Register Number

Part III — BUSINESS MATHEMATICS

(English Version)

Time Allowed : 3 Hours] [Maximum Marks : 200

SECTION – A

N. B. : i) Answer *all* the 40 questions.

ii) Choose and write the correct answer from the four choices given. $40 \times 1 = 40$

1. If $AB = BA = |A|I$, then the matrix B is

   a) the inverse of A

   b) the transpose of A

   c) the adjoint of A

   d) $2A$.

2. The inverse of $\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$ is

   a) $\begin{pmatrix} 0 & \frac{1}{2} \\ 2 & 0 \end{pmatrix}$

   b) $\begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix}$

   c) $\begin{pmatrix} 0 & -\frac{1}{2} \\ \frac{1}{2} & 1 \end{pmatrix}$

   d) $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$.

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3. For what value of k the matrix A, where $A = \begin{pmatrix} 2 & k \\ 3 & 5 \end{pmatrix}$ has no inverse ?

a) $\dfrac{3}{10}$ b) $\dfrac{10}{3}$

c) 3   d) 10.

4. The rank of an $n \times n$ matrix each of whose elements is 1, is

a) 1   b) 2

c) n d) n^2.

5. The number of Hawkin-Simon conditions for the viability of an input-output model is

a) 1   b) 3

c) 4   d) 2.

6. The eccentricity of a conic is $\dfrac{1}{\sqrt{2}}$. The conic is

a) a parabola   b) an ellipse

c) a circle   d) a hyperbola.

7. Latus rectum of $y^2 = 4ax$ is

a) $2a$ b) $3a$

c) $4a$ d) a.

8. Eccentricity of the hyperbola $\dfrac{x^2}{4} - \dfrac{y^2}{5} = 1$ is

   a) $\dfrac{3}{2}$

   b) $\dfrac{9}{4}$

   c) $\dfrac{5}{4}$

   d) $4.$

9. If a is the length of the semi-transverse axis of rectangular hyperbola $xy = c^2$, then the value of c^2 is

   a) a^2

   b) $2a^2$

   c) $\dfrac{a^2}{2}$

   d) $\dfrac{a^2}{4}.$

10. The average fixed cost of the function $C = 2x^3 - 3x^2 + 4x + 8$ is

    a) $\dfrac{2}{x}$

    b) $\dfrac{4}{x}$

    c) $\dfrac{-3}{x}$

    d) $\dfrac{8}{x}.$

11. The demand for some commodity is given by $q = -3p + 15$ ($0 < p < 5$), where p is the unit price. The elasticity of demand is

    a) $\dfrac{9p^2 + 15}{p}$

    b) $\dfrac{9p - 45}{p}$

    c) $\dfrac{15p - 9}{p}$

    d) $\dfrac{p}{-p + 5}.$

12. If $y = 2x^2 + 3x$, the instantaneous rate of change of y at $x = 4$ is

    a) 16

    b) 19

    c) 30

    d) $4.$

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13. The slope of the tangent at (2, 8) on the curve $y = x^3$ is

a) 3

b) 12

c) 6

d) 8.

14. The point at which the tangent to the curve $y^2 = x$ makes an angle $\frac{\pi}{4}$ with the

x-axis is

a) $\left(\frac{1}{2}, \frac{1}{4}\right)$

b) $\left(\frac{1}{2}, \frac{1}{2}\right)$

c) $\left(\frac{1}{4}, \frac{1}{2}\right)$

d) (1, – 1).

15. If $u = e^{x^2 + y^2}$, then $\frac{\partial u}{\partial x}$ is equal to

a) $y^2 u$

b) $x^2 u$

c) $2xu$

d) $2yu$.

16. $f(x, y) = \dfrac{x^{\frac{1}{2}} + y^{\frac{1}{2}}}{x^{\frac{1}{3}} + y^{\frac{1}{3}}}$ is a homogeneous function of degree

a) $\frac{1}{2}$

b) $\frac{1}{3}$

c) $\frac{1}{6}$

d) $\frac{1}{5}$.

17. The marginal revenue is Rs. 40 and the average revenue is Rs. 60. The elasticity of demand with respect to price is

a) 1

b) 0

c) 2

d) 3.

18. The marginal productivity of labour (L) for the production function $P = 15k - L^2 + 2kL$, when $L = 3$ and $k = 4$ is

   a)   21

   b)   12

   c)   2

   d)   3.

19. If $f(x)$ is an even function then $\int\limits_{-a}^{a} f(x)\, dx =$

   a) $2\int\limits_{0}^{a} f(x)\, dx$

   b) $\int\limits_{0}^{a} f(x)\, dx$

   c) $-2a$

   d) $2a$.

20. The area bounded by the curve $y = e^x$, the x-axis and the lines $x = 0$ and $x = 2$ is

   a) $e^2 - 1$

   b) $e^2 + 1$

   c) e^2

   d) $e^2 - 2$.

21. The marginal revenue of a firm is $MR = 15 - 8x$. Then the revenue function is

   a) $15x - 4x^2 + k$

   b) $\dfrac{15}{x} - 8$

   c) -8

   d) $15x - 8$.

22. The order and the degree of the differential equation $\left(\dfrac{dy}{dx}\right)^2 - 3\dfrac{d^3y}{dx^3} + 7\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} = x + \log x$ are

   a)   1 and 3

   b)   3 and 1

   c)   2 and 3

   d)   3 and 2.

23. The solution of $\dfrac{dy}{dx} = e^{x-y}$ is

    a) $e^y\, e^x = c$ b) $y = \log c\, e^x$

    c) $y = \log\left(e^x + c\right)$ d) $e^{x+y} = c$.

24. The solution of the equation of the type $\dfrac{dy}{dx} + py = 0$, (p is the function of x) is

    given by

    a) $y\, e^{\int p\, dx} = c$ b) $y \int p\, dx = c$

    c) $x\, e^{\int p\, dx} = y$ d) $y = cx$.

25. The particular integral of the differential equation $\dfrac{d^2 y}{dx^2} - 6\dfrac{dy}{dx} + 9y = e^{3x}$ is

    a) $\dfrac{e^{3x}}{2!}$ b) $\dfrac{x^2\, e^{3x}}{2!}$

    c) $\dfrac{x\, e^{3x}}{2!}$ d) $9e^{3x}$.

26. $E =$

    a) $1 + \Delta$ b) $1 - \Delta$

    c) $\nabla + 1$ d) $\nabla - 1$.

27. Five data relating to x and y are to be fit in a straight line. It is found that $\sum x = 0$

    and $\sum y = 15$. Then the y-intercept of the line of best fit is

    a)   1   b)   2

    c)   3   d)   4.

28. The mean and variance of a binomial distribution are

   a) np, npq

   b) pq, npq

   c) $np, \sqrt{npq}$

   d) np, nq.

29. If X is a Poisson variate with $P(X = 1) = P(X = 2)$, the mean of the Poisson variate is equal to

   a) 1

   b) 2

   c) -2

   d) 3.

30. If $X \sim N(\mu, \sigma^2)$, the maximum probability at the point of inflexion of normal distribution is

   a) $\dfrac{1}{\sqrt{2\pi}} e^{\frac{1}{2}}$

   b) $\dfrac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}}$

   c) $\dfrac{1}{\sigma\sqrt{2\pi}}$

   d) $\dfrac{1}{\sqrt{2\pi}}$.

31. If a random variable X has the following probability distribution :

X	-1	-2	1	2
$P(x)$	$\dfrac{1}{3}$	$\dfrac{1}{6}$	$\dfrac{1}{6}$	$\dfrac{1}{3}$

   then the expected value of X is

   a) $\dfrac{3}{2}$

   b) $\dfrac{1}{6}$

   c) $\dfrac{1}{2}$

   d) $\dfrac{1}{3}$.

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32. The *Z* value that is used to establish a 95% confidence interval for the estimate of a population parameter is

   a) 1·28

   b) 1·65

   c) 1·96

   d) 2·58.

33. Probability of rejecting null hypothesis when it is true is

   a) Type I error

   b) Type II error

   c) Sampling error

   d) Standard error.

34. Which of the following statements is *True* ?

   a) Point estimate gives a range of values.

   b) Sampling is done only to estimate a statistic.

   c) Sampling is done to estimate the population parameter.

   d) Sampling is not possible for an infinite population.

35. The number of ways in which one can select 2 customers out of 10 customers is

   a) 90

   b) 60

   c) 45

   d) 50.

36. A *time* series is a set of data recorded

     a)   periodically   b)   at equal time interval

     c)   at successive points of time   d)   all of these.

37. The terms 'prosperity', 'recession', 'depression' and 'recovery' are in particular attached to

     a)   secular trend   b)   seasonal fluctuation

     c)   cyclic movements   d)   irregular variation.

38. The weights used in Paasche's formula belong to

     a)   the base period   b)   the current period

     c)   any arbitrary chosen period   d)   none of these.

39. If X and Y are two variates, there can be at most

     a)   one regression line   b)   two regression lines

     c)   three regression lines   d)   none of these.

40. The lines of regression intersect at the point

     a) (X, Y) b) $(\bar{X}, \bar{Y})$

     c) $(0, 0)$ d)   none of these.

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SECTION – B

N. B. : Answer any *ten* questions. $10 \times 6 = 60$

41. Show that the equations $x + y + z = -3$, $3x + y - 2z = -2$, $2x + 4y + 7z = 7$ are not consistent.

42. Verify $(AB)^{-1} = B^{-1} A^{-1}$, when $A = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix}$.

43. Find the equation of the ellipse whose focus is $(1, -2)$, directrix is $3x - 2y + 1 = 0$ and $e = \dfrac{1}{\sqrt{2}}$.

44. Find the equilibrium price and equilibrium quantity for the following demand and supply functions :

    $Q_d = 4 - 0 \cdot 06\, p$ and $Q_s = 0 \cdot 6 + 0 \cdot 11\, p$.

45. At what points on the curve $3y = x^3$, the tangents are inclined at $45°$ to the x-axis ?

46. Find EOQ for the data given below. Also verify that carrying cost is equal to ordering cost at EOQ.

Item	Monthly requirements	Ordering cost per order	Carrying cost per unit
A	9,000	Rs. 200	Rs. 3·60

47. The marginal cost function of manufacturing x units of a commodity $6 + 10x - 6x^2$. Find the total cost and average cost, given that the total cost of producing 1 unit is 15.

48. Solve : $\dfrac{dy}{dx} + \dfrac{2xy}{1+x^2} = \dfrac{1}{\left(1+x^2\right)^2}$, given that $y = 0$, when $x = 1$.

49. Solve : $\dfrac{d^2y}{dx^2} + 4\dfrac{dy}{dx} + 4y = 2e^{-3x}$.

50. Find the missing term from the following data :

X	1	2	3	4
$f(x)$	100	—	126	157

51. If $f(0) = 5$, $f(1) = 6$, $f(3) = 50$, $f(4) = 105$, find $f(2)$ by using Lagrange's formula.

52. On an average if one vessel in every ten is wrecked, find the probability that out of five vessels expected to arrive, at least four will arrive safely.

53. A random sample of 500 apples was taken from large consignment and 45 of them were found to be bad. Find the limits at which the bad apples lie at 99% confidence level.

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54. Obtain the trend values by the method of semi-average :

Year :	1987	1988	1989	1990	1991	1992	1993
Production (in tonnes)	90	110	130	150	100	150	200

55. Calculate the cost of living index number using Family Budget method for the following data taking the base year as 1995 :

Commodity	Weight	Price (per unit)	
		1995	1996
A	40	16·00	20·00
B	25	40·00	60·00
C	5	0·50	0·50
D	20	5·12	6·25
E	10	2·00	1·50

SECTION – C

N. B. : Answer any *ten* questions. $10 \times 10 = 100$

56. If $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{pmatrix}$, verify that $A \, (\, \text{Adj}\, A\,) = (\,\text{Adj}\, A\,)\, A = |A|\, I$.

57. In an economy, there are two industries P and Q and the following table gives the supply and demand positions in crores of rupees :

Producer	User		Final Demand	Total Output
	P	Q		
P	10	25	15	50
Q	20	30	10	60

Determine the outputs, when the final demand changes to 35 for P and 42 for Q.

58. Find the equation to the asymptotes of the hyperbola

$8x^2 + 10xy - 3y^2 - 2x + 4y - 2 = 0$.

59. Find the elasticity of demand, when the demand is $q = \dfrac{20}{p+1}$ and $p = 3$. Interpret the result.

60. A certain manufacturing concern has total cost function $C = 15 + 9x - 6x^2 + x^3$. Find x, when the total cost is minimum.

61. If $z = e^{x^3 + y^3}$, then prove that $x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y} = 3z \log z$ (Use Euler's theorem).

62. Evaluate : $\displaystyle\int_0^3 \dfrac{\sqrt{x}}{\sqrt{x} + \sqrt{3-x}}\, dx$

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63. The demand and supply functions under pure competition are $P_d = 16 - x^2$ and $P_s = 2x^2 + 4$. Find the consumers' surplus and producers' surplus at the market equilibrium price.

64. Solve : $\dfrac{dy}{dx} = \dfrac{xy}{x^2 - y^2}$.

65. Using Lagrange's formula find y, when $x = 4$ from the following table :

x	0	3	5	6	8
y	276	460	414	343	110

66. Suppose that the life in hours of a certain part of radio tube is a continuous random variable X with p.d.f. given by

$$f(x) = \begin{cases} \dfrac{100}{x^2}; & \text{when } x \geq 100 \\ 0; & \text{elsewhere} \end{cases}$$

   i) What is the probability that all of three such tubes in a given radio set will have to be replaced during the first of 150 hours of operation ?

   ii) What is the probability that none of three of the original tubes will have to be replaced during that first 150 hours of operation ?

67. Find the probability that at most 5 defective bolts will be found in a box of 200 bolts, if it is known that 2% of such bolts are expected to be defective. $\left(e^{-4} = 0.01832\right)$

68. The mean I.Q. of a sample of 1600 children was 99. Is it likely that this was a random sample from a population with mean I.Q. 100 and Standard Deviation 15 ? (Test at 5% level of significance)

69. Obtain the two regression lines from the following :

X	6	2	10	4	8
Y	9	11	5	8	7

70. From the following data, calculate price index number by —

a) Laspeyre's method

b) Paasche's method

c) Fisher's method.

Commodity	Base Year		Current Year	
	Price	Quantity	Price	Quantity
A	2	40	6	50
B	4	50	8	40
C	6	20	9	30
D	8	10	6	20
E	10	10	5	20

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A	2	40	6	50
B	4	50	8	40
C	6	20	6	30
D	8	10	6	20
E	10	10	5	20