Cambridge International Examinations
Cambridge Pre-U Certificate

MATHEMATICS (STATISTICS WITH PURE MATHEMATICS) (SHORT COURSE) **1347/01**

Paper 1 Pure Mathematics **For Examination from 2016**

SPECIMEN PAPER

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF21)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 65.

The syllabus is approved for use in England, Wales and Northern Ireland as a Cambridge International Level 3 Pre-U Certificate.

This document consists of **3** printed pages and **1** blank page.

1 A curve has equation $y = x^2 - 6x - 2$.

(i) Write this equation in the form $y = (x - a)^2 - b$. [2]

(ii) Hence write down the minimum value of y and the equation of the line of symmetry of the curve. [2]

(iii) Describe fully the transformation(s) required to map the curve $y = x^2$ onto the curve $y = x^2 - 6x - 2$. [2]

2 A curve has equation $y = 5x^2 - x^3 - 7$.

(i) Find $\frac{dy}{dx}$. [2]

(ii) Hence find the equation of the tangent to the curve at the point where $x = 4$, simplifying your answer. [3]

3 Expand and simplify $(2 + x)^5 + (2 - x)^5$. [3]

4 A curve satisfies $\frac{dy}{dx} = 4x + 3$, and passes through the point (2, 19). Find the equation of the curve. [4]

5 Write as a single logarithm

$$\ln(x + 1) + \ln(x - 1) - 2\ln x.$$ [3]

6 A factory stocks replacement parts for its machinery, and these replacement parts must be reordered at regular time intervals of t months. The total cost £C per year is given by

$$C = 800t + \frac{20000}{t}.$$

Find the minimum total cost per year, and prove that this total cost is a minimum. [5]

7 A rectangular field has length x metres and width y metres, with $x \geqslant y$. Its area is 12 000 square metres and its perimeter is P metres.

(i) In the case $P = 460$, obtain two equations satisfied by x and y, and hence find the dimensions of the field. [5]

(ii) By considering the discriminant of a suitable quadratic equation, show that P cannot be less than $80\sqrt{30}$. [4]

8 **(i)** Show that the equation $x = \frac{10}{x} - \frac{9}{x^3}$ has exactly two positive solutions, $x = 1$ and $x = 3$, and find the other solutions. [4]

(ii) The region R is to the right of the y-axis and is bounded by the curve $y = \frac{10}{x} - \frac{9}{x^3}$ and the line $y = x$. Find an exact expression for the area of R. [5]

9 **(a)** The population of species A, N_A thousand, in a particular region at the end of a time period m years (starting from 1 January 2000) is given by

$$N_A = \sum_{r=1}^{m} 15r\left(\frac{6-r}{5}\right).$$

Find the value of N_A when $m = 5$. [2]

(b) The population of species B, N_B thousand, in the same region at time t years is given by

$$N_B = ab^t,$$

where a and b are constants. It is given that $N_B = 15$ when $t = 0$, and $N_B = 16.873$ when $t = 3$.

(i) Find the value of a and the value of b. [2]

(ii) Find the value of t when $N_B = 20$. [2]

(iii) Write the formula for N_B in the form $N_B = c\mathrm{e}^{kt}$, where c and k are constants to be found. Hence find the rate of increase of the population when $N_B = 20$. [4]

10 $ABCD$ is a rhombus. The points A and C have coordinates (2, 1) and (6, 9) respectively, and AB has equation $7x + 4y = 18$.

(i) Find the coordinates of the mid-point M of AC. [1]

(ii) The diagonal BD is perpendicular to AC and passes through M. Find the equation of BD. [3]

(iii) Use an algebraic method to find the coordinates of B. Find also the coordinates of D. [3]

(iv) By finding the exact lengths of AC and BM, find the area of $ABCD$. [4]

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