Level 3 Cambridge Technical in Engineering

05822/05823/05824/05825/05873

Formula Booklet

Unit 1	**Mathematics for engineering**
Unit 2	**Science for engineering**
Unit 3	**Principles of mechanical engineering**
Unit 4	**Principles of electrical and electronic engineering**
Unit 23	**Applied mathematics for engineering**

This booklet contains formulae which learners studying the above units and taking associated examination papers may need to access.

Other relevant formulae may be provided in some questions within examination papers. However, in most cases suitable formulae will need to be selected and applied by the learner. Clean copies of this booklet will be supplied alongside examination papers to be used for reference during examinations.

Formulae have been organised by topic rather than by unit as some may be suitable for use in more than one unit or context.

Note for teachers

This booklet does not replace the taught content in the unit specifications or contain an exhaustive list of required formulae. You should ensure all unit content is taught before learners take associated examinations.

1. Trigonometry and Geometry

1.1 Geometry of 2D and 3D shapes

1.1.1 Circles and arcs

Circle: radius r

Area of a circle $= \pi r^2$

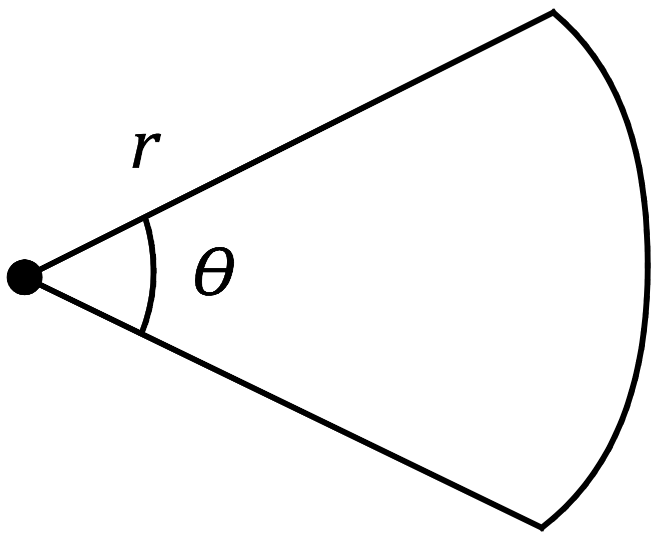
Circumference of a circle $= 2\pi r$

Co-ordinate equation of a circle:

radius r, centre (a, b)

$$(x - a)^2 + (y - b)^2 = r^2$$

Arc and sector: radius r, angle θ



Arc length $= \boldsymbol{\theta r}$,   for θ expressed in radians

Area of sector $= \frac{1}{2}, r^2\theta$ for $\boldsymbol{\theta}$ expressed in radians

Arc length $= \frac{\theta}{180}, \pi r$ for $\boldsymbol{\theta}$ expressed in degrees

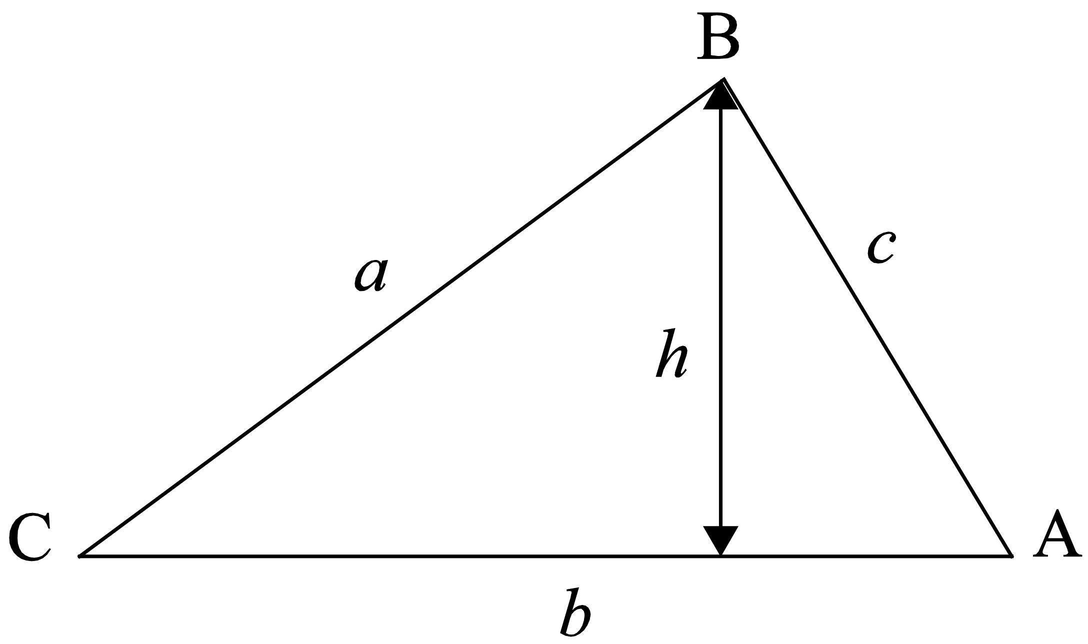
Area of sector $= \frac{\theta}{360}, \pi r^2$ for $\boldsymbol{\theta}$ expressed in degrees

Converting between radians and degrees

$$x \textbf{ radians} = \frac{180x}{\pi} \textbf{ degrees}$$

$$x \textbf{ degrees} = \frac{\pi x}{180} \textbf{ radians}$$

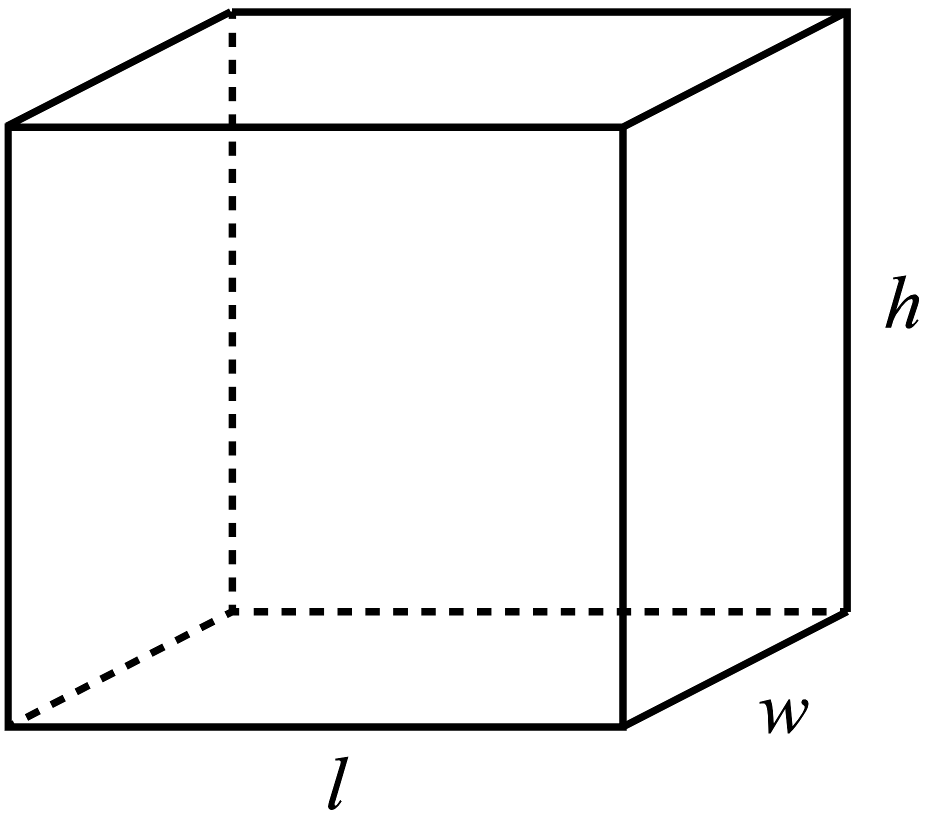
1.1.2 Triangles



$$\textbf{Area} = \frac{1}{2}bh \textbf{ or } \frac{1}{2}bc\sin A$$

1.2 Volume and Surface area of 3D shapes

Cuboid

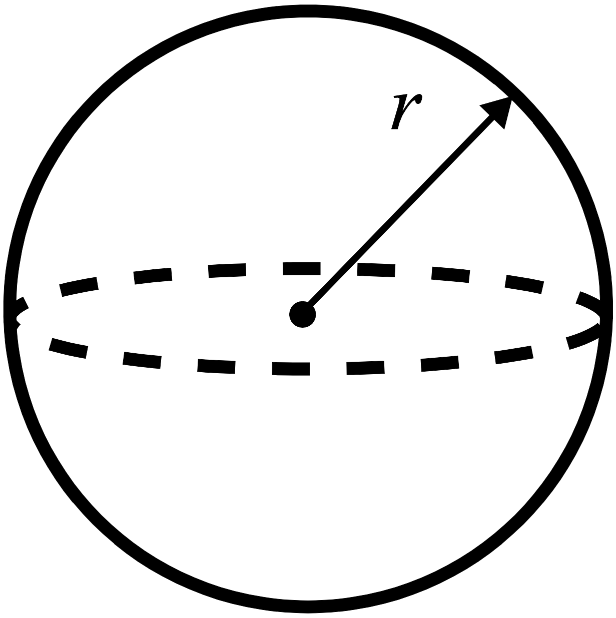


Surface area $= \boldsymbol{2lw + 2wh + 2hl}$

$= \boldsymbol{2(lw + wh + hl)}$

Volume $= \boldsymbol{lwh}$

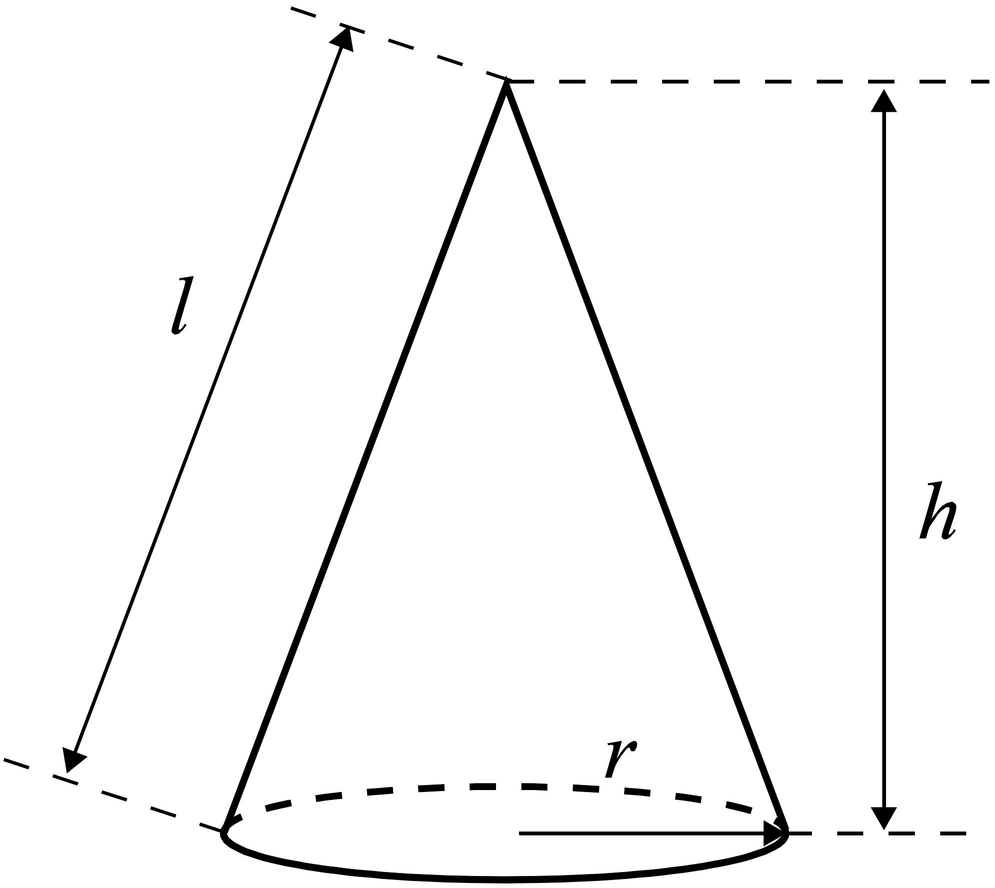
Sphere



Surface area $= \boldsymbol{4\pi r^2}$

Volume $= \boldsymbol{\frac{4}{3}\pi r^3}$

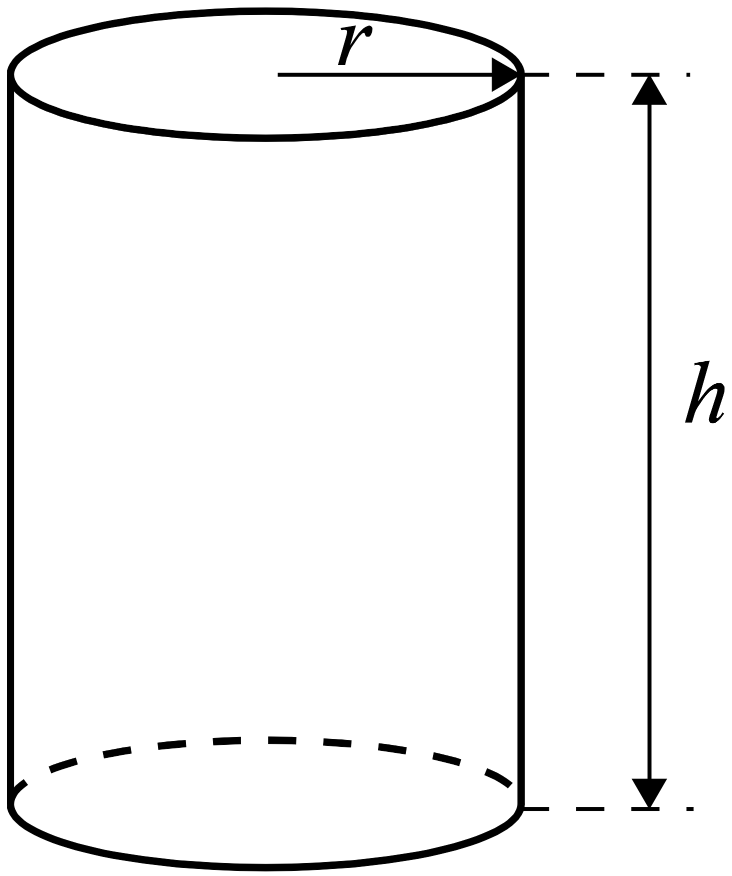
Cone



Surface area $= \boldsymbol{\pi r^2 + \pi r l}$

Volume = $\boldsymbol{\frac{1}{3}\pi r^2 h}$

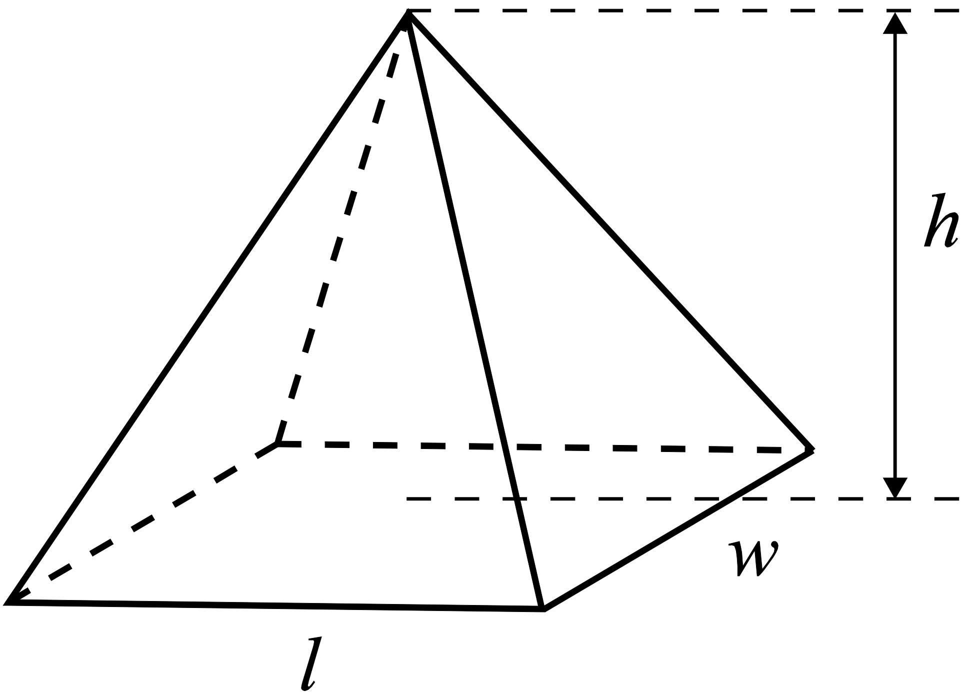
Cylinder



Surface area $= 2\pi r^2 + 2\pi r h$

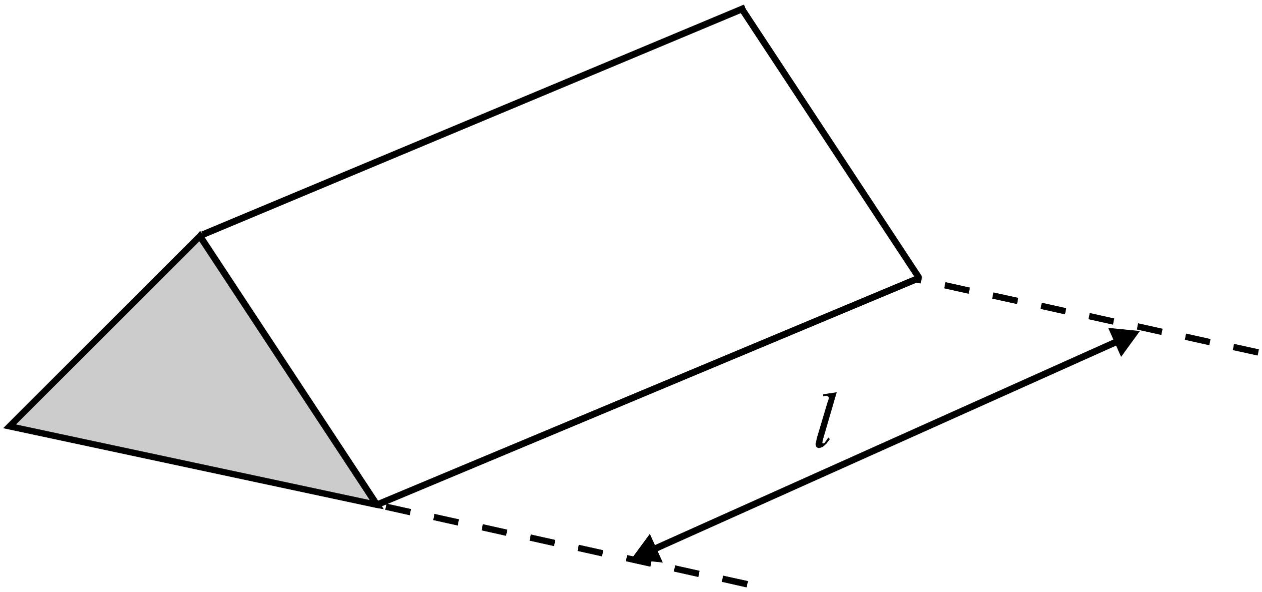
Volume $= \pi r^2 h$

Rectangular Pyramid



Volume $= \dfrac{lwh}{3}$

Prism

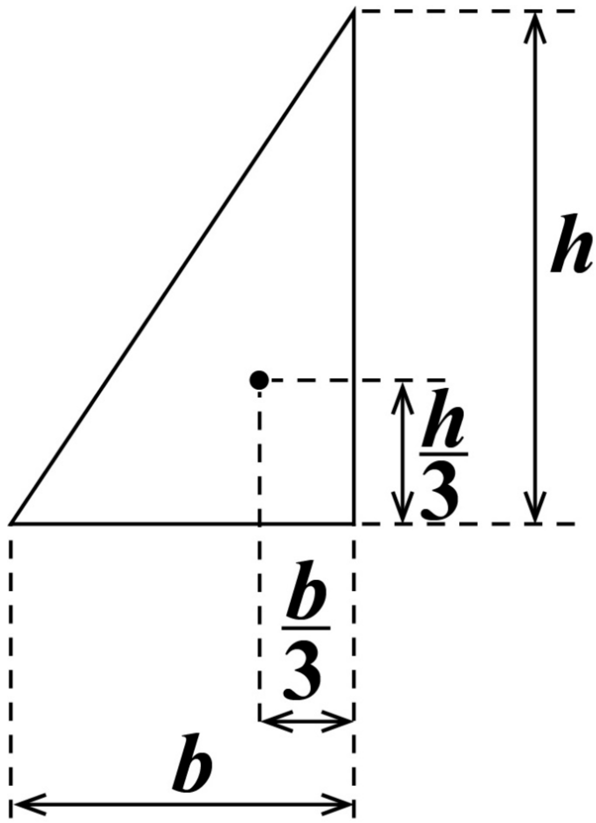


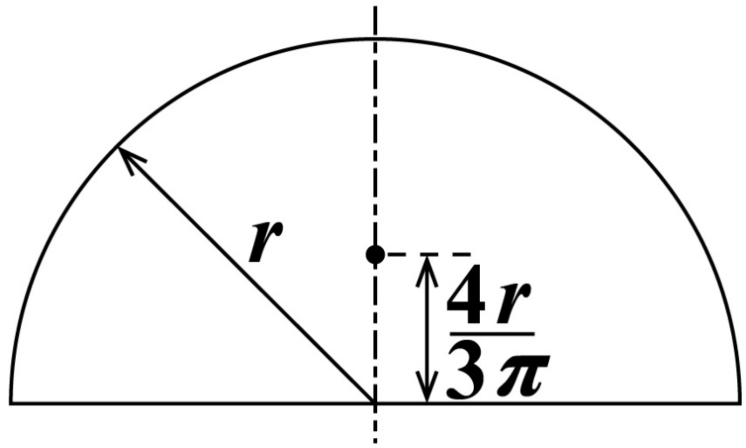
Volume = area of shaded cross-section × l

Density

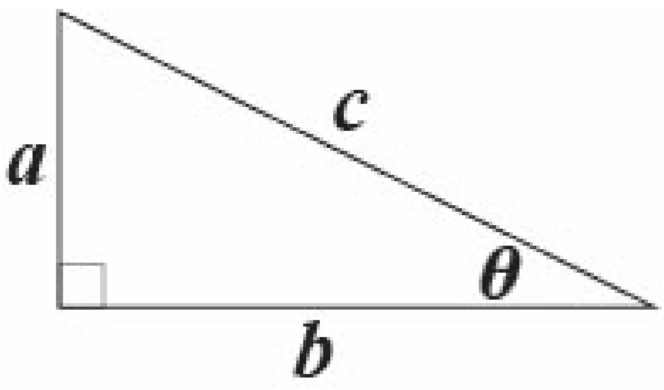
Density = $\dfrac{\text{mass}}{\text{volume}}$

1.3 Centroids of planar shapes





1.4 Trigonometry

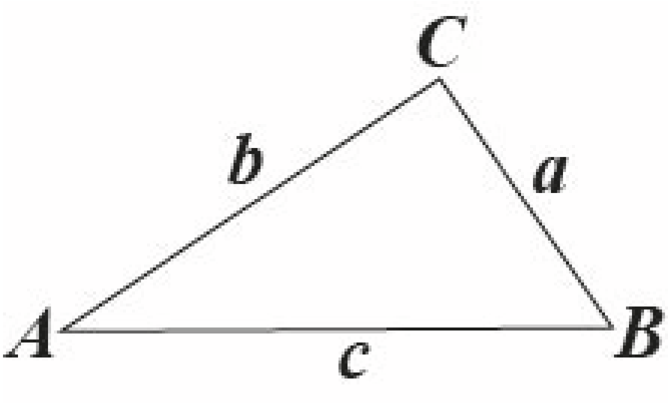


$$\sin\theta = \frac{a}{c}$$

$$\cos\theta = \frac{b}{c}$$

$$\tan\theta = \frac{a}{b}$$

Pythagoras' rule: $c^2 = a^2 + b^2$



Sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

1.4.1 Trigonometric identities

Basic trigonometric values

$$\sin 60° = \frac{\sqrt{3}}{2}$$

$$\cos 60° = \frac{1}{2}$$

$$\tan 60° = \sqrt{3}$$

$$\sin 45° = \cos 45° = \frac{1}{\sqrt{2}}$$

$$\tan 45° = 1$$

$$\sin 30° = \frac{1}{2}$$

$$\cos 30° = \frac{\sqrt{3}}{2}$$

$$\tan 30° = \frac{1}{\sqrt{3}}$$

Trigonometric identities

$\sin A = \cos(90° - A)$ for angle A in degrees

$\cos A = \sin(90° - A)$ for angle A in degrees

$$\sin A = \cos\left(A - \frac{\pi}{2}\right)$$

$$\cos A = -\sin\left(A - \frac{\pi}{2}\right)$$

$$\tan A = \frac{\sin A}{\cos A}$$

$$\sin^2 A + \cos^2 A = 1$$

$$\sin(-A) = -\sin A$$

$$\cos(-A) = \cos A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

2. Calculus

2.1 Differentiation

$\mathrm{f}(x)$	$\dfrac{\mathrm{df}(x)}{\mathrm{d}x}$
c	0
x^{n}	$nx^{n\text{-}1}$
$\sin(ax)$	$a\cos(ax)$
$\cos(ax)$	$a\sin(ax)$
$\tan(ax)$	$-a\sec^{2}(ax)$
e^{ax}	$a\mathrm{e}^{ax}$
$1\mathrm{n}(ax)$	$\dfrac{1}{x}$
a^{x}	$a^{x}\, 1\mathrm{n}\, a$
$\log_{a} x$	$\dfrac{1}{x 1\mathrm{n} a}$

2.1.1 Differentiation of the product of two functions

If $y = u \times v$ $$\frac{\mathrm{d}y}{\mathrm{d}x} = u\frac{\mathrm{d}v}{\mathrm{d}x} + v\frac{\mathrm{d}u}{\mathrm{d}x}$$

2.1.2 Differentiation of the quotient of two functions

If $y = \frac{u}{v}$ $$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$$

2.1.3 Differentiation of a function of a function

If $y = u(v)$ $$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}u}{\mathrm{d}v}\ \frac{\mathrm{d}v}{\mathrm{d}x}$$

2.2 Integration

2.2.1 Indefinite integrals

$\mathrm{f}(x)$	$\int \mathrm{f}(x)\,\mathrm{d}x\ (+c)$
a	ax
x^n for $n \neq -1$	$\frac{x^{n+1}}{n+1}$
$\frac{1}{x}$	$1\mathrm{n}\,\lvert x \rvert$
e^{ax}	$\frac{\mathrm{e}^{ax}}{a}$
a^x	$\frac{a^x}{\ln a}$
$\sin(ax)$	$\frac{-\cos(ax)}{a}$
$\cos(ax)$	$\frac{\sin(ax)}{a}$

2.2.2 Definite integral

$$\int_{a}^{b} \mathbf{f}\,(\boldsymbol{x})\,\mathbf{d}\boldsymbol{x} = [\mathbf{F}(\boldsymbol{x})]_{a}^{b} = \mathbf{F}(\boldsymbol{b}) - \mathbf{F}(\boldsymbol{a})$$

2.2.3 Integration by parts

$$\int \boldsymbol{u}\frac{\mathbf{d}\boldsymbol{v}}{\mathbf{d}\boldsymbol{x}}\,\mathbf{d}\boldsymbol{x} = \boldsymbol{u}\boldsymbol{v} - \int \boldsymbol{v}\frac{\mathbf{d}\boldsymbol{u}}{\mathbf{d}\boldsymbol{x}}\,\mathbf{d}\boldsymbol{x}$$

3. Algebraic formulae

3.1 Solution of quadratic equation

$$ax^2 + bx + c = 0, \quad a \neq 0$$

$$\Rightarrow \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

3.2 Exponentials/Logarithms

$$y = \mathrm{e}^{ax} \quad \Rightarrow \quad \ln y = ax$$

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4. Measurement (see page 25)

$$\text{Absolute error} = \text{indicated value} - \text{true value}$$

$$\text{Relative error} = \frac{\text{absolute error}}{\text{true value}}$$

$$\text{Absolute correction} = \text{true value} - \text{indicated value}$$

$$\text{Relative correction} = \frac{\text{absolute correction}}{\text{true value}}$$

5. Statistics

For a sample, size N, $x_1,\ x_2,\ x_3, \ldots, x_N$,

sample mean $$\bar{x} = \frac{x_1 + x_2 + x_3 + \cdots + x_N}{N}$$

standard deviation

$$S = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \bar{x})^2} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}x_i^2 - (\bar{x})^2}$$

5.1 Probability

For events A and B, with probabilities of occurrence P(A) and P(B),

P(A or B) = P(A) + P(B) – P(A and B)

If A and B are mutually exclusive events,

P(A and B) = 0

P(A or B) = P(A) + P(B)

If A and B are independent events,

P(A and B) = P(A) × P(B)

6. Mechanical equations

6.1 Stress and strain equations

$$\text{axial stress } (\sigma) = \frac{\text{axial force}}{\text{cross sectional area}}$$

$$\text{axial strain } (\xi) = \frac{\text{change in length}}{\text{original length}}$$

$$\text{shear stress } (\tau) = \frac{\text{shear force}}{\text{shear area}}$$

$$\text{Young's modulus } (E) = \frac{\text{stress}}{\text{strain}}$$

Working or allowable stress =

$$\frac{\text{Ultimate stress}}{\text{Factor of Safety (FOS)}}$$

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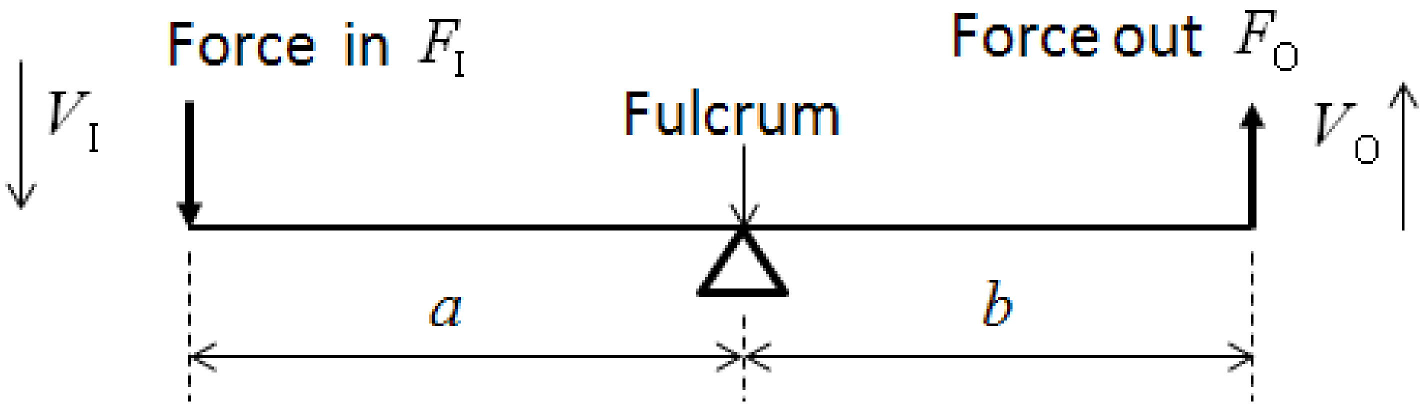
6.2 Mechanisms (see page 31)

Mechanical advantage (MA) $= \dfrac{\text{output force (or torque)}}{\text{input force (or torque)}}$

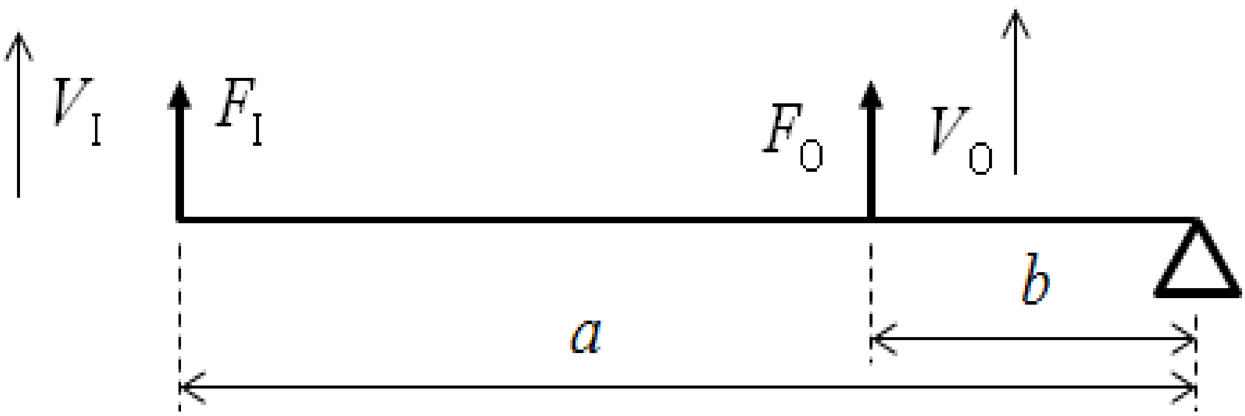
Velocity ratio (VR) $= \dfrac{\text{velocity of output from a mechanism}}{\text{velocity of input to a mechanism}}$

6.2.1 Levers

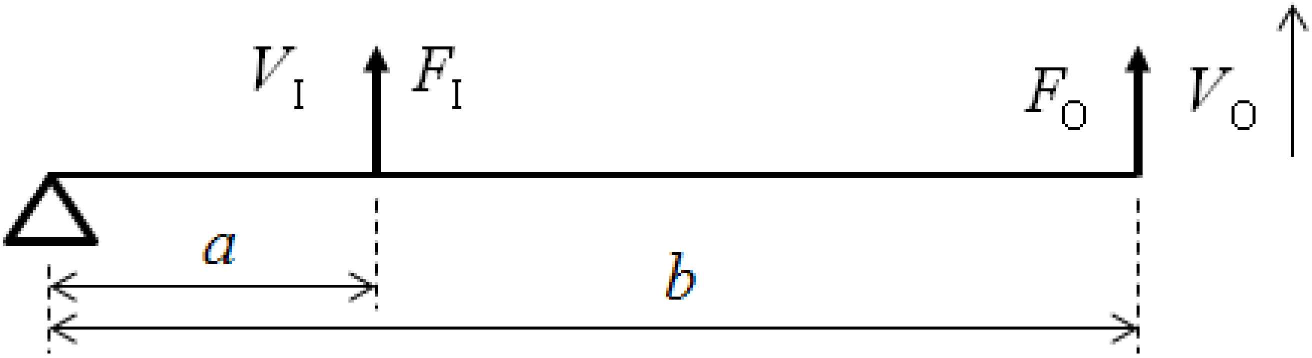
Class one lever



Class two lever



Class three lever



$$\mathbf{MA} = \frac{\boldsymbol{F_0}}{\boldsymbol{F_\mathrm{I}}} = \frac{\boldsymbol{a}}{\boldsymbol{b}} \qquad \boldsymbol{VR} = \frac{\boldsymbol{V_0}}{\boldsymbol{V_\mathrm{I}}} = \frac{\boldsymbol{b}}{\boldsymbol{a}}$$

6.2.2 Gear systems

$$MA = \frac{\text{Number of teeth on output gear}}{\text{Number of teeth on input gear}}$$

$$VR = \frac{\text{Number of teeth on input gear}}{\text{Number of teeth on output gear}}$$

6.2.3 Belt and pulley systems

$$MA = \frac{\text{Diameter of output pulley}}{\text{Diameter of input pulley}}$$

$$VR = \frac{\text{Diameter of input pulley}}{\text{Diameter of output pulley}}$$

6.3 Dynamics (see pages 35 and 36)

Newton's equation force = mass × acceleration $(F = ma)$

Gravitational potential energy (W_p) =
mass × gravitational acceleration × height (mgh)

Kinetic energy (W_k) = ½ mass × velocity^2 ($\frac{1}{2}mv^2$)

Work done = force × distance (Fs)

Instantaneous power = force × velocity (Fv)

Average power = work done / time ($\frac{W}{t}$)

Friction Force ≤ coefficient of friction x normal contact force ($F \leq \mu N$)

Momentum of a body = mass x velocity (mv)

Pressure = force / area ($\frac{F}{A}$)

6.4 Kinematics

Constant acceleration formulae

a – acceleration

s – distance

t – time

u – initial velocity

v – final velocity

$v^2 = u^2 + 2as$
$s = ut + \frac{1}{2}at^2$
$v = u + at$
$s = \frac{1}{2}(u+v)t$
$s = vt - \frac{1}{2}at^2$

6.5 Fluid mechanics (see page 39)

Pressure due to a column of liquid

= height of column × gravitational acceleration × density of liquid ($hg\rho$)

Up-thrust force on a submerged body

= volume of submerged body × gravitational acceleration × density of liquid ($Vg\rho$)

6.5.1 Energy equations

Non-flow energy equation

$U_1 + Q = U_2 + W$ so $Q = (U_2 - U_1) + W$

where Q = energy entering the system
W = energy leaving the system
U_1 = initial energy in the system
U_2 = final energy in the system.

Steady flow energy equation

$Q = (W_2 - W_1) + W$

where Q = heat energy supplied to the system
W_1 = energy entering the system
W_2 = energy leaving the system
W = work done by the system.

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7. Thermal Physics

p – pressure
V – volume
C – constant
T – absolute temperature
n – number of moles of a gas
R – the gas constant

Boyle's law $pV = C \quad p_1V_1 = p_2V_2$

Charles' law $\frac{V}{T} = C \quad \frac{V_1}{T_1} = \frac{V_2}{T_2}$

Pressure law $\frac{p}{T} = C \quad \frac{p_1}{T_1} = \frac{p_2}{T_2}$

Combined gas law $\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$

Ideal gas law $pV = nRT$

Characteristic gas law

$pV = mRT$ **where m = mass of specific gas and R = specific gas constant**

Efficiency $\eta = \dfrac{\text{work output}}{\text{work input}}$

7.1 Heat formulae

Latent heat formula

Heat absorbed or emitted during a change of state, $Q = mL$

where Q = Energy, L = latent heat of transformation, m = mass

Sensible heat formula

Heat energy, $Q = mc\Delta T$

where Q = Energy, m = mass,
c = specific heat capacity of substance,
ΔT is change in temperature

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8. Electrical equations
(see pages 47-51)

Q = charge
V = voltage
I = current
R = resistance
ρ = resistivity
P = power
E = electric field strength (capacitors)
C = capacitance
L = inductance
t = time
l = length
τ = time constant
W = energy
A = cross sectional area
Φ = magnetic flux

N = number of turns
Θ = angle (in radians)
f = Frequency (in cycles per second)
$\omega = 2\pi f$
X_L, X_C = inductive reactance, capacitive reactance
Z = impedance
$\emptyset$ = phase angle
E = emf (motors)
I_a = armature current
I_f = field current
I_l = load current
R_a = armature resistance
R_f = field resistance
n = speed (motors)
T = torque
η = efficiency

Charge and potential energy	$Q = It$ $V = W/Q$ $W = Pt$
Drift velocity (current)	$I = nAv\mathrm{e}$
Power	$P = VI$ $P = I^2R$ $P = V^2/R$
Resistance and Ohms law	**Series resistance:** $R = R_1 + R_2 + R_3 + \ldots$ **Parallel resistance:** $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$ **Ohms law:** $R = V/I \quad V = IR \quad I = V/R$
Resistivity	$\rho = RA/l$
Electric field and capacitance	$E = V/d$ $C = Q/V$ $W = \frac{1}{2}QV$

Inductance and self-inductance	$L = \Phi N / I$ $W_L = \frac{1}{2} LI^2$
RC circuits	$\tau = RC$ $v = v_0 e^{-t/RC}$
AC waveforms	$v = V \sin\theta$ $i = I \sin\theta$ $v = V \sin\omega t$ $i = I \sin\omega t$
AC circuits – resistance and reactance	$R = V/I$ $X_L = V/I$ and $X_L = 2\pi f L$ $X_C = V/I$ and $X_C = \dfrac{1}{2\pi f C}$
Series RL and RC circuits	$Z = \sqrt{(R^2 + X_L^2)}$ and $\cos\phi = R/Z$ $Z = \sqrt{(R^2 + X_C^2)}$ and $\cos\phi = R/Z$

Series RLC circuits	*When* $X_L > X_C$ $Z = \sqrt{[R^2 + (X_L - X_C)^2]}$ and $\cos\phi = R/Z$ *When* $X_C > X_L$ $Z = \sqrt{[R^2 + (X_C - X_L)^2]}$ and $\cos\phi = R/Z$ *When* $X_L = X_C$ $Z = R$
DC motor	$V = E + I_a R_a$
DC generator	$V = E - I_a R_a$
DC Series wound self-excited GENERATOR	$V = E - I_a R_t$ *Where* $R_t = R_a + R_f$
DC Shunt wound self-excited GENERATOR	$V = E - I_a R_a$ *Where* $I_a = I_f + I_l$ $I_f = V/R_f$ $I_l = P/V$

DC Series wound MOTOR	$V = E + I_a R_t$ $Where\ R_t = R_a + R_f$ $E \propto \Phi n$
DC Shunt wound MOTOR - No-load conditions:	$V = E_1 + I_a R_a$ $Where\ I_a = I_l - I_f$ $I_f = V/R_f$
DC Shunt wound MOTOR - Full load conditions:	$V = E_2 + I_a R_a$ $Where\ I_a = I_l - I_f$ $E_1/E_2 = n_1/n_2$ $T_1/T_2 = (\Phi_1 I_{a1})/(\Phi_2 I_{a2})$
Speed control of DC motors - Shunt motor	$V = E + I_a R_a$ $n = (V - I_a R_a)/(k\Phi)$
DC Machine efficiency	η = **output/input** η = **1 – (losses/input)**