A

Register Number

Part III — MATHEMATICS

(English Version)

Time Allowed : 3 Hours] [Maximum Marks : 200

SECTION – A

N. B. : i) *All* questions are compulsory.

ii) Each question carries *one* mark.

iii) Choose the most suitable answer from the given four alternatives. $40 \times 1 = 40$

1. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then

a) $\vec{a}$ is parallel to $\vec{b}$ b) $\vec{a}$ is perpendicular to $\vec{b}$

c) $|\vec{a}| = |\vec{b}|$ d) $\vec{a}$ and $\vec{b}$ are unit vectors.

2. The shortest distance of the point (2, 10, 1) from the plane

$\vec{r} \cdot (3\vec{i} - \vec{j} + 4\vec{k}) = 2\sqrt{26}$ is

a) $2\sqrt{26}$ b) $\sqrt{26}$

c) 2 d) $\frac{1}{\sqrt{26}}$.

[Turn over

3. The point of intersection of the lines $\frac{x-6}{-6} = \frac{y+4}{4} = \frac{z-4}{-8}$ and $\frac{x+1}{2} = \frac{y+2}{4} = \frac{z+3}{-2}$ is

a) $(0, 0, -4)$

b) $(1, 0, 0)$

c) $(0, 2, 0)$

d) $(1, 2, 0)$.

4. The projection of $3\vec{i} + \vec{j} - \vec{k}$ on $4\vec{i} - \vec{j} + 2\vec{k}$ is

a) $\frac{9}{\sqrt{21}}$

b) $\frac{-9}{\sqrt{21}}$

c) $\frac{81}{\sqrt{21}}$

d) $\frac{-81}{\sqrt{21}}$.

5. The centre and radius of the sphere $|\vec{r} - (2\vec{i} - \vec{j} + 4\vec{k})| = 5$ are

a) $(2, -1, 4)$ and 5

b) $(2, 1, 4)$ and 5

c) $(-2, 1, 4)$ and 6

d) $(2, 1, -4)$ and 5.

6. The distance between the foci of the ellipse $9x^2 + 5y^2 = 180$ is

a) 4

b) 6

c) 8

d) 2.

7. The directrices of the hyperbola $x^2 - 4(y-3)^2 = 16$ are

a) $y = \pm \frac{8}{\sqrt{5}}$

b) $x = \pm \frac{8}{\sqrt{5}}$

c) $y = \pm \frac{\sqrt{5}}{8}$

d) $x = \pm \frac{\sqrt{5}}{8}$.

8. The point of intersection of tangents at t_1 and t_2 to the parabola $y^2 = 4ax$ is

a) $\left[a(t_1 + t_2),\ at_1 t_2 \right]$

b) $\left[at_1 t_2,\ a(t_1 + t_2) \right]$

c) $\left[at^2,\ 2at \right]$

d) $\left[at_1 t_2,\ a(t_1 - t_2) \right]$.

9. The slope of the tangent to the curve $y = 3x^2 + 3 \sin x$ at $x = 0$ is

a) 3 b) 2

c) 1 d) -1.

10. The function $f(x) = x^2$ is decreasing in

a) $(-\infty, \infty)$ b) $(-\infty, 0)$

c) $(0, \infty)$ d) $(-2, \infty)$.

11. The area between the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and its auxiliary circle is

a) $\pi b (a - b)$

b) $2 \pi a (a - b)$

c) $\pi a (a - b)$

d) $2 \pi b (a - b)$.

12. The volume generated by rotating the triangle with vertices at (0, 0), (3, 0) and (3, 3) about x-axis is

a) 18π

b) 2π

c) 36π

d) 9π.

13. $\int_a^b f(x)\, dx$ is

a) $2 \int_0^a f(x)\, dx$

b) $\int_a^b f(a - x)\, dx$

c) $\int_a^b f(b - x)\, dx$

d) $\int_a^b f(a + b - x)\, dx$.

14. The integrating factor of $\frac{dy}{dx} + 2\frac{y}{x} = e^{4x}$ is

a) $\log x$ b) x^2

c) e^x d) x.

15. The complementary function of $\left(D^2 + 1\right) y = e^{2x}$ is

a) $(Ax + B)\, e^x$

b) $A \cos x + B \sin x$

c) $(Ax + B)\, e^{2x}$

d) $(Ax + B)\, e^{-x}$.

16. If p is true and q is unknown, then

a) $\sim p$ is true

b) $p \vee (\sim p)$ is false

c) $p \wedge (\sim p)$ is true

d) $p \vee q$ is true.

17. If $f(x) = \begin{cases} kx^2 & ; \ 0 < x < 3 \\ 0 & ; \ \text{elsewhere.} \end{cases}$

is a probability density function, then the value of k is

a) $\frac{1}{3}$ b) $\frac{1}{6}$

c) $\frac{1}{9}$ d) $\frac{1}{12}$.

18. Given $E(x + c) = 8$ and $E(x - c) = 12$, then the value of c is

a) -2 b) 4

c) -4 d) 2.

19. In a Poisson distribution, if $P(x = 2) = P(x = 3)$ then the value of its parameter λ is

a) 6 b) 2

c) 3 d) 0.

20. Which of the following are correct ?

I. $E(aX + b) = a\,E(X) + b$

II. $\mu_2 = \mu_2' - (\mu_1')^2$

III. μ_2 = Variance

IV. $Var(aX + b) = a^2\, var(X)$.

a) All b) I, II and III

c) II and III d) I and IV.

21. If $A = [2 \;\; 0 \;\; 1]$, then the rank of AA^T is

a) 1 b) 2

c) 3 d) 0.

22. If A is a matrix of order 3, then det (kA) is

a) k^3 det (A)

b) k^2 det (A)

c) k det (A)

d) det (A).

23. If $A = \begin{bmatrix} 0 & 0 \\ 0 & 5 \end{bmatrix}$, then A^{12} is

a) $\begin{bmatrix} 0 & 0 \\ 0 & 60 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 0 \\ 0 & 5^{12} \end{bmatrix}$

c) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

24. In a homogeneous system $\rho(A) <$ (the number of unknowns) then the system has

a) only trivial solution

b) trivial solution and infinitely many non-trivial solutions

c) only non-trivial solutions

d) no solution.

25. If $\vec{a}$ and $\vec{b}$ include an angle 120° and their magnitudes are 2 and $\sqrt{3}$, then $\vec{a} \cdot \vec{b}$ is equal to

a) $\sqrt{3}$

b) $-\sqrt{3}$

c) 2

d) $-\frac{\sqrt{3}}{2}$.

26. If $x^2 + y^2 = 1$ then the value of $\frac{1 + x + iy}{1 + x - iy}$ is

a) $x - iy$ b) $2x$

c) $-2iy$ d) $x + iy$.

27. The polar form of the complex number $\left(i^{25}\right)^3$ is

a) $\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$

b) $\cos\pi + i\sin\pi$

c) $\cos\pi - i\sin\pi$

d) $\cos\frac{\pi}{2} - i\sin\frac{\pi}{2}$.

28. The value of $i + i^{22} + i^{23} + i^{24} + i^{25}$ is

a) i b) $-i$

c) 1 d) -1.

29. Which of the following is incorrect regarding n^{th} roots of unity ?

a) The number of distinct roots is n

b) The roots are in G.P. with common ratio $cis\left(\frac{2\pi}{n}\right)$

c) The arguments are in A.P. with common difference $\frac{2\pi}{n}$

d) Product of the roots is 0 and the sum of the roots is ± 1.

30. If the line $4x + 2y = c$ is a tangent to the parabola $y^2 = 16x$ then c is

a) -1 b) -2

c) 4 d) -4.

31. Which of the following curves is concave down ?

a) $y = -x^2$ b) $y = x^2$

c) $y = e^x$ d) $y = x^2 + 2x - 3$.

32. One of the conditions of Rolle's theorem is

a) f is defined and continuous on (a, b)

b) f is differentiable on $[a, b]$

c) $f(a) = f(b)$

d) f is differentiable on $(a, b]$.

33. If $u = \frac{1}{\sqrt{x^2 + y^2}}$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is equal to

a) $\frac{1}{2}u$ b) u

c) $\frac{3}{2}u$ d) $-u$.

34. If $x = r\cos\theta$, $y = r\sin\theta$, then $\frac{\partial r}{\partial x} =$

a) $\sec\theta$ b) $\sin\theta$

c) $\cos\theta$ d) $\text{cosec}\,\theta$.

35. The value of $\int_0^{\pi/4} \cos^3 2x\,dx$ is

a) $\frac{2}{3}$ b) $\frac{1}{3}$

c) 0 d) $\frac{2\pi}{3}$.

36. Solution of $\frac{dx}{dy} + mx = 0$, where $m < 0$ is

a) $x = ce^{my}$ b) $x = ce^{-my}$

c) $x = my + c$ d) $x = c$.

37. The order and degree of the differential equation $\frac{d^2y}{dx^2} - y + \left(\frac{dy}{dx} + \frac{d^3y}{dx^3}\right)^{3/2} = 0$ are

a) 2, 3 b) 3, 3

c) 3, 2 d) 2, 2.

38. Which of the following is a tautology ?

a) $p \vee q$ b) $p \wedge q$

c) $p \vee \sim p$ d) $p \wedge \sim p$.

39. Which of the following is not a binary operation on R ?

a) $a * b = ab$

b) $a * b = a - b$

c) $a * b = \sqrt{ab}$

d) $a * b = \sqrt{a^2 + b^2}$.

40. The value of $[3] +_{11} ([5] +_{11} [6])$ is

a) [0] b) [1]

c) [2] d) [3].

SECTION – B

N. B. : i) Answer any *ten* questions.

ii) Question No. **55** is compulsory and choose any *nine* questions from the remaining.

iii) Each question carries *six* marks. $10 \times 6 = 60$

41. Solve the following system of linear equations by determinant method.

$2x - 3y = 7,\ 4x - 6y = 14.$

42. If $A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$.

43. Find the equation of the sphere on the join of the points A and B having position vectors $2\vec{i} + 6\vec{j} - 7\vec{k}$ and $2\vec{i} - 4\vec{j} + 3\vec{k}$ and respectively as a diameter.

44. i) A force of magnitude 5 units acting parallel to $2\vec{i} - 2\vec{j} + \vec{k}$ displaces the point of application from (1, 2, 3) to (5, 3, 7). Find the work done.

ii) The volume of a parallelopiped whose edges are represented by $-12\vec{i} + \lambda\vec{k},\ 3\vec{j} - \vec{k},\ 2\vec{i} + \vec{j} - 15\vec{k}$ is 546. Find the value of λ.

45. Find the square root of $-7 + 24i$.

46. Solve the equation $x^4 - 4x^3 + 11x^2 - 14x + 10 = 0$, if one root is $1 + 2i$.

47. Evaluate $\lim_{x \to 1} x^{\frac{1}{x-1}}$.

48. i) Verify Rolle's theorem for the function $f(x) = \sin x, \quad 0 \le x \le \pi$.

ii) Prove that e^x is strictly increasing function on R.

49. If $z = ye^{x^2}$, where $x = 2t$, $y = 1 - t$, then find $\frac{dz}{dt}$.

50. Evaluate $\int_0^3 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{3 - x}}\, dx$.

51. Solve $\frac{dy}{dx} + 2y \tan x = \sin x$.

52. Prove that $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$.

53. A game is played with a single fair die. A player wins Rs. 20 if a 2 turns up, Rs. 40 if a 4 turns up, loses Rs. 30 if a 6 turns up, while he neither wins nor loses if any other face turns up. Find the expected sum of money he can win.

54. Four coins are tossed simultaneously. What is the probability of getting (i) exactly 2 heads, (ii) at least 2 heads, and (iii) at most 2 heads ?

55. a) Let G be a group, $a, b \in G$. Then prove that $(a * b)^{-1} = b^{-1} * a^{-1}$.

OR

b) Find the equations of the tangent and normal to the parabola $x^2 + 2x - 4y + 4 = 0$ at the point $(0, 1)$.

SECTION – C

N. B. : i) Answer any *ten* questions.

ii) Question No. **70** is compulsory and choose any *nine* questions from the remaining.

iii) Each question carries *ten* marks. $10 \times 10 = 100$

56. Examine the consistency of the following system of equations. If it is consistent, solve them. (Use rank method)

$x + y - z = 1, \quad 2x + 2y - 2z = 2, \quad -3x - 3y + 3z = -3.$

57. Show that the lines $\frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0}$ and $\frac{x-4}{2} = \frac{y}{0} = \frac{z+1}{3}$ intersect and hence find the point of intersection.

58. Find the vector and cartesian equations of the plane passing through the points with position vectors $3\vec{i} + 4\vec{j} + 2\vec{k}$, $2\vec{i} - 2\vec{j} - \vec{k}$ and $7\vec{i} + \vec{k}$.

59. Solve $x^7 + x^4 + x^3 + 1 = 0$.

60. On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4 m when it is 6 m away from the point of projection. Finally it reaches the ground 12 m away from the starting point. Find the angle of projection.

61. Find the eccentricity, centre, foci and vertices of the ellipse $16x^2 + 9y^2 - 32x + 36y - 92 = 0$ and draw the diagram.

62. Prove that the line $5x + 12y = 9$ touches the hyperbola $x^2 - 9y^2 = 9$ and find its point of contact.

63. Find the intervals of concavity and the points of inflexion of the curve $y = 12x^2 - 2x^3 - x^4$.

64. Trace the curve $y = x^3 + 1$.

65. Find the area between the curve $y = x^2 - x - 2$, x-axis and the lines $x = -2$ and $x = 4$.

66. Prove that the curved surface area of a sphere of radius r intercepted between two parallel planes at the distances a and b from the centre of the sphere is $2\pi r(b - a)$ and hence deduce the surface area of the sphere ($b > a$).

67. Radium disappears at a rate proportional to the amount present. If 5% of the original amount disappears in 50 years, how much will remain at the end of 100 years ? [Take A_0 as the initial amount]

68. Show that the set $\{[1], [3], [4], [5], [9]\}$ forms an Abelian group under multiplication modulo 11.

69. Find c, μ and σ^2 of the normal distribution whose probability density function is given by $f(x) = ce^{-x^2 + 3x}$, $-\infty < x < \infty$.

70. a) Solve : $\left(D^2 - 1 \right) y = \cos 2x - 2 \sin 2x$.

OR

b) A car A is travelling from west to east at 50 km/hr and car B is travelling from south towards north at 60 km/hr. Both are headed for the intersection of the two roads. At what rate are the cars approaching each other when car A is 0·3 km and car B is 0·4 km from the intersection ?.