types of batteries are tested for their length of life and the following data are obtained :

	No. of samples	Mean life (in hours)	Variance
Type A	9	600	121
Type B	8	640	144

Is there a significant difference in the two means ?

84. Three processes A, B and C are tested to see whether their outputs equivalent. The following observations of outputs are made :

A :	10	12	13	11	10	14	15	13
B :	9	11	10	12	13			
C :	11	10	15	14	12	13		

Carry out the analysis of variance and state your conclusion.

85. Obtain seasonal fluctuation from the following time series :

Quarterly output of coal for four years

| Quarter | Year | | | |
	2000	2001	2002	2003
I	65	58	70	60
II	58	63	59	55
III	56	63	56	51
IV	61	67	52	58

86. A magazine distributor assigns probabilities to the demand for a magazine as follows :

Copies demanded :	2	3	4	5
Probability :	0·4	0·3	0·2	0·1

A copy of magazine which he sells at Rs. 8 costs Rs. 6. How many should he stock to get the maximum possible expected profit if the distributor can return back unsold copies for Rs. 5 each ?

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