

Cambridge Technicals Engineering

Unit 23: Applied mathematics for engineering

Level 3 Cambridge Technical Certificate/Diploma in Engineering
05823 - 05825 & 05873

Mark Scheme for June 2022

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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MARKING INSTRUCTIONS

PREPARATION FOR MARKING

TRADITIONAL

Before the Standardisation meeting you must mark at least 10 scripts from several centres. For this preliminary marking you should use **pencil** and follow the **mark scheme**. Bring these **marked scripts** to the meeting.

MARKING

1. Mark strictly to the mark scheme.
2. Marks awarded must relate directly to the marking criteria.
3. The schedule of dates is very important. It is essential that you meet the traditional 40% Batch 1 and 100% Batch 2 deadlines. If you experience problems, you must contact your Team Leader (Supervisor) without delay.
4. If you are in any doubt about applying the mark scheme, consult your Team Leader by telephone or by email.
5. **Crossed Out Responses**
Where a candidate has crossed out a response and provided a clear alternative then the crossed out response is not marked. Where no alternative response has been provided, examiners may give candidates the benefit of the doubt and mark the crossed out response where legible.

Rubric Error Responses – Optional Questions

Where candidates have a choice of questions across a whole paper or a whole section and have provided more answers than required, then all responses are marked and the highest mark allowable within the rubric is given. (The underlying assumption is that the candidate has penalised themselves by attempting more questions than necessary in the time allowed.)

Multiple Choice Question Responses

When a multiple choice question has only a single, correct response and a candidate provides two responses (even if one of these responses is correct), then no mark should be awarded (as it is not possible to determine which was the first response selected by the candidate). When a question requires candidates to select more than one option/multiple options, then local marking arrangements need to ensure consistency of approach.

Contradictory Responses

When a candidate provides contradictory responses, then no mark should be awarded, even if one of the answers is correct.

Short Answer Questions (requiring only a list by way of a response, usually worth only **one mark per response**)

Where candidates are required to provide a set number of short answer responses then only the set number of responses should be marked. The response space should be marked from left to right on each line and then line by line until the required number of responses have been considered. The remaining responses should not then be marked. Examiners will have to apply judgement as to whether a 'second response' on a line is a development of the 'first response', rather than a separate, discrete response. (The underlying assumption is that the candidate is attempting to hedge their bets and therefore getting undue benefit rather than engaging with the question and giving the most relevant/correct responses.)

Short Answer Questions (requiring a more developed response, worth **two or more marks**)

If the candidates are required to provide a description of, say, three items or factors and four items or factors are provided, then mark on a similar basis – that is downwards (as it is unlikely in this situation that a candidate will provide more than one response in each section of the response space.)

Longer Answer Questions (requiring a developed response)

Where candidates have provided two (or more) responses to a medium or high tariff question which only required a single (developed) response and not crossed out the first response, then only the first response should be marked. Examiners will need to apply professional judgement as to whether the second (or a subsequent) response is a 'new start' or simply a poorly expressed continuation of the first response.

6. Always check the pages (and additional lined pages if present) at the end of the response in case any answers have been continued there. If the candidate has continued an answer there then add an annotation to confirm that the work has been seen.
7. There is a NR (No Response) option. Award NR (No Response)
 - if there is nothing written at all in the answer space
 - OR if there is a comment which does not in anyway relate to the question (e.g. 'can't do', 'don't know')
 - OR if there is a mark (e.g. a dash, a question mark) which isn't an attempt at the questionNote: Award 0 marks - for an attempt that earns no credit (including copying out the question)
8. Assistant Examiners will email a brief report on the performance of candidates to your Team Leader (Supervisor) by the end of the marking period. Your report should contain notes on particular strength displayed as well as common errors or weaknesses. Constructive criticism of the question paper/mark scheme is also appreciated.

9. Annotations

Annotation	Meaning
✓ and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
DM1	Method mark dependent on previous M mark
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
λ	Omission sign
Other abbreviations in mark scheme	Meaning
oe	Or equivalent
Soi	Seen or implied
www	Without wrong working
ecf	Error carried forward

10. Subject-specific marking instructions

Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. These annotations must be in the body of the work and **not** anywhere near the right hand margin of each page.

Mark in using a red pen.

Put the mark for each subquestion near to and to the right of the mark for the question. Total all marks for the question and put this total in a ring at the bottom right of each question.

Transfer these marks to the box on the front page.

Total the marks for the paper. I suggest that all unringed marks are then totalled to make sure that the final mark is correct.

An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

DM

A method mark which is dependent on a previous method mark.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

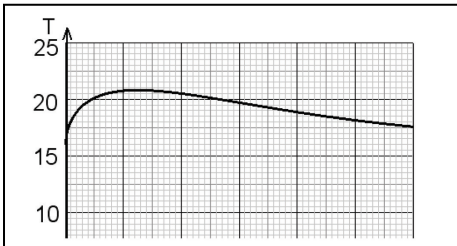
Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

Question	Answer	Marks	Guidance
1 (i)	Attempt angle between board/leg: $\sin \alpha = \frac{0.8}{1.1}$ [$\alpha = 46.7$] OR Attempt angle between vertical/leg: $\cos \phi = \frac{0.8}{1.1}$ [$\phi = 43.3$] $\theta = 180 - 2 \times \text{their } 46.7$ OR $\theta = 2 \times \text{their } 43.3$ $= 86.7 (^{\circ})$	M1 M1 A1	Must be attempting to use appropriate right angled Δ for first method mark Second M1 available for obtaining θ from their angle ie M0M1 possible 3sf or better
		3	
1 (ii)	$AC^2 = 0.4^2 + 0.4^2 - 2(0.4)(0.4)\cos \theta$ $AC = \sqrt{0.4^2 + 0.4^2 - 2(0.4)(0.4)\cos 86.7}$ $= 0.549$ (m)	M1 M1 A1ft	Appropriate substitutions with cosine rule OR sine rule Solved using their θ to obtain distance AC 2dp or better Award A1 if correct follow through from their (i)
		3	
1 (iii)	$AQ_2 = \text{their } 0.549 + 0.07$ [= 0.619] $\cos \theta = \frac{2 \times 0.4^2 - 0.6191^2}{2 \times 0.4^2}$ $\theta = 101.4$ $\alpha = (180 - \text{their new } \theta)/2$ [= 39.3] Minimum $h = 1.1 \sin \alpha = 0.697$ (m) State Q_2	M1 M1 A1 M1 A1ft B1	Appropriate substitutions with cosine rule OR sine rule cao 2dp or better Follow through their new α obtained in this part only
		6	
		12	

Question		Answer	Marks	Guidance
2	(i)	$400 \times 9.8 = 0.02 N^2 1.22$ $N = \sqrt{\frac{3920}{0.0244}} = 400.82 \text{ (rpm)}$	M1 A1	AWRT 401
			2	
2	(ii)	Net force = $0.02 \times 450^2 \times 1.22 - 400 \times 9.8 = 1021 \text{ (N)}$ Use of $F=ma$ $a = 1021/400 = 2.55 \text{ (ms}^{-2}\text{)}$	B1 M1 A1	B1 for the upward element Must use net force now Accept s3f
			3	
2	(iii)	At 2500 m the density of air has reduced to $1.22 - 0.032 \times 10 = 0.9 \text{ kg m}^{-3}$ $400 \times 9.8 = 0.02 N^2 0.9$ $N = \sqrt{((400 \times 9.8)/(0.02 \times 0.9))} \approx 466.66... \text{ (rpm)}$	B1 M1 A1	First mark is for the correct density of air soi (may be amongst other incorrect work) Accept 3sf
			3	
2	(iv)	$400 \times 9.8 = 0.02 \times 530^2 \times \rho$ $\rho = 0.6977$ $\rho_{diff} = 1.22 - 0.6977 = 0.522$ $h = 0.522/0.032 \times 250 = 4080$	M1 A1 M1 A1	awrt 0.7 Accept awrt 4080
			4	
			12	

Question		Answer	Marks	Guidance
3	(i)	$m_1 g \sin \theta_1 + m_2 g \sin \theta_2 = 30g$ $m_1 g \cos \theta_1 - m_2 g \cos \theta_2 = 0$	B3	Allow equations with or without g on both sides. 1 mark for each reasonable attempt at equations. Each must include m_1 and m_2 and +/- sin/cos of both angles 1 mark for fully correct equations.
			3	
3	(ii)	$0.7071m_1 + 0.5 m_2 = 30$ $0.7071m_1 - 0.866m_2 = 0$ $1.366 m_2 = 30$ $m_2 = 30/1.366 = \mathbf{21.961..}$ $m_1 = (30 - 0.5 * 21.961) / 0.7071 = \mathbf{26.897..}$	B1 M1 A1	All trig substitutions correct soi by later working or answers Attempt to solve their equations (elimination, trial and error, matrix solution) Accept 2sf here SCB3 correct answers without working or incomplete working
			3	
3	(iii)	$25 \sin 45 + Q \sin \phi = 30; \quad 25 \cos 45 - Q \cos \phi = 0$ $Q \sin \phi = 30 - \frac{25}{\sqrt{2}}; \quad Q \cos \phi = \frac{25}{\sqrt{2}}$ $\tan \phi = (30\sqrt{2} - 25) / 25$ $\phi = 34.88^\circ$ $Q = 21.548 \text{ (kg)}$	B1,B1 M1 M1 A1 A1	Note: $\phi = \tan^{-1}(0.6971)$ awrt 35 awrt 21.5
			6	
			12	

Question	Answer	Marks	Guidance						
4 (a)	$\int_{0_0}^{T_1} I(t)dt = \int_0^{10} 15e^{-t/10} dt$ $15[-10e^{-t/10}]_0^{10}$ $-150(e^{-1} - 1) =$ $94.818... \text{ (Ah)}$	B1 M1 M1 A1	Integration set up with limits soi Integrate to obtain appropriate form Apply limits awrt 95 SCB4 correct answers without working or incomplete working						
		4							
4 (b) (i)	<table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td style="padding: 2px;">Discharge current I</td> <td style="padding: 2px; text-align: center;">3</td> <td style="padding: 2px; text-align: center;">7</td> </tr> <tr> <td style="padding: 2px;">Discharge time t</td> <td style="padding: 2px; text-align: center;">29.53</td> <td style="padding: 2px; text-align: center;">10.68</td> </tr> </table>	Discharge current I	3	7	Discharge time t	29.53	10.68	B2	awrt 29.5, awrt 10.7
Discharge current I	3	7							
Discharge time t	29.53	10.68							
		2							
4 (b) (ii)	$9 = 16 \left(\frac{80}{8 \times 16} \right)^k$ $\log 9 = \log 16 + k \log \left(\frac{5}{8} \right)$ $k = \frac{\log 9 - \log 16}{\log 0.625} = \frac{\log 0.5625}{\log 0.625} =$ $1.224... = 1.22$	M1 M1 A1	Accept any valid alternative method to solve for k Accept any logarithms method, such as: $\log_{5/8} \frac{9}{16}$						
		3							
		9							

Question		Answer	Marks	Guidance										
5	(i)	<table border="1"> <tr> <td>Time (t hours)</td> <td>0</td> <td>2</td> <td>6</td> <td>12</td> </tr> <tr> <td>Room Temperature ($T^\circ\text{C}$)</td> <td>16 given</td> <td>20.74</td> <td>19.69</td> <td>17.57</td> </tr> </table>	Time (t hours)	0	2	6	12	Room Temperature ($T^\circ\text{C}$)	16 given	20.74	19.69	17.57	B1,B1,B1	Allow 20.7, 19.7, 17.6 SC B2 all correct to 2sf
		Time (t hours)	0	2	6	12								
Room Temperature ($T^\circ\text{C}$)	16 given	20.74	19.69	17.57										
		3												
5	(ii)	$T = 16 + tq = 16 + \frac{2500 e^{-t/5} \sqrt{t}}{500} = 16 + 5e^{-t/5} \sqrt{t} = 16 + 5e^{-t/5} t^{1/2}$	B1	Correct T as a function of t only, allow unsimplified										
		$\frac{dT}{dt} = 5 \left(\frac{e^{-t/5} t^{-1/2}}{2} - \frac{e^{-t/5} t^{1/2}}{5} \right)$	M1,M1	Differentiation of $e^{-\frac{t}{5}}$ to appropriate form Differentiation of \sqrt{t} to appropriate form										
		$\left(\frac{e^{-t/5} t^{-1/2}}{2} - \frac{e^{-t/5} t^{1/2}}{5} \right) = 0$	A1	Correct derivative, any equivalent form										
		$\left(t = \frac{5}{2} = 2.5 \right)$	M1	Set their derivative to zero										
		$T = 16 + 5e^{-t/5} t^{1/2} = 16 + 5e^{-1/2} \sqrt{2.5} \approx 20.8$	M1 A1	Solve using legitimate process to obtain a value for t T correct www, allow 2sf										
			7											
5	(iii)		B1	Correct shape starting at (0,16)										
			B1	Suitable scales and with max close to $t = 2$										
			B1	All correct with max at $t = 2.5$, no FT from ii										
			3											
			13											

Question		Answer	Marks	Guidance	
6	(i)	Example: Using $\cos(2A) = \cos^2 A - \sin^2 A$ $\cos(2A) = (\cos^2 A + \sin^2 A) - 2\sin^2 A$ $2\sin^2 A = 1 - \cos(2A)$ $\sin^2 A = \frac{1}{2}(1 - \cos 2A)$	B1 M1 A1	Note: allow development of any appropriate proof Any relevant identity substituted into given expression or any standard identity manipulated as a starting point Simplification towards given answer Convincing completion	
			3		
6	(ii)	(A)	Clear use of $f = 50$ To obtain given answer $T = 1/50$	B1 1	May be implied by sight of ratio $100\pi : 2\pi$
6	(ii)	(B)	$RMS = \sqrt{\frac{1}{1/50} \int_0^{1/50} 330^2 \sin^2 100\pi t \, dt}$ $= \sqrt{\frac{1}{1/50} \int_0^{1/50} 330^2 \frac{1}{2}(1 - \cos 200\pi t) \, dt}$ $\frac{330}{\sqrt{2}} \sqrt{50 \left[t - \frac{\sin 200\pi t}{200\pi} \right]_0^{1/50}}$ $\frac{330}{\sqrt{2}} \sqrt{50\{(1/50 - 0) - (0 - 0)\}}$ $\frac{330}{\sqrt{2}} (\approx 233) \text{ (V)}$	B1 M1* M1* A1 DM1 A1	Correct substitution of V, ω, T into given expression Substitute identity $\sin^2 A = \frac{1}{2}(1 - \cos 2A)$ from (i), may be amongst other errors, and condone $A \neq 100\pi$ Integrate <i>their</i> $\cos mt$ and obtain form $\frac{1}{m} \sin mt$ Integration fully correct Limits applied with clear sight of $\sin 2n\pi = 0$ Final answer awrt 230
			6		
			10		

Question	Answer	Marks	Guidance
7 (i)	$v = -10i + 15j$ $\sqrt{(-10)^2 + (15)^2} \approx 18.03$ (miles/hour)	M1 A1	Velocity vector soi Allow 2sf Any equivalent form including exact surd eg $\sqrt{325}$
		2	
7 (ii)	$(10 - 10ti) + 15tj - (5ti + 20tj)$ $(10-15t)i + (-5t)j$	M1 A1	<u>Subtraction</u> of position vectors (either way round) soi
		2	
7 (iii)	Allow restarts here for full marks $\sqrt{(-5)^2 + (-5)^2} \approx 7.071$ (miles)	M1 A1ft	Substitute $t = 1$ into their relative position vector, or into both position vectors FT from their ii
		2	
7 (iv)	Separation = $\sqrt{(-15t + 10)^2 + (-5t)^2}$ Separation ² = $(-15t + 10)^2 + (-5t)^2$ $\frac{dS}{dt} = \frac{1}{2}(250t^2 - 300t + 100)^{-1/2} \times (500t - 300)$ $t = \frac{3}{5} = 0.6 = (36 \text{ min})$ Distance = $\sqrt{(-15 \times 0.6 + 10)^2 + (-5 \times 0.6)^2}$ $\sqrt{1+9} = \sqrt{10} \approx 3.162$ (miles)	M1 A1 M1 A1 M1 A1	Form appropriate expression for distance or distance squared, using their relative position vector from ii Correct expression for either, unsimplified Note: $S = \sqrt{250t^2 - 300t + 100}$ Differentiate expression of correct form using chain rule Set their derivative to zero and obtain a value for t eg $(500t - 300) = 0$ Accept surd or 2sf
		6	
		12	

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