OCR
Oxford Cambridge and RSA

Cambridge Technicals
Engineering

Unit 23: Applied mathematics for engineering

Level 3 Cambridge Technical Certificate/Diploma in Engineering
05822 - 05825

Mark Scheme for June 2019

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations

Annotation	**Meaning**
✓and ✖	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
DM1	Method mark dependent on previous M mark
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
λ	Omission sign
Other abbreviations in mark scheme	**Meaning**
oe	Or equivalent
Soi	Seen or implied
www	Without wrong working
ecf	Error carried forward

Subject-specific marking instructions

Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. These annotations must be in the body of the work and **not** anywhere near the right hand margin of each page.
Mark in using a red pen.
Put the mark for each subquestion near to and to the right of the mark for the question. Total all marks for the question and put this total in a ring at the bottom right of each question.
Transfer these marks to the box on the front page.
Total the marks for the paper. I suggest that all unringed marks are then totalled to make sure that the final mark is correct.

An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

DM
A method mark which is dependent on a previous method mark.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

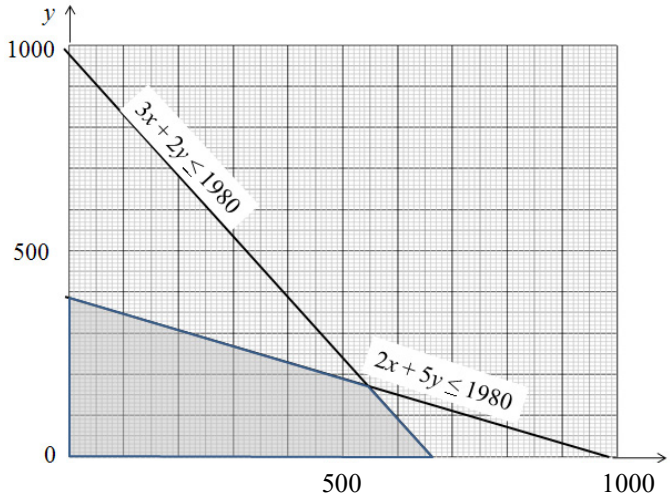
The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

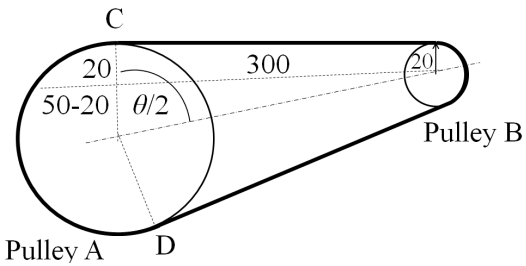
Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

Question			Answer	Marks	Guidance
1	(i)		$l = \frac{C}{d^2}$ $100 = \frac{C}{2.5^2} \rightarrow \quad C = 625$	B1 [1]	
1	(ii)		$d^2 = \frac{625}{80} \quad (= 2.8^2)$ $x^2 + 2.5^2 = \frac{625}{80}$ $x^2 = \sqrt{\frac{625}{80} - 2.5^2} \quad \left(= \sqrt{\frac{25}{16}}\right)$ 1.25 or $\frac{5}{4}$ [m]	B1 M1 A1 [3]	soi 2.795^2 or $\frac{125}{16}$ Exact answer required
1	(iii)		(PQ) = $\sqrt{2^2 + 2.5^2}$ (SQ) = $\sqrt{PQ^2 + 2.5^2}$ lux at corner = $\left(\frac{625}{16.5}\right)$ = 37.9 [lux]	B1 M1 A1 [3]	Horizontal distance soi Expect 3.2 or $\sqrt{10.25}$ or $\frac{\sqrt{41}}{2}$ FT their PQ from Pythagoras Expect $\sqrt{16.5}$ or 4.06m

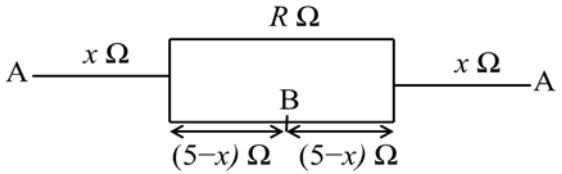
Question			Answer	Marks	Guidance
1	(iv)	(*A*)	$100 + 2\left(\frac{625}{2.5^2+1.5^2}\right)$ = 247 [lux] awrt	**M1** **A1** **[2]**	
1	(iv)	(*B*)	$BQ^2 = 2^2 + 4^2 + 2.5^2$ $\frac{625}{2^2+1^2+2.5^2}$ + 37.9 + $\frac{625}{2^2+4^2+2.5^2}$ = 117 [lux] awrt	**M1** **M1** **A1** **[3]**	Sum of 3 luminances with attempt at distances FT their iii 117.265

Question			Answer	Marks	Guidance
2	(i)		(Time on) **machine A** = $3x + 2y$ (Time on) **machine B** = $2x + 5y$ $33 \times 60 = 1980$ (min)	**B1** **B1** **[2]**	Either equation or inequality linked to the correct machine (maximum) minutes in 33 hour week stated
2	(ii)			**B1** **B1** **B1** **[3]**	Reasonable attempt at both lines Suitable scales and both lines correct Correct line for $3x + 2y \leq 1980$ → from (660,0) to (0,990) Correct line for $2x + 5y \leq 1980$ → from (990,0) to (0, 396) Correct region shaded

Question			Answer	Marks	Guidance
2	(iii)		At the **intersection** of the two lines **Both** machines are working at full capacity oe	**B1** **B1** **[2]**	SC B1 for: 'x and y must be within the shaded area' OR 'any other point in the shaded area would result in fewer total products'
2	(iv)		$\begin{bmatrix}3 & 2\\2 & 5\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}1980\\1980\end{bmatrix}$	**B1** **[1]**	3 matrices required. Multiplication LHS
2	(v)		$=\frac{1}{11}\begin{bmatrix}5 & -2\\-2 & 3\end{bmatrix}\begin{bmatrix}1980\\1980\end{bmatrix}$ $=\frac{1}{11}\begin{bmatrix}5\times1980 & -2\times1980\\-2\times1980 & +3\times1980\end{bmatrix}$ $\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}540\\180\end{bmatrix}$	**B1** **B1** **M1** **A1** **[4]**	Must use matrices to score any marks B1 for $\frac{1}{11}$ or 11 soi B1 for $\begin{bmatrix}5 & -2\\-2 & 3\end{bmatrix}$ soi Attempt at multiplication

Question			Answer	Marks	Guidance
3	(i)				
			$50 - 20 = 30$	B1	Difference in radii soi
			$(C\hat{O}D) = \tan^{-1}\left(\frac{300}{30}\right)$ oe	M1	oe using sin / cos
			$= 84.3°$	A1	FT double their answer
			= 168.6° AG	A1√	
				[4]	
3	(ii)		100π and 40π	B1	Both pulley circumferences soi
			Length of belt on pulley A: A: $\frac{191.4}{360} \times 100\pi$ oe	M1	
			B: $\frac{168.6}{360} \times 40\pi$ oe	M1	
			$= 600 + \frac{191.4}{360} \times 100\pi + \frac{168.6}{360} \times 40\pi$	A1	Awrt 826 or 0.826 (m)
			= 825.9 [mm]	[4]	

Question			Answer	Marks	Guidance
3	(iii)		$\frac{3000}{60} = 50$ $50 \times 100\pi$ 15.7 [m/s)]	**B1** **M1** **A1** **[3]**	Revolutions per second soi Distance travelled by belt in mm / s Expect 15707.96 ALT B1 M1 $\omega = \frac{3000}{60} \times 2\pi$ (rad) A1 $v = 50\omega =$ 15707.96 = 15.7
3	(iv)		$\frac{50}{20} \times 3000 = 7500$ [RPM]	**B1** **[1]**	Ratio 2.5 from consideration of pulley circumferences or radii ALT Velocity ratio 2.5 $\times$ 3000

Question			Answer	Marks	Guidance
4	(i)		$x + R + x = 2x + R = 14$ AG	B1	Addition of 3 resistances in series seen, either as a formula or in a diagram
			When the cores are connected at B the circuit should be modelled as follows.  	B1	Answer must show evidence of correct modelling either as a diagram or clear analysis using correct formulae below.
			Resistance across central section $\frac{1}{r} = \frac{1}{R} + \frac{1}{2(5-x)} = \frac{2(5-x)+R}{R2(5-x)} \Rightarrow r = \frac{2R(5-x)}{2(5-x)+R}$	B1	Must be convincing
			Total resistance $2x + \frac{2R(5-x)}{2(5-x)+R} = 9.5$	B1 [4]	Leading to answer given

Question			Answer	Marks	Guidance
4	(ii)				Accept equivalent steps throughout with substitution for x in terms of R: $x = \frac{14-R}{2}$
			$R = 14 - 2x$ substituted into second equation: $2x + \frac{2(14-2x)(5-x)}{2(5-x)+(14-2x)} = 9.5$	M1	Rearrange and attempt substitution into second equation
			$2x(24-4x) + 2(14-2x)(5-x) = 9.5(24-4x)$ oe	M1	Appropriate simplification and correct process to clear the fraction
			$48x - 8x^2 + 140 - 48x + 4x^2 = 228 - 38x$	DM1	Expand *their* terms to obtain quadratic
			$4x^2 - 38x + 88 = 0$ oe	A1	Correct 3 term quadratic in x or R $R^2 - 9R + 18 = 0$ oe
				M1	Correct process to solve their 3 term quadratic and obtain at least one real root
			$x = 4$ $[x = 5.5]$	A1	$R = 6$ $[R = 3]$
			$R = 6$ $[R = 3]$	A1√	FT their $x < 5$ to obtain R $x = 4$ $[x = 5.5]$
			$x = 4$ [km], R =6 [ohms] only	A1	Correct final answer stated clearly
				[8]	

Question			Answer	Marks	Guidance
5	(i)		$t = 0 \rightarrow (e^{kt} = e^0) = 1$ $t = 1 \rightarrow (e^{kt} = e^k) = 0.5$	B1	Sight of 1 and 0.5 both required oe $1 = 2e^k$
			$t = 1 \rightarrow e^k = 0.5$	B1	Must see this step explicitly as the answer is given
			$k = \ln(0.5)$ AG	[2]	
5	(ii)		When $t = 0, x = b$		
			b=0.05	B1	
			$\frac{dx}{dt} = e^{kt}(a\omega \cos \omega t - b\omega \sin \omega t)$ $+ ke^{kt}(a \sin \omega t + b \cos \omega t)$	M1	Attempts differentiation as a product $uv' + vu'$ May use $k = \ln(0.5)$ throughout
			Substitutes $\frac{dx}{dt} = 0, t = 0, \omega = 2\pi$	DM1	Attempt to evaluate a
			$a = 0.0055$	A1	5.5×10^{-3}
				[4]	

Question			Answer	Marks	Guidance
5	(iii)		Compares $a\sin\omega t + b\cos\omega t$ and $C\sin\omega t\cos\theta + C\cos\omega t\sin\theta$ to state $a = C\cos\theta$ $b = C\sin\theta$ Indicates division and use of identity $\tan\theta = \frac{\sin\theta}{\cos\theta}$ to show $\theta = \tan^{-1}\left(\frac{b}{a}\right)$ Sight of $\sin^2\theta + \cos^2\theta = 1$ $a^2 + b^2 = C^2(\cos^2\theta + \sin^2\theta)$ $C = \sqrt{a^2 + b^2}$	**B1** **B1** **B1** **B1** **[4]**	Condone absence of e^{kt} throughout Missing C can score B0B1B1B0
5	(iv)		$C = \sqrt{(0.0055)^2 + 0.05^2} = 0.0503$ $\theta = \tan^{-1}\left(\frac{0.05}{0.0055}\right) = 1.46$ [radians]	**B1FT** **B1FT** **[2]**	Ft their a and b from (ii) No need to form the expression

Question			Answer	Marks	Guidance
6	(i)		Long wall $(A_3) = 5h$ Two ends $(A_2) = 3(h+3)$ Roof $(A_1) = 5\sqrt{3^2+(3-h)^2}$	B1 B1 B1 [3]	Accept unsimplified answers for each area eg $2\times\frac{1}{2}(h+3)\times 3$
6	(ii)		$k(3\times 5\sqrt{3^2+(3-h)^2}+0.3\times 3(h+3)+0.8\times 5h)$ $T = k(15\sqrt{9+(3-h)^2}+4.9h+2.7)$ AG	M1 A1 [2]	Substitution of their values for A_1, A_2, A_3 into formula, and attempt at simplification cao
6	(iii)		Using the given substitution $x = 3-h$ $T = k(15\sqrt{9+x^2}+4.9(3-x)+2.7)$ $\frac{dT}{dx} = k(\frac{15}{2}(9+x^2)^{-1/2}\times 2x - 4.9)$ $k\left(\frac{15x}{\sqrt{9+x^2}}-4.9\right)=0$ $15x = 4.9\sqrt{9+x^2}$ $225x^2 = 24.01(9+x^2)$ oe $x = 1.04$ $h = 1.96$	M1 A1 M1 M1 A1 B1FT [6]	Accept differentiation of function in h throughout $\frac{1}{2}u^{-\frac{1}{2}}\times u'$ term attempted Correct derivative (condone absence of k) Equate their derivative including $(9+x^2)^{-\frac{1}{2}}$ term to zero and attempt simplification Square both sides to obtain equation in x^2 1.03688 FT 3 –their x, if their $x < 3$

Question	Answer	Marks	Guidance
7 (i)	$\frac{\mathrm{d}T}{3000-10(T-T_a)}=\frac{\mathrm{d}t}{10^6}$ oe	M1	Separate variables
	$\frac{\ln(3000-10(T-T_a))}{-10}$	M1	Integrate LHS side to obtain appropriate expression involving ln Condone $\pm$ for M1
	$\frac{\ln(3000-10(T-T_a))}{-10}=\frac{t}{10^6}$ (+A)	A1	Both sides correct, condone absence of constant of integration
	$t=-10^5\ln(3000-10(T-T_a))+B$ oe	A1 [4]	Arbitrary constant required now
(ii)			Must start with an expression of the correct form to score any marks in (ii)
	$3000-10(T-T_a)=Ce^{-\frac{t}{10^5}}$ oe	M1	Attempts process to remove logarithms, condone missing C or wrong treatment of C
		B1	$e^{-\frac{t}{10^5}}$ term present
		A1	All correct
	$T=T_a+300-De^{-\frac{t}{10^5}}$	DM1	Rearrange their equation to make T the subject
	When $t=0$ $T=30$ $\Rightarrow D=T_a+270$	M1	Apply conditions to evaluate D, which must now be part of the $e^{-\frac{t}{10^5}}$ term
	$T=T_a+300-(T_a+270)e^{-\frac{t}{10^5}}$ oe	A1 [5]	

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