OCR
Oxford Cambridge and RSA

Level 3 Cambridge Technical in Engineering

05823/05824/05825

Unit 23: Applied mathematics for engineering

Sample Assessment Material

Date - Morning/Afternoon

Time allowed: 2 hours

You must have:

- the formula booklet for Level 3 Cambridge Technical in Engineering (inserted)
- a ruler (cm/mm)
- a scientific calculator

First Name		Last Name	
Centre Number		Candidate Number	
Date of Birth			

INSTRUCTIONS

- Use black ink. You may use an HB pencil for graphs and diagrams.
- Complete the boxes above with your name, centre number, candidate number and date of birth.
- Answer **all** the questions.
- Write your answer to each question in the space provided. Additional paper may be used if required but you must clearly show your candidate number, centre number and question number(s).
- The acceleration due to gravity is denoted by g ms^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$

INFORMATION

- The total mark for this paper is **80**.
- The marks for each question are shown in brackets **[ ]**.
- Where appropriate, your answers should be supported with working.

  Marks may be given for a correct method even if the answer is incorrect.
- An answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- This document consists of **20** pages. Any blank pages are indicated.

Answer **all** questions.

1 Fig. 1 shows a metal plate measuring 20cm by 10cm. Four holes, A, B, C and D have been drilled in positions as shown. A straight line has been scribed between the centres of holes A and B. Another line has been scribed between the centres of holes C and D. These two lines cross at point E.

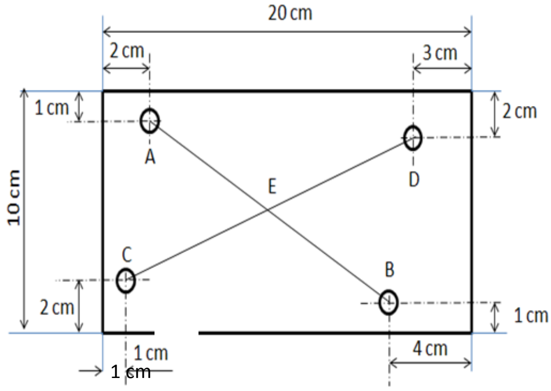


Fig. 1 (Not drawn to scale)

(a) The bottom left-hand corner of the plate is the origin with coordinates $(0, 0)$ within an x-y plane and the coordinates of the centre of hole A are $(2, 9)$.

Determine the coordinates of the centres of holes B, C and D.

B ....

....

C ....

....

D ....

.... **[3]**

(b) The coordinates (X, Y), of point E, are to be determined.

Show that these coordinates satisfy the linear simultaneous equations:

$$3X - 8Y = -13$$
$$4X + 7Y = 71$$

....

....

....

....

....

....

....

.... **[4]**

(c) The simultaneous equation in part **(b)** can be expressed in matrix notation as:

$$\mathbf{A} \,.\, \mathbf{z} = \mathbf{c}$$

where $\mathbf{A} = \begin{bmatrix} 6 & -16 \\ 4 & 7 \end{bmatrix}$, $\mathbf{z} = \begin{bmatrix} X \\ Y \end{bmatrix}$ and $\mathbf{c} = \begin{bmatrix} -26 \\ 71 \end{bmatrix}$

Calculate the inverse matrix A^{-1}.

....

....

....

....

....

.... **[3]**

(d) Calculate the values of X and Y.

....

....

....

....

....

....

....

.... **[4]**

2 Solar panels are sometimes mounted on horizontal roofs. Fig. 2 shows panels tilted at an angle, θ to the roof. To maximise the number of panels, the distance, d, between each panel should be small. However, there is no advantage in allowing d to be very small because each panel will partly obscure the sun's radiation from the panel behind.

Fig. 2 shows three panels of length l separated so that no part of any panel is shaded by the panel in front when the sun has an angle of elevation α with the roof. If the sun is lower in the sky, then each panel will be in partial or total shade.

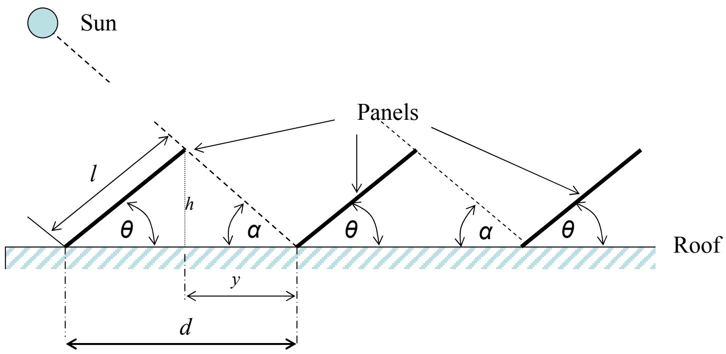


Fig. 2

(a) Show that in Fig. 2 $y = d - l\cos\theta$.

....

....

....

.... **[2]**

(b) Express the height, h, of the left-hand panel in Fig. 2 in terms of:

(i) l and θ

....

.... **[1]**

(ii) y and α.

....

.... **[1]**

(c) The utilisation, u, of the roof space is defined as:

$$u = \frac{l}{d}$$

With reference to parts **(a)** and **(b)** show that:

$$\alpha = \tan^{-1}\left(\frac{u\sin\theta}{1-u\cos\theta}\right)$$

....

....

....

....

....

....

....

.... **[4]**

(d) Calculate the utilisation, u, when $\alpha = 60^{o}$ and $\theta = 45^{o}$.

....

....

....

....

....

....

....

.... **[4]**

3 Fig. 3 shows a diagram of an electrical circuit that has a resistor with resistance R, a capacitor with capacitance C, a constant DC supply voltage, V, and a single-pole two-way switch.

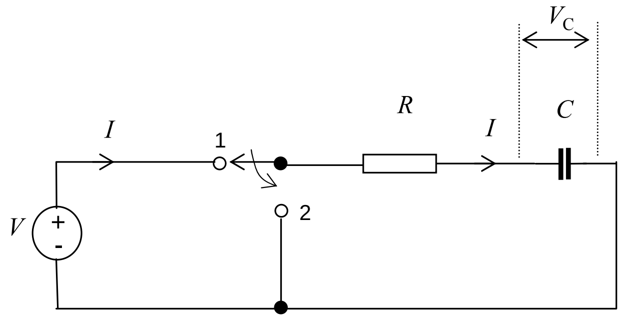


Fig. 3

The differential equation connecting V_C and t is

$$\frac{dV_C}{dt} + \frac{V_C}{RC} = \frac{V}{RC}$$

where t is time and V_C is the potential difference across the capacitor.

(a) By separating variables and integrating show that:

$$V_C = V - Ke^{-t/RC}$$

where K is a constant.

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

.... **[8]**

(b) Assume that in Fig. 3, $C = 5 \times 10^{-6}$, $R = 20 \times 10^{3}$ and $V = 20$. With the switch in position 1 the capacitor is fully charged and V_C = 20.

At time $t = 0$, the switch is then moved to position 2 and the capacitor is allowed to discharge fully through the resistor. The equation that now defines potential difference across the capacitor is:

$$V_C = 20e^{-t/RC}$$

Calculate the time it will take for the potential difference across the capacitor to reduce to 10 volts.

....

....

....

....

....

....

....

....

....

....

....

.... **[6]**

4 Fig. 4 shows a projectile launched at an angle of α^{o} from the edge of a cliff. The cliff is H m above sea level and the initial velocity of the projectile is V_o ms^{-1}.

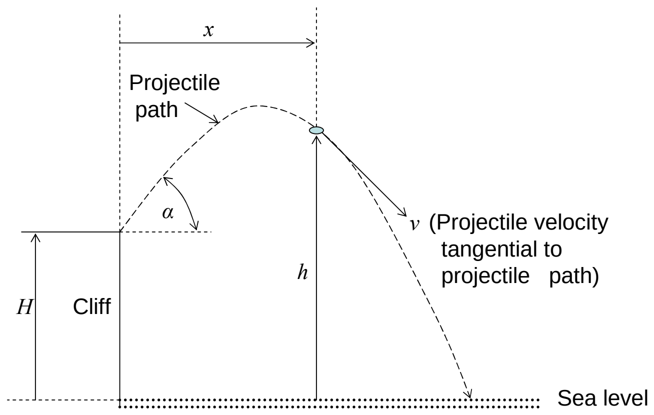


Fig. 4

The height of the projectile above sea level while it is in flight is given by the formula:

$$h = -\frac{gt^2}{2} + V_o t \sin\alpha + H$$

where:

h is the height of the projectile above sea level in metres

t is the time in flight in seconds

g is the acceleration due to gravity in ms^{-2}

For this question assume that g = 9.8.

The horizontal distance, x metres, travelled by the projectile at time, t seconds, after launch is given by the formula:

$$x = V_o t \cos\alpha$$

(a) V_o = 25, α = 45° and H = 20.

(i) Calculate the time in seconds it will take the projectile to reach sea level.

....

....

....

....

....

....

....

.... **[4]**

(ii) Calculate the time after launch that the projectile will reach its maximum height.

....

....

....

....

....

....

....

.... **[4]**

(b) Using the given formulae for h and x, determine a formula in the form $h = \mathrm{f}(x)$ that relates the height of the projectile, h m, to the horizontal distance travelled, x m, while it is in flight.

....

....

....

....

....

.... **[3]**

(c) The projectile is launched at an angle of $\alpha = 45°$ from the top of a 20 m high cliff and reaches sea level at a distance of 100 m from the base of the cliff.

Calculate the speed at which the projectile was launched.

....

....

....

....

....

.... **[3]**

5 Fig. 5 shows the profile of a car testing track. The track has a straight, horizontal section of length 300 m starting at point A and ending at point B. At point B the track continues in a straight line with a constant upward gradient of 20°.

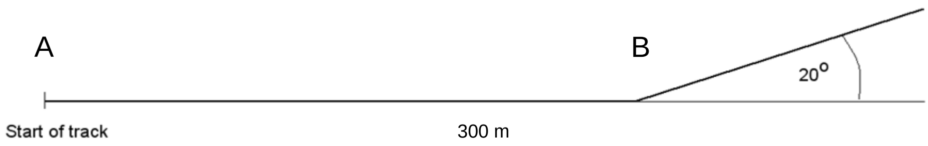


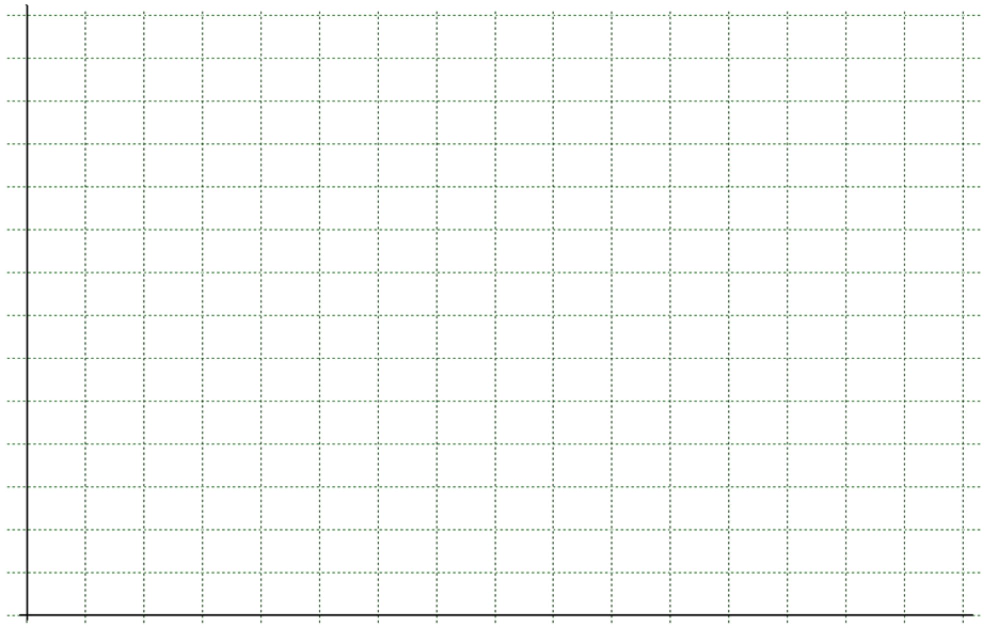
Fig. 5

A car starts from rest at point A and is driven with a constant acceleration of a ms^{-2} until it reaches point B. The time taken, t s, for the car to reach certain distances along the track is recorded in Table 1.

Distance along track (m)	50	100	200	300
Time (s)	6.12	8.66	12.25	15

Table 1

(a) (i) Use the information in Table 1 to draw a graph showing time on the horizontal axis and distance travelled on the vertical axis.



[2]

(ii) Use your graph to estimate the time it will take the car to travel along the last 150 m of the horizontal track to point B.

....

....

....

.... **[2]**

(iii) Calculate the constant acceleration of the car, a ms^{-2}.

....

....

....

....

....

.... **[3]**

(iv) Calculate the speed of the car when it reaches point B.

....

....

....

.... **[2]**

(b) An object of mass m kg, travelling at v ms^{-1} at a height of h m above a fixed origin has kinetic energy and relative potential energy as follows:

- Kinetic Energy = $\frac{1}{2} m v^2$
- Relative Potential Energy = $m g h$
- where g ms^{-2} is acceleration due to gravity.

You may assume that $g = 9.8$.

When the car reaches point B at the speed calculated in part **(a) (iv),** the engine is turned off and the car is allowed to "freewheel" as it continues in a straight line up the gradient.

Assume that no energy is lost due to aerodynamic drag or other forms of friction while the car is on its journey. By equating the initial energy of the car with its final energy calculate the distance the car will travel along the gradient before it comes to a complete rest.

....

....

....

....

....

....

....

....

....

.... **[5]**

6 A plane figure, bounded by the curve $y=f(x)$ and where the x-axis ordinates are $x = a$ and $x = b$, rotates completely about its y-axis.

The volume V generated is given by:

$$V = 2\pi \int_a^b xy \mathrm{d}x .$$

A metal component shown in Fig. 6a is to be turned on a CNC lathe. The top curved surface, shown in the cross-section in Fig. 6b, is generated by the curve with equation:

$$y = e^{x/4} \text{ for } 1 \le x \le 2 .$$

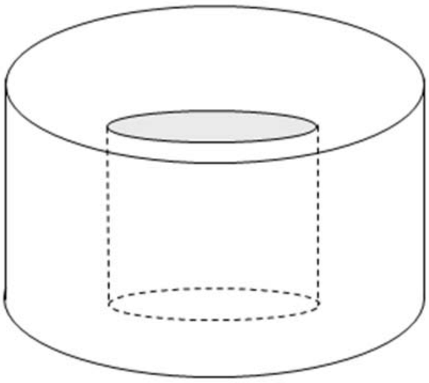


Fig. 6a

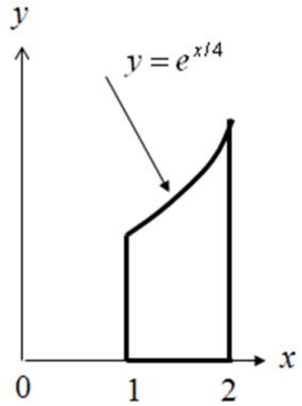


Fig. 6b

Calculate the volume of material in the finished component.

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

.... **[12]**

END OF QUESTION PAPER

BLANK PAGE

PLEASE DO NOT WRITE ON THIS PAGE

PLEASE DO NOT WRITE ON THIS PAGE

Oxford Cambridge and RSA

SPECIMEN

Sample Assessment Material
Level 3 Cambridge Technicals in Engineering

UNIT 23: Applied mathematics for engineering

MARK SCHEME

Duration: 2 hours

MAXIMUM MARK 80

This document consists of 11 pages

Question			Answer	Marks	Guidance
1	(a)		B: (16, 1) C: (1, 2) D: (17, 8)	1 1 1	Correct answers only (All three marks are gained for the application of knowledge from Unit 1 LO2 how to use co-ordinate geometry)
1	(b)		$\frac{X-1}{Y-2}=\frac{17-1}{8-2}=\frac{8}{3}$ $3X-3=8Y-16$; $3X-8Y=-13$; $\frac{X-16}{Y-1}=\frac{2-16}{9-1}=-\frac{7}{4}$ $4X-64=-7Y+7$; $4X+7Y=71$	1 1 1 1	Allow Error Carried Forward (ECF) (The latter three marks are gained for the application of knowledge from Unit 1 LO1 how to solve linear simultaneous equations with two unknowns)
1	(c)		Det(**A**) = 3 × 7 – (– 8)×4 = 53 $\mathbf{A}^{-1}=\frac{1}{53}\begin{bmatrix}7 & 8\\ -4 & 3\end{bmatrix}$	1 2	1 mark for $\begin{bmatrix}7 & 8\\ -4 & 3\end{bmatrix}$ plus 1 mark for determinant
1	(d)		$\begin{bmatrix}X\\ Y\end{bmatrix}=\frac{1}{53}\begin{bmatrix}7 & 8\\ -4 & 3\end{bmatrix}\begin{bmatrix}-13\\ 71\end{bmatrix}=$ $\begin{bmatrix}9\\ 5\end{bmatrix}$ Coordinates of point E are (9, 5).	4	Allow ECF. (1) (1) (1) $\begin{bmatrix}X\\ Y\end{bmatrix}=\frac{1}{53}\begin{bmatrix}7 & 8\\ -4 & 3\end{bmatrix}\begin{bmatrix}-13\\ 71\end{bmatrix}=$ $\begin{bmatrix}9\\ 5\end{bmatrix}$ (1) Accept solution by matrix methods, elimination or substitution.

Question			Answer	Marks	Guidance
2	(a)		$\cos\theta = \dfrac{d-y}{l}$ $l\cos\theta = d-y$ $y = d - l\cos\theta$	1 1	
2	(b)	(i)	$h = l\sin\theta$	1	Correct answer only
2	(b)	(ii)	$h = y\tan\alpha$	1	Correct answer only
2	(c)		$\tan\alpha = \dfrac{h}{y} = \dfrac{l\sin\theta}{d - l\cos\theta}$ $= \dfrac{l\sin\theta}{d(1 - l\cos\theta/d)}$ $= \dfrac{l/d\sin\theta}{1 - l\cos\theta/d} = \dfrac{u\sin\theta}{1 - u\cos\theta}$ $\alpha = \tan^{-1}\left(\dfrac{u\sin\theta}{1 - u\cos\theta}\right)$	1 1 1 1	

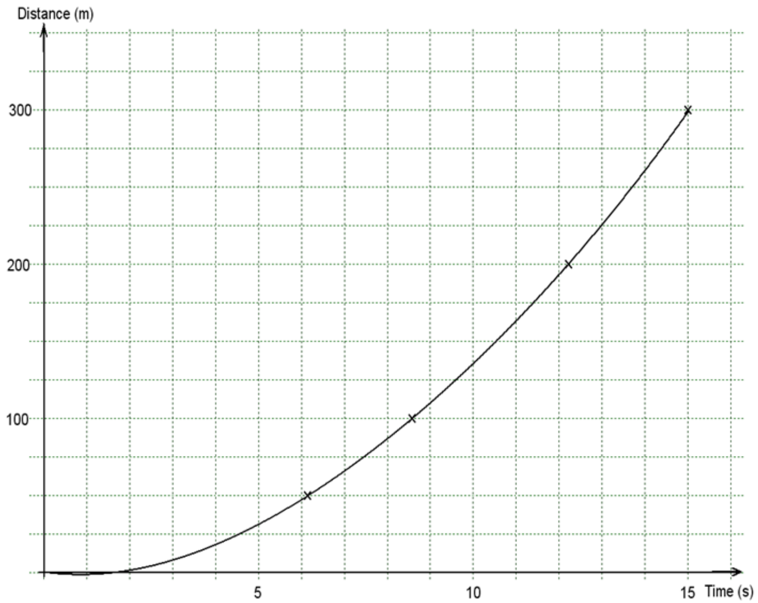
Question			Answer	Marks	Guidance
2	(d)		$\tan\alpha = \left(\dfrac{u\sin\theta}{1-u\cos\theta}\right)$		
			$\tan\alpha(1-u\cos\theta) = u\sin\theta$		
			$\tan\alpha - u\cos\theta\tan\alpha = u\sin\theta$	1	
			$\tan\alpha = u\cos\theta\tan\alpha + u\sin\theta$	1	
			$\tan\alpha = u(\cos\theta\tan\alpha + \sin\theta)$		
			$u = \tan\alpha/(\cos\theta\tan\alpha + \sin\theta)$	1	
			$u = \tan 60/(\cos 45 \tan 60 + \sin 45) = 0.8966$	1	

Question			Answer	Marks	Guidance
3	(a)		$\frac{dV_c}{dt}+\frac{V_c}{RC}=\frac{V}{RC}$	1	Award one mark for each correct step as shown, allowing for ECF.
			$RC\frac{dV_c}{dt}+V_c=V$	1	
			$RC\frac{dv_c}{dt}=V-V_c$	1	
			$\int\frac{RC}{V-V_c}\mathrm{d}V_c=\int\mathrm{d}t$	1	
			$-RC\ln(V-V_c)=t+A$	1	
			$\ln(V-V_c)=-\frac{t}{RC}+B$	1	
			$V-V_c=e^{-\frac{t}{RC}+B}=e^{-\frac{t}{RC}}e^B=Ke^{-\frac{t}{RC}}$	1	
			$V_c=V-Ke^{-t/RC}$	1	

Question			Answer	Marks	Guidance
3	(b)		$V_c = 20e^{-t/RC}$		Award marks for correct steps as shown allowing for ECF.
			$20e^{-t/RC} = 10$	1	
			$e^{-t/RC} = 0.5$	1	
			$-t/RC = \ln 0.5$	1	(These two marks are gained for the application of knowledge from Unit 1 LO3 how to use inverse functions and log laws)
			$-t = RC \ln 0.5$		
			$t = -RC \ln 0.5$	1	
			$t = -20000 \times 5 \times 10^{-6} \ln(0.5)$	1	
			$= \dfrac{-\ln(0.5)}{10} = 69.3147\text{ms}$	1	

Question			Answer	Marks	Guidance
4	(a)	(i)	$-\frac{gt^2}{2}+V_o t\sin\alpha+H=0$ $-\frac{9.8t^2}{2}+25t\sin 45+20=0$ $-4.9t^2+17.678+20=0$ $t=\frac{-17.678\pm\sqrt{(17.678)^2-4\times(-4.9)\times 20}}{2\times(-4.8)}$ $t=4.512$ seconds	1 1 1 1	
4	(a)	(ii)	$h=-\frac{gt^2}{2}+V_o t\sin\alpha+H$ $\frac{dh}{dt}=-gt+V_0\sin\alpha$ For stationary point $\frac{dh}{dt}=-gt+V_0\sin\alpha=0$ $t=\frac{V_0\sin\alpha}{g}=\frac{25\sin 45}{9.8}=1.804\text{ s}$	1 1 1 1	

Question			Answer	Marks	Guidance
4	(b)		$x = V_o t \cos\alpha$ $t = \dfrac{x}{V_o \cos\alpha}$	1	
			Substitute *t* in $h = -\dfrac{gt^2}{2} + V_o t \sin\alpha + H$	1	
			$h = -\dfrac{g}{2}\left(\dfrac{x}{V_o \cos\alpha}\right)^2 + x\tan\alpha + H$	1	
4	(c)		$-\dfrac{g}{2}\left(\dfrac{x}{V_o \cos\alpha}\right)^2 + x\tan\alpha + H = 0$ $-\dfrac{9.8}{2}\left(\dfrac{100}{V_o \cos 45}\right)^2 + 100\tan 45 + 20 = 0$	1	
			$\dfrac{-98000}{V_o^2} + 100 + 20 = 0$	1	
			$V_0 = \sqrt{\dfrac{98000}{120}} = 28.577$	1	

Question			Answer	Marks	Guidance
5	(a)	(i)		1 1	Axis scale and labels General shape trough given points
5	(a)	(ii)	Time for first 150 m ≈ 10.5 Time for last 150 m = 15 – (their 10.5) = 4.5 s	 1 1	Accept values between 10.25 and 10.75 Allow ECF
5	(a)	(iii)	$a = 2\,s/t^2$ $= 2 \times 300/15^2 =$ 2.66.. m s^{-2}	1 1 1	Accept any correct pair of *s* and *t*
5	(a)	(iv)	$v = at =$ 2.66×15 = 40 m s^{-1}	1 1	Allow ECF

Question			Answer	Marks	Guidance
5	(b)		Kinetic Energy = $m\ v^2/2$ Relative Potential Energy = $m\ g\ h$ When the car comes to rest Final PE = Initial KE		Knowledge of correct formulae can be implied by correct calculations
			$m\ g\ h = m\ v^2/2$	1	
			$h = v^2/2g = (40^2)/(2\times9.8) = 81.63$ m	1	Allow ECF
			Distance along gradient = h / $\sin(\theta)$	1	
			= (81.63)/sin(20)	1	Accept (80.81)/sin(20) OR
			= 238.68m	1	236.27

Question			Answer	Marks	Guidance
6			$\int xe^{x/4}\mathrm{d}x \quad \int uv' = uv - \int vu'$	1	Integral requires integration by parts
			$\int xe^{x/4}\mathrm{d}x = x4e^{x/4} -$	2	
			$\int 4e^{x/4}\mathrm{d}x$	2	
			$= 4xe^{x/4} - 16e^{x/4}$ $= 4e^{x/4}(x-4)$	1 1	
			$V = 2\pi\int_1^2 x\ e^{x/4}\mathrm{d}x = 2\pi\left[4e^{x/4}(x-4)\right]_1^2$	2	
			$= 2\pi\left[(4e^{1/2}(-2)) - (4e^{1/4}(-3))\right]$	2	
			$= 2\pi\left[-8e^{1/2} + 12e^{1/4}\right] = 2\pi \times 2.219$ =13.939 cubic units	1	