OCR
Oxford Cambridge and RSA

Monday 3 June 2019 – Morning

LEVEL 3 CAMBRIDGE TECHNICAL IN ENGINEERING

05823/05824/05825/05873 Unit 23: Applied mathematics for engineering

Time allowed: 2 hours
C305/1906

You must have:
- the formula booklet for Level 3 Cambridge Technical in Engineering (inserted)
- a ruler (cm/mm)
- a scientific calculator

Please write clearly in black ink.

Centre number | | | | | | Candidate number | | | | |

First name(s) ______________________

Last name ______________________

Date of Birth | D | D | M | M | Y | Y | Y | Y |

INSTRUCTIONS
- Use black ink. You may use an HB pencil for graphs and diagrams.
- Answer **all** the questions.
- Write your answer to each question in the space provided.
- If additional answer space is required, you should use the lined page(s) at the end of this booklet. The question number(s) must be clearly shown.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total mark for this paper is **80**.
- The marks for each question are shown in brackets **[ ]**.
- Where appropriate, your answers should be supported with working. Marks may be given for a correct method even if the answer is incorrect.An answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- This document consists of **20** pages.

FOR EXAMINER USE ONLY	
Question No	Mark
1	/12
2	/12
3	/12
4	/12
5	/12
6	/11
7	/9
Total	**/80**

Answer **all** the questions.

1 Illuminance is a measure of the amount of light reflected by a surface. The SI unit for illuminance is the lux. The illuminance, l lux, produced by a single light source, such as an unshaded domestic lamp, is inversely proportional to the square of the distance, d m, between the source and the point at which the illuminance is observed. This may be expressed as

$$l = \frac{C}{d^2}, \text{ where } C \text{ is a constant.}$$

Fig. 1 shows the horizontal floor of a rectangular room which is 5 m long and 4 m wide, and a single light source, S, at a height of 2.5 m above the centre of the floor. The light source produces an illuminance of 100 lux at the point P on the floor which is directly below S. Q is a point on the floor at one corner of the room. R is a point on the floor a distance x m from P.

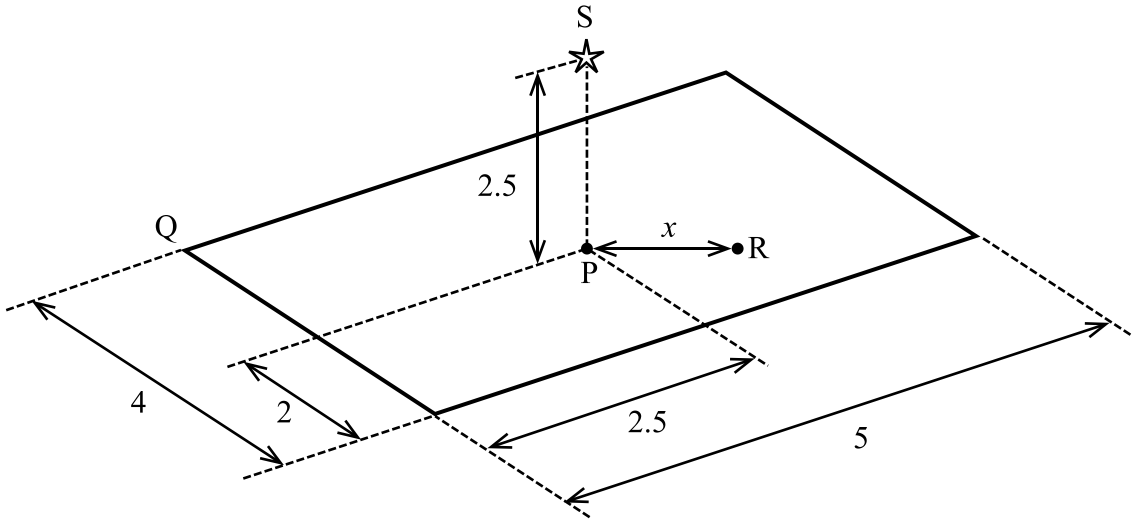

Fig. 1

(i) Calculate the value of C.

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(ii) The illuminance at R is 80 lux. Calculate x.

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(iii) Calculate the illuminance at Q.

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When more than one light source is present, the illuminance at any point is the sum of the illuminance produced by each source. Two additional light sources, A and B, each identical to S, are now installed as shown in Fig. 2. These are positioned so that A is 1 m from one of the shorter walls of the room, and B is 1 m from the other. They are each 2 m from the longer walls and 2.5 m above the floor. ASB forms a straight line parallel to the longer walls of the room.

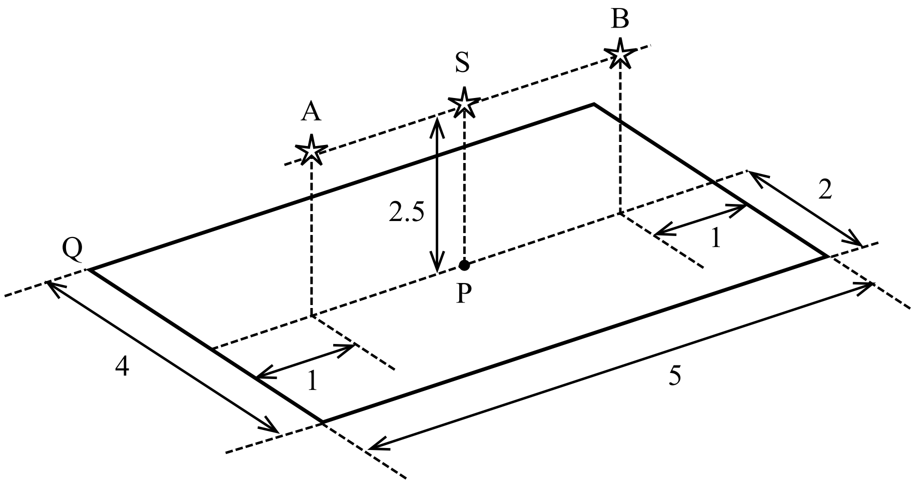

Fig. 2

(iv) Calculate

(A) the total illuminance at P,

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(B) the total illuminance at Q.

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2 A manufacturing company produces two products, X and Y. Each product involves the use of two machines, A and B. To produce one unit of product X requires the exclusive use of machine A for 3 minutes and machine B for 2 minutes. To produce one unit of product Y requires the exclusive use of machine A for 2 minutes and machine B for 5 minutes. Each machine can be operated for no more than 33 hours each week.

(i) If x and y are the number of units of products X and Y produced each week respectively, explain why x and y must satisfy the following two conditions.

$$3x + 2y \leq 1980$$
$$2x + 5y \leq 1980$$

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(ii) Represent these two conditions graphically on the grid below and shade the area for which they are **both simultaneously** satisfied. Do not shade any other areas.

[3]

(iii) The manufacturer wishes to maximise the total number of products produced each week. With reference to your answer to part **(ii)** explain why, in this case, the maximum total number of products is found when the following two simultaneous equations are satisfied.

$$3x + 2y = 1980$$
$$2x + 5y = 1980$$

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.... **[2]**

(iv) Represent these equations in matrix notation.

[1]

(v) Using matrix methods, calculate the values of x and y.

[4]

3 Fig. 3 shows a belt and pulley system containing pulley A with radius 50 mm and pulley B with radius 20 mm. The system is arranged so that the straight sections of the belt between the two pulleys each measure 300 mm.

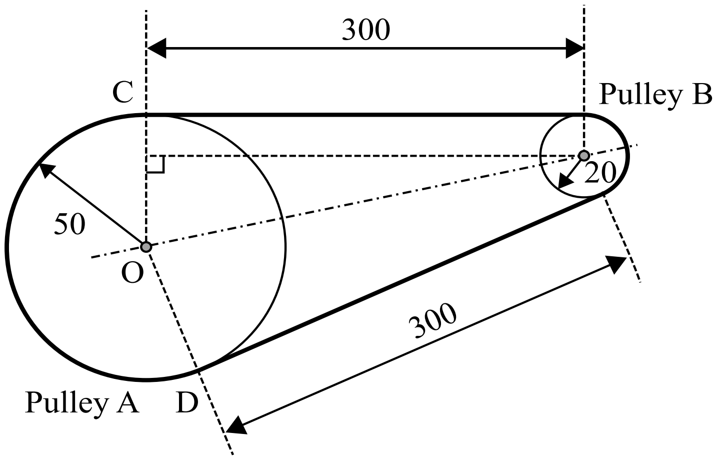

Fig. 3

(i) The centre of pulley A is at point O and the straight parts of the belt meet pulley A tangentially at points C and D. Show that the angle COD is approximately 168.6°.

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(ii) Calculate the total length of the belt.

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Pulley A rotates at 3000 RPM.

(iii) Calculate the linear speed of the belt in metres per second.

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(iv) Calculate the rotational speed of pulley B, giving your answer in RPM.

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4 A twin core electrical cable of length 5 km connects two buildings, A and B. It is known that the resistance of each core is 1 Ω per km. Damage has occurred in the cable at a distance of *x* km from building A; this creates a resistance of *R* Ω across the two cores at that point. Engineers are trying to find the values of *R* and *x*. They measure the resistance across the two cores in building A while ensuring that the two cores in building B are not connected together. They find this resistance to be 14 Ω. This situation is shown in Fig. 4a and can be modelled by a circuit in which three resistors are connected in series.

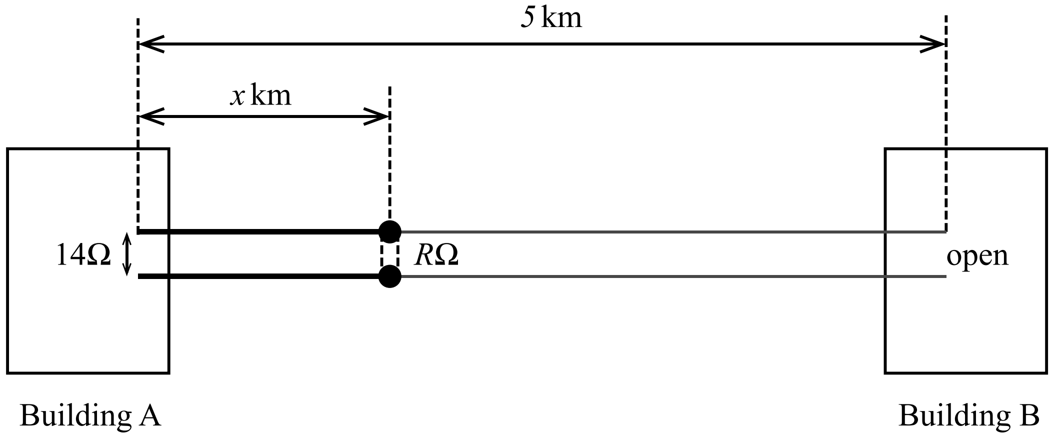

Fig. 4a

The engineers now connect the two cores together in building B and find that the resistance across the two cores measured in building A is 9.5 Ω. This situation is shown in Fig. 4b and can be modelled by a circuit in which five resistors are connected in a combined serial and parallel arrangement.

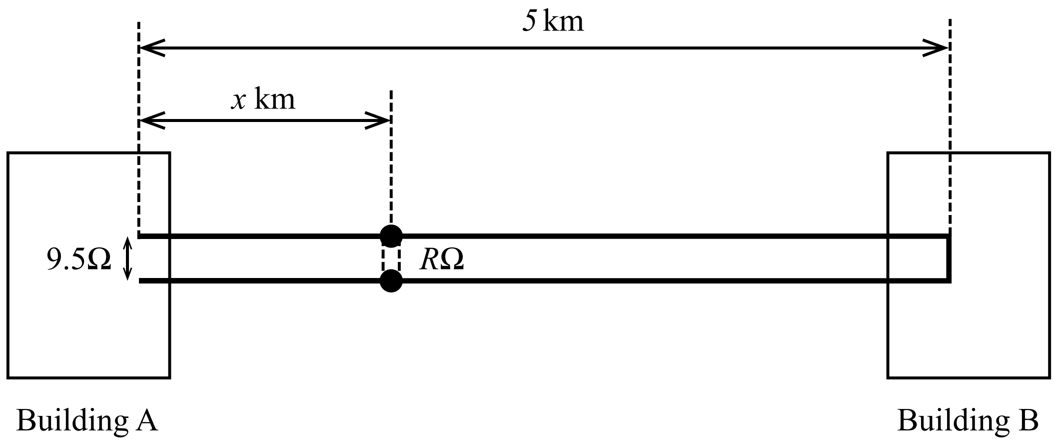

Fig. 4b

(i) Using the equations for resistors connected in series and in parallel given on page 15 of the formula booklet, show that the values of x and R satisfy the following two simultaneous equations.

$$2x + R = 14$$

$$2x + \frac{2R(5-x)}{2(5-x)+R} = 9.5$$

.... **[4]**

(ii) Use the equations in part **(i)** to calculate the values of x and R. These equations are repeated here.

$$2x + R = 14$$

$$2x + \frac{2R(5-x)}{2(5-x)+R} = 9.5$$

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Question 5 begins on page 12

5 A spring and damper system is being developed in a laboratory. A schematic diagram of the system, supporting a load, while at rest in equilibrium in its natural stationary, unforced position is shown in Fig. 5. This position is called the reference level.

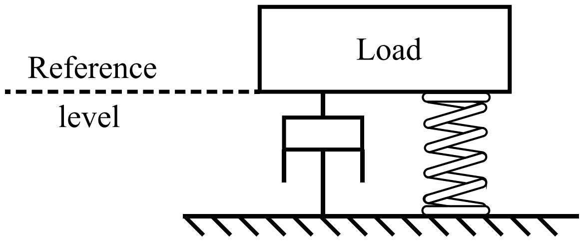

Fig. 5

The load is then raised above its reference level and released. This causes the load to fall and then oscillate about its reference position according to the following formula.

$$x = e^{kt}(a \sin \omega t + b \cos \omega t)$$

where x m is the distance of the load from its reference level, measured upwards
k is an exponential decay constant,
t s is the time after the load was released,
ω rad s^{-1} is the frequency of the oscillation,
a and b are constants which depend on the initial conditions.

(i) The value of k is such that the value of e^{kt} is halved every second. By considering the values of e^{kt} when $t = 0$ and when $t = 1$, show that $k = \text{In}\,(0.5)$.

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(ii) The frequency of the oscillation is 2π radians per second. Given that $x = 0.05$ and $\frac{dx}{dt} = 0$ when $t = 0$, calculate the values of a and b.

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(iii) Show that $x = e^{kt}\,(a \sin \omega t + b \cos \omega t)$ may be written as

$$x = Ce^{kt} \sin(\omega t + \theta), \text{ where } C = \sqrt{a^2 + b^2} \text{ and } \theta = \tan^{-1} \frac{b}{a}.$$

You may wish to use the formula $\sin(\omega t + \theta) = \sin \omega t \cos\theta + \cos \omega t \sin\theta$.

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(iv) Using your values of a and b from part **(ii)**, express the oscillatory motion of the mass in the form

$x = Ce^{kt} \sin(\omega t + \theta)$ giving approximate values for C and θ.

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6 A lean-to extension with a glazed sloping roof is to be constructed on the side of a building as shown in Fig. 6.

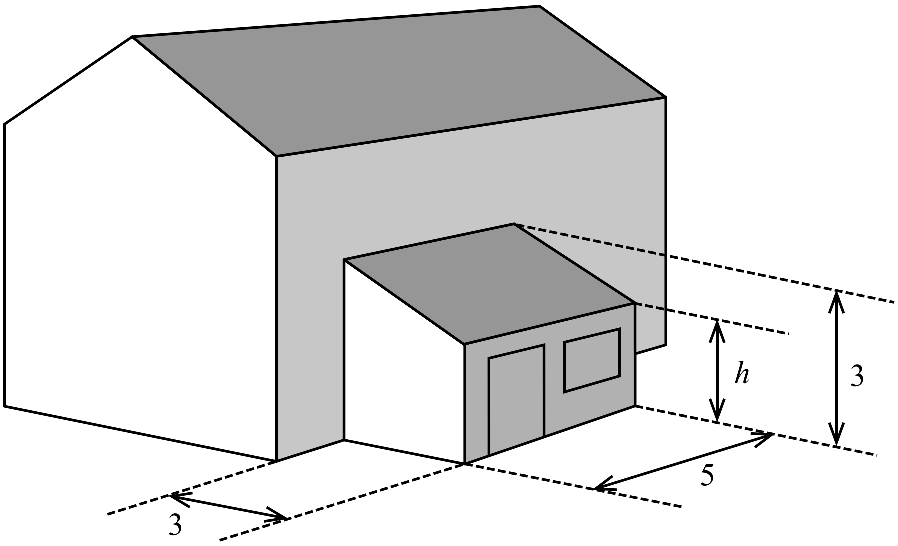

Fig. 6

The sloping roof of the lean-to is to meet the building at a height of 3 m. The lean-to is to have a width of 3 m and a length of 5 m. With a knowledge of the thermal transmittance properties of the materials used, engineers have estimated that the total heat loss, T Joules, from inside the lean-to is given by

$$T = k\,(3A_1 + 0.3A_2 + 0.8A_3),$$

where A_1 m^2 is the area of the roof,

A_2 m^2 is the combined area of the two end walls,

A_3 m^2 is the area of the front wall including the door and window,

k is a constant.

The height of the front wall, h m, is to be chosen so that the total heat loss is minimised.

(i) Write down expressions in terms of h and the given measurements for the areas A_1, A_2 and A_3.

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(ii) Show that

$$T = k(15\sqrt{9+(3-h)^2} + 4.9h + 2.7).$$

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(iii) Using calculus, calculate the value of h so that heat loss from the lean-to is minimised.

(You may find it helpful to use the substitution $x = 3 - h$.)

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7 In this question you should use the result

$$\int \frac{1}{A-B(x-C)}\,\mathrm{d}x = -\frac{\ln(A-B(x-C))}{B}, \text{ where } A, B \text{ and } C \text{ are constants.}$$

A 3 kW thermostatically controlled electric immersion heater is used to heat water in a domestic hot water tank. t seconds after the immersion heater is switched on, the temperature of the water is T °C. While the water is being heated, heat is lost through the cylinder insulation at a rate proportional to the difference between the temperature of the water in the tank and the ambient room temperature T_a °C. While the water is being heated, and until the water reaches a temperature of 60 °C, T satisfies the following differential equation.

$$10^6 \frac{\mathrm{d}T}{\mathrm{d}t} = 3\times10^3 - 10(T - T_a)$$

Assume that T_a remains constant.

(i) Find an expression for t, giving your answer in terms of T, T_a and an arbitrary constant.

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(ii) Given that $T = 30$ when $t = 0$, find an expression for T as a function of t and T_a .

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END OF QUESTION PAPER

ADDITIONAL ANSWER SPACE

If additional answer space is required, you should use the following lined pages. The question number(s) must be clearly shown – for example 2(ii) or 6(i).

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