OCR
Oxford Cambridge and RSA

Cambridge Technicals
Engineering

Unit 23: Applied mathematics for engineering

Level 3 Cambridge Technical Certificate/Diploma in Engineering
05822 - 05825

Mark Scheme for January 2019

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations

Annotation	**Meaning**
✓and ✖	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
DM1	Method mark dependent on previous M mark
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
λ	Omission sign
Other abbreviations in mark scheme	**Meaning**
oe	Or equivalent
Soi	Seen or implied
www	Without wrong working
ecf	Error carried forward

Subject specific marking instructions

Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. These annotations must be in the body of the work and **not** anywhere near the right hand margin of each page.
Mark in using a red pen.
Put the mark for each subquestion near to and to the right of the mark for the question. Total all marks for the question and put this total in a ring at the bottom right of each question.

Transfer these marks to the box on the front page.
Total the marks for the paper. I suggest that all unringed marks are then totalled to make sure that the final mark is correct.

An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

DM
A method mark which is dependent on a previous method mark.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

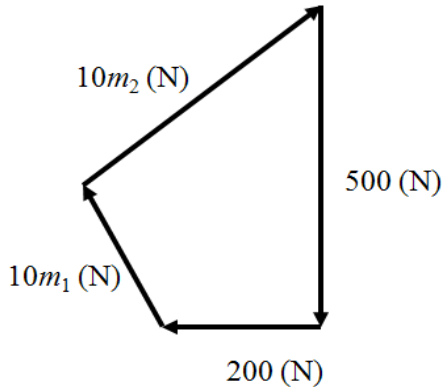
When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

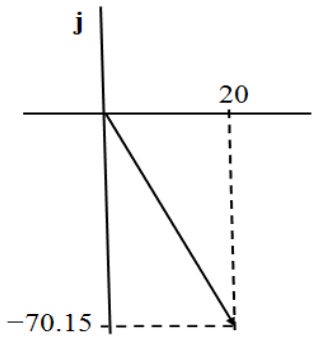
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

Question			Answer	Marks	Guidance
1	(i)		50 000×9.8 $490000 = 100V^2$; $V^2 = 4900$ $V = \sqrt{4900} = 70\,(\text{m s}^{-1})$	**M1** **M1** **A1** **[3]**	Weight soi Equates their weight to $100V^2$ and attempts to solve for V Award M0M1A0 max 1/3 if g is missing and $100V^2 = 50\,000$ seen followed by attempt to solve
1	(ii) (*A*)		Using $F = ma$, $a = F/m$ a=150 000/50 000 = 3 m s^{-2} Using $v = u + at$ with $u = 0$ t = 70 / 3 ≈ 23.3 (s)	**M1** **A1** **M1** **A1FT** **[4]**	Applies N2L Solve to find acceleration Appropriate equations of motion used FT their a and V
1	(ii) (*B*)		Using $s = (u+v)\times t/2$ with $u = 0$ oe s= 70×23.33/2 ≈ 817 (m)	**M1** **A1** **[2]**	Correct use of equations of motion to determine distance
1	(ii) (*C*)		Examples If friction and air resistance had been included …. … the aircraft would take a longer time (A) to reach the required speed and the distance (B) required would be longer.	 **B1** **B1** **[2]**	 Two different simplifying assumptions mentioned Effect on both A and B stated Accept mention of any other factors that may change results such as wind speed and direction, inclination of the runway, varying thrust etc. Also accept; if acceleration is not constant then the formulae used would not be valid giving incorrect results.

Question			Answer	Marks	Guidance
2	(i)			**B1** **B1** **[2]**	Diagram must show four forces in a closed loop all of which must point in correct directions Magnitudes correct (note $g = 10$) and labelled appropriately Forces should be the 4 separate forces, unresolved Wrong shape polygon gets 0/2
2	(ii)		$F_1 = 10\ m_1(-\cos 60\ \mathbf{i} + \sin 60\ \mathbf{j})$ $F_2 = 10\ m_2(\cos 40\ \mathbf{i} + \sin 40\ \mathbf{j})$ $F_3 = -200\mathbf{i}$ $F = -500\mathbf{j}$	**B1** **B1** **B1** **B1** **[4]**	$-5m_1\boldsymbol{i} + 8.66m_1\boldsymbol{j}$ $7.66m_2\boldsymbol{i} + 6.43m_2\boldsymbol{j}$ B4 for all 6 components clearly identified within a single force in terms of $\boldsymbol{i}$ and $\boldsymbol{j}$ Deduct [1] for use of wrong value of g (eg 9.8 or 9.81)
2	(iii)		Set up equations Horizontal $-\cos 60\ m_1 g + \cos 40\ m_2 g - 20g = 0$ Vertical $\sin 60\ m_1 g + \sin 40\ m_2 g - 50g = 0$ $m_1 = 25.8390\ldots$ $m_2 = 42.9734\ldots$	**M1** **M1** **A1** **A1** **[4]**	Allow components with or without g here Solution by any correct method Accept AWRT to 26 and 43 No penalty for wrong or missing g in this part

Question			Answer	Marks	Guidance
2	(iv)		$F_1 \times F_2 = (10m_1 \times 10m_2 \times \sin 80) \approx 109352$	M1 A1FT [2]	FT their m_1 and m_2 from iii used

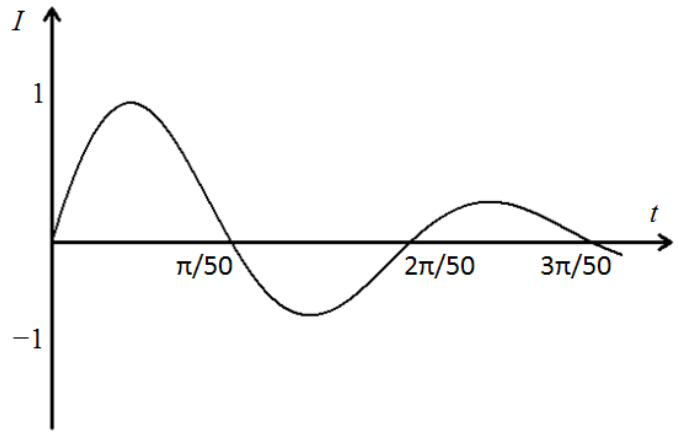
Question			Answer	Marks	Guidance
3	(i)		$50\times 2\pi$ (rad/s) ; $\omega = 100\pi$ (314.1593..)	**B1** **[1]**	Exact or rounded
3	(ii)	(*A*)	$Z_T = 20 + 100\pi \times 30\times 10^{-3}\,\mathrm{j} + \dfrac{1}{100\pi \times 40\times 10^{-6}\,\mathrm{j}}$ $= 20 + 3\pi\mathrm{j} - \dfrac{\mathrm{j}}{0.004\pi}$ $= 20 + (3\pi - \dfrac{1}{0.004\pi})\mathrm{j}$ oe	**B2,1,0** **M1** **A1** **[4]**	B2 all 3 terms, deduct 1 mark for each error FT their ω. Exact or rounded values Method is for correctly rationalising $20 - 70.1527\,\mathrm{j}$ AWRT 70.2
3	(ii)	(*B*)		 **B1 FT** **B1 FT** **[2]**	Allow FT from part (ii) B1 for their correct quadrant of the complex plane B1 indication of their 20 and their –70j on axes
3	(ii)	(*C*)	$r = \sqrt{20^2 + 70.1527^2}$ $\alpha = \tan^{-1}\dfrac{70.1527}{20}$	**M1** **M1**	Modulus Argument

Question			Answer	Marks	Guidance
			Express in the form $Z_T = 72.95(\cos(-74.1) + j\sin(-74.1))$ oe	**A1** **[3]**	AWRT to 73 and AWRT -74 Accept answer given in radians
3	**(ii)**	**(D)**	$Z_T = 72.9479\,e^{4.99j}$	**B1** **[1]**	Angle must be in radians here
4	**(i)**	**(A)**	$\sin\frac{\theta}{2} = \frac{c}{2r} \Rightarrow \theta = 2\sin^{-1}\frac{c}{2r}$ AG	**B1 B1** **[2]**	Must see expression for $\sin\left(\frac{\theta}{2}\right)$ for the first mark Likely to see $\frac{c/2}{r}$ in the first step
4	**(i)**	**(B)**	Area of sector $= \frac{1}{2}r^2\theta$ Area of triangle $= \frac{1}{2}r^2\sin\theta$ Area of shaded region $= \frac{1}{2}r^2(\theta - \sin\theta)$ AG	**B1** **B1** **[2]**	

Question			Answer	Marks	Guidance
4	(ii)		**EITHER** Angle between two adjacent apexes = 60^o	**B1**	Angle identified
			$\cos\frac{60}{2}=\frac{d}{2r}\Rightarrow r=\frac{d}{2\cos 30}=\frac{d}{2(\sqrt{3}/2)}=\frac{d}{\sqrt{3}}$	**B1**	Side length $\frac{d}{\sqrt{3}}$ oe SOI
			Total area $=6\times\frac{1}{2}\sin 60(r^2)=3\left(\frac{\sqrt{3}}{2}\right)\left(\frac{d^2}{3}\right)$	**M1**	Accept any method that correctly finds the area of six equilateral triangles using their side length
			$\frac{\sqrt{3}d^2}{2}$ AG	**A1**	
				[4]	
			OR Area of hexagon = area of circle with radius r minus 6×shaded region		
			$\pi r^2-6(\frac{1}{2}r^2(\frac{\pi}{3}-\sin 60))$	**B1** **B1**	Angle identified Correct segment expression
				M1	Complete method
			$\pi r^2-\pi r^2+3r^2\sin 60 = 3\frac{d^2}{3}\frac{\sqrt{3}}{2}=\frac{\sqrt{3}d^2}{2}$ AG	**A1**	
				[4]	

Question			Answer	Marks	Guidance
4	(iii)		Hexagon area = $200\sqrt{3}$ oe Circle area = 400π oe and handle = 1200 Total area = $\pi 20^2 - 20^2\frac{\sqrt{3}}{2} - \frac{1}{2}20^2(\frac{\pi}{3} - \frac{\sqrt{3}}{2}) + 60\times 20$ Volume = $5\times$area = $2000(\frac{5\pi}{6} - \frac{\sqrt{3}}{4} + 3) \approx (10370 \text{ mm}^3)$	B1 B1 M1 M1 A1 [5]	Total area of spanner = Circle radius 20 – hexagon – area of segment + rectangle Soi by $\frac{\sqrt{3}}{2}20^2$ Soi by $\pi 20^2$ and 60×20 3 of these terms required for M1. Segment area may be missing. All 4 terms for second M1
5	(i)		$\frac{dv}{dt} = Ae^{-ct}$	B1 [1]	Recognition that $a = \frac{dv}{dt}$
5	(ii)		$v = -\frac{A}{c}e^{-ct} + C$ When $t = 0$ $-\frac{A}{c} + C = v_0 \Rightarrow C = v_0 + \frac{A}{c}$ $v = -\frac{A}{c}e^{-ct} + v_0 + \frac{A}{c} = \frac{A}{c}(1 - e^{-ct}) + v_0$	M1 M1 A1 [3]	Must contain C Attempt to find c using initial condition Accept $v = -\frac{A}{c}e^{-ct} + v_0 + \frac{A}{c}$ OE
5	(iii)		$s = \frac{A}{c^2}e^{-ct} + \left(v_0 + \frac{A}{c}\right)t + k$	M1 M1 M1	Integration of exponential term to give correct form Integration of their other term + new constant of integration

Question			Answer	Marks	Guidance
			when $t = 0$ $\quad s = \frac{A}{c}(\frac{1}{c}) + k = 0 \Rightarrow k = -\frac{A}{c^2}$ $s = \frac{A}{c^2}e^{-ct} + (v_0 + \frac{A}{c})t - \frac{A}{c^2}$ oe	**A1** **[4]**	Uses initial conditions to find constant of integration
5	**(iv)**		$34 = \frac{8}{0.3}(1 - e^{-0.3t}) + 10$ $\frac{(34-10)\times 0.3}{8} = (1 - e^{-0.3t}) + 10$ $0.9 - 1 = -e^{-0.3t}$ $t = \frac{\ln 0.1}{-0.3} = (7.67528....)$	**M1** **A1** **[2]**	Substitute all values into their expression for v and attempt to solve AWRT 7.7

Question			Answer	Marks	Guidance
6	(i)			 B1 B1 B1 [4]	Condone missing values on I axis Clear indication of sinusoidal oscillation with origin (0,0) Damping evident in amplitudes of the maxima/minima Crosses the t axis 3 times in the interval to 0.2 All in correct proportions. t values need not be given as long as there is some indication of horizontal scale [Crossing t axis at π/50 (0.06283), 2π/50 (0.12566) and 3π/50 (0.18849)] [Maximum peaks approximately half way between t values 0 and π/50 and between 2π/50 and 3π/50] [Minimum value approximately between t values π/50 and 2π/50]
6	(ii)		$\frac{dI}{dt} = e^{-10t}\,50\cos(50t) - 10e^{-10t}\sin(50t)$ $50\cos(50t) = 10\sin(50t)$ $\tan(50t) = \frac{50}{10} = 5$ $t = 0.027468..(\text{s})$	M1 A1 M1 M1 A1 [5]	Differentiates using product rule to obtain correct form for both elements Equates to zero and attempts to solve Solves equation of form $\tan(50t) = k$ Accept AWRT to 0.027

Question			Answer	Marks	Guidance
6	(iii)		$50t = \pi + 1.373$	M1	Finds solution in 3rd quadrant, FT their $tan^{-1}(k)$
			$[t = 0.090299....]$	M1	Solve for t, FT their k
			$I = -0.39748...$	A1	Accept anything that rounds to −0.4
				[3]	

Question			Answer	Marks	Guidance
7	(i)				Power stroke $p = 39V^2 - 32V + 7$
			$39V^2 - 32V + 5 = 0$	M1	Form a 3-term quadratic
				M1	Solve quadratic to obtain at least one solution
			Solution (0.6105 and) 0.2099	A1	Correct solution(s)
			0.21 only	A1	Reject 0.6105 (assume rejected if 0.6105.. not seen, crossed out or 0.21 highlighted, circled)
				[4]	
7	(ii)		$13V^3 - 16V^2 + 7V$	M1	Integrates power-stroke function to obtain correct form
				A1	
			$-\frac{1}{4}e^{-12V}$	M1	Integrates compression-stroke function to obtain correct form
				A1	
					Applies limits 0.1 and 0.4 correctly
			$[0.519 - 0.073]$	M1	Subtracts areas
			Or $[1.074 - 0.628]$	M1	Or subtracts integrands and then applies limits 0.1 and 0.4 correctly
					AWRT 0.446
			W = 0.44576 (kJ) (445.76 J)	A1	
				[7]	

OCR (Oxford Cambridge and RSA Examinations)
The Triangle Building
Shaftesbury Road
Cambridge
CB2 8EA

OCR Customer Contact Centre

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Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

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Head office
Telephone: 01223 552552
Facsimile: 01223 552553