Cambridge Pre-U Syllabus

Cambridge International Level 3 Pre-U Certificate in **MATHEMATICS**

For examination in 2010, 2011 and 2012

UNIVERSITY of CAMBRIDGE
International Examinations

Cambridge Pre-U

Mathematics (9794)

Cambridge International Level 3
Pre-U Certificate in Mathematics (Principal)

For examination in 2010, 2011 and 2012

QAN 500/3789/4

Support

CIE provides comprehensive support for all its qualifications, including the Cambridge Pre-U. There are resources for teachers and candidates written by experts. CIE also endorses a range of materials from other publishers to give a choice of approach. More information on what is available for this particular syllabus can be found at www.cie.org.uk

Syllabus Updates

This syllabus is for teaching from 2008 and is valid for examination in 2010, 2011 and 2012.

If there are any changes to this syllabus CIE will write to Centres to inform them. This syllabus will also be published annually on the CIE website (www.cie.org.uk/cambridgepreu). The version of the syllabus on the website should always be considered as the definitive version.

Further copies of this, or any other Cambridge Pre-U syllabus, can be obtained by either downloading from our website www.cie.org.uk/cambridgepreu

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Cambridge International Level 3 Pre-U Certificate

Mathematics

9794

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Introduction

Cambridge Pre-U syllabuses aim to equip candidates with the skills required to make a success of their subsequent studies at university, involving not only a solid grounding in each specialist subject at an appropriate level, but also the ability to undertake independent and self-directed learning and to think laterally, critically and creatively. The Cambridge Pre-U curriculum is underpinned by a core set of educational principles:

- A programme of study which supports the development of well-informed, open and independent-minded individuals capable of applying their skills to meet the demands of the world as they will find it and over which they may have influence.
- A curriculum which retains the integrity of subject specialisms and which can be efficiently, effectively and reliably assessed, graded and reported to meet the needs of universities.
- A curriculum which is designed to recognise a wide range of individual talents, interests and abilities and which provides the depth and rigour required for a university degree course.
- A curriculum which encourages the acquisition of specific skills and abilities, in particular the skills of problem solving, creativity, critical thinking, team working and effective communication.
- The encouragement of 'deep understanding' in learning – where that deep understanding is likely to involve higher order cognitive activities.
- The development of a perspective which equips young people to understand a range of different cultures and ideas and to respond successfully to the opportunity for international mobility.

All Cambridge Pre-U syllabuses are linear. A candidate taking a Principal Subject must take all the components together at the end of the course in the same examination session.

Studying Mathematics involves the acquisition of skills that can be applied in a wide range of contexts. The syllabus is designed to encourage teaching and learning which enable candidates to develop a positive attitude towards the subject by developing an understanding of Mathematics and mathematical processes in a way that promotes confidence and enjoyment. Throughout this course candidates are expected to develop two parallel strands of Mathematics, Pure Mathematics and Applications of Mathematics.

The study of Mathematics encourages the development of logical thought and problem solving skills. Whilst it is a satisfying subject in its own right, Mathematics is also a prerequisite for further study in an increasing range of subjects. For this reason, candidates following this course will be expected to apply their mathematical knowledge in the contexts of both Mechanics and Probability and will also be presented with less familiar scenarios. This syllabus provides a sound foundation in Mathematics for higher education courses or other career pathways.

Aims

1. To enable candidates to develop a range of mathematical skills and techniques, appreciating their applications in a wide range of contexts, and to apply these techniques to problem solving in familiar and less familiar contexts.
2. To enable candidates to develop an understanding of how different branches of mathematics are connected.
3. To enable candidates to recognise how a situation may be represented mathematically and understand how mathematical models can be refined.
4. To encourage candidates to use mathematics as an effective means of communication, through the use of correct mathematical language and notation and through the construction of sustained logical arguments, including an appreciation of the limitations of calculator use in relation to obtaining exact solutions.

Assessment Objectives

Candidates will be expected to:

AO1	manipulate mathematical expressions accurately; round answers to an appropriate degree of accuracy and understand the limitations of solutions obtained using calculators.
AO2	construct rigorous mathematical arguments and proofs through the use of precise statements and logical deduction, including extended arguments for problems presented in unstructured form.
AO3	recall, select and apply their knowledge of mathematical facts, concepts and techniques in a variety of contexts.
AO4	understand how mathematics can be used to model situations in the real world and solve problems in relation to both standard models and less familiar contexts, interpreting their results.

Scheme of Assessment

Candidates taking the Cambridge Pre-U Principal Subject qualification in Mathematics take both papers together in the same session.

Component	Component Title	Duration	Weighting (%)	Type of assessment
Paper 1	Pure Mathematics and Probability	3 hours	50	Written paper, externally set and marked
Paper 2	Pure Mathematics and Mechanics	3 hours	50	Written paper, externally set and marked

Weightings of Assessment Objectives

The question papers will be constructed so that, for the assessment as a whole, the relative weightings of the Assessment Objectives fall within the ranges shown in the following table.

Assessment Objective	AO1	AO2	AO3	AO4
Percentage weighting	35–40	8–13	35–40	15–20

Description of Papers

For both papers, knowledge of the content of GCSE/IGCSE or O Level Mathematics is assumed.

Paper 1 and Paper 2 may contain questions on any topics from the Pure Mathematics syllabus content.

Paper 1 and Paper 2 – Pure Mathematics

Quadratics
Algebra
Functions
Coordinate Geometry
Circular Measure
Trigonometry
Sequences and Series
Logarithms and Exponentials
Differentiation
Integration
Vector Geometry
Differential Equations
Complex Numbers
Numerical Methods

Paper 1 – Pure Mathematics and Probability

Pure Mathematics (see list above)

Probability

Analysis of Data
Probability Laws
Permutations and Combinations
Discrete Random Variables
The Normal Distribution

The paper will consist of a mixture of short, medium and longer questions with a total of 120 marks, of which approximately 80 marks will relate to Pure Mathematics and 40 marks to the Applications of Mathematics. In addition to the topics listed, candidates will be expected to apply their knowledge of Pure Mathematics to questions set in less familiar contexts. Candidates will be expected to answer all questions.

Paper 2 – Pure Mathematics and Mechanics

Pure Mathematics (see list above)

Mechanics

- Kinematics of Motion in a Straight Line
- Force and Equilibrium
- Friction
- Newton's Laws of Motion
- Linear Momentum and Impulse
- Motion of a Projectile

The paper will consist of a mixture of short, medium and longer questions with a total of 120 marks, of which approximately 80 marks will relate to Pure Mathematics and 40 marks to the Applications of Mathematics. In addition to the topics listed, candidates will be expected to apply their knowledge of Pure Mathematics to questions set in less familiar contexts. Candidates will be expected to answer all questions.

USE OF CALCULATORS

The use of scientific calculators will be permitted in all papers. Graphic calculators will not be permitted. Candidates will be expected to be aware of the limitations inherent in the use of calculators.

MATHEMATICAL TABLES AND FORMULAE

Candidates will be provided with a booklet of mathematical formulae and tables for use in the examination.

Curriculum Content

Paper 1 and Paper 2: PURE MATHEMATICS CONTENT

Throughout, candidates should be familiar with the logical processes and conventional symbolic machinery involved in a mathematical development or proof by direct argument. They should also understand the methods of proof by exhaustion and contradiction, and of disproof by counterexample.

Candidates should be able to:

PURE MATHEMATICS

Quadratics

(a) carry out the process of completing the square for a quadratic polynomial $ax^2 + bx + c$, and understand the relationship between this form and the graph of the associated curve;

(b) find the discriminant of a quadratic polynomial $ax^2 + bx + c$, and understand how this relates to the number of real roots of the equation $ax^2 + bx + c = 0$;

(c) manipulate expressions involving surds;

(d) solve quadratic equations, and linear and quadratic inequalities in one unknown;

(e) solve, by substitution, a pair of simultaneous equations, of which one is linear and the other is quadratic;

(f) recognise and solve equations that are quadratic in some function.

Algebra

(a) understand the meaning of $|x|$, and use relations such as $|a| = |b| \Leftrightarrow a^2 = b^2$ and $|x - a| < b \Leftrightarrow a - b < x < a + b$ in the course of solving equations and inequalities;

(b) divide a polynomial, of degree not exceeding 4, by a linear or quadratic polynomial, and identify the quotient and the remainder (which may be zero);

(c) use the factor theorem and the remainder theorem;

(d) recall an appropriate form for expressing rational functions in partial fractions and carry out the decomposition, in cases where the denominator is no more complicated than

$(ax + b)(cx + d)(ex + f)$

$(ax + b)(cx + d)^2$

$(ax + b)(c^2x^2 + d^2)$

and where the degree of the numerator does not exceed that of the denominator.

Functions

(a) understand the terms function, domain, codomain, range, one-one function, inverse function and composite function;

(b) identify the range of a given function in simple cases and find the composite of two given functions;

(c) determine whether a function is one-one and find the inverse of a one-one function in simple cases;

(d) understand the relationship between the graphs of a one-one function and its inverse.

Coordinate Geometry

(a) find the length, gradient and mid-point of a line segment, given the coordinates of the end points;

(b) find the equation of a straight line given sufficient information (e.g. two points, or one point and the gradient);

(c) understand and use the relationships between the gradients of parallel and perpendicular lines;

(d) interpret and use linear equations in context;

(e) understand the relationship between a graph and its associated algebraic equation and use the relationship between points of intersection of graphs and solutions of equations (including, for simple cases, the relationship between tangents and repeated roots);

(f) recognise and use the equation of a circle;

(g) understand and use the transformations of graphs given by $y = f(x) + a$, $y = f(x + a)$, $y = af(x)$, $y = f(ax)$, $y = -f(x)$, $y = f(-x)$ and simple combinations of these.

Circular Measure

(a) understand the definition of radian measure and use the relationship between radians and degrees;

(b) use the formulae $s = r\theta$ and $A = \frac{1}{2} r^2\theta$ in solving problems concerning the arc length and sector area of a circle.

Trigonometry

(a) sketch and use the graphs of the sine, cosine and tangent functions (for angles of any size and using degrees or radians), e.g. $y = \sin 3x$, or $y = \cos (x + 30°)$;

(b) use the exact values of sine, cosine and tangent of 30°, 45°, 60° and related angles;

(c) solve problems using the sine and cosine rules; use the formula $\frac{1}{2}ab\sin C$ for the area of a triangle;

(d) find all the solutions, lying in specified intervals, of simple trigonometric equations, e.g. solve $2\tan 3\theta = -1$, for $0 \leqslant \theta < 2\pi$;

(e) understand the use of $\sin^{-1}x$, $\cos^{-1}x$ and $\tan^{-1}x$ to denote the principal values of the inverse trigonometric functions;

(f) understand the relationship of the secant, cosecant and cotangent functions to cosine, sine and tangent and use properties and graphs of all six trigonometric functions for angles of any magnitude;

(g) use trigonometric identities for the simplification of expressions and in the course of solving equations, and select an identity or identities appropriate to the context, showing familiarity in particular with the use of

$\frac{\sin\theta}{\cos\theta} = \tan\theta$ and $\sin^2\theta + \cos^2\theta = 1$,

$\sec^2\theta = 1 + \tan^2\theta$ and $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$,

the expansions of $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$,

the formulae for $\sin 2A$, $\cos 2A$ and $\tan 2A$,

the expressions of $a\sin\theta + b\cos\theta$ in the forms $R\sin(\theta \pm \alpha)$ and $R\cos(\theta \pm \alpha)$.

Sequences and Series

(a) use formulae for the nth term of a sequence, and for the sum of a finite series;
(b) understand and use the sigma notation;
(c) recognise arithmetic and geometric progressions;
(d) use the formulae for nth term and sum of the first n terms to solve problems involving arithmetic and geometric progressions;
(e) use the condition for the convergence of a geometric progression and the formula for the sum to infinity of a convergent geometric progression;
(f) use the binomial expansion of
$(a + b)^n$, where n is a positive integer,
$(1 + x)^n$, where n is a rational number and $|x| < 1$ and be able to adapt this method to the expansion of expressions of the form $(a + bx^c)^n$, stating the range of values for which such an expansion is valid;
(g) understand the use of recurrence relations in defining some sequences, and recognise alternating, periodic, convergent and divergent sequences.

Logarithms and Exponentials

(a) understand the relationship between logarithms and indices and use the laws of logarithms;
(b) use logarithms – for example, to solve equations of the form $a^x = b$ and related equations or inequalities;
(c) understand the definition and properties of e^x and $\ln x$, including their relationship as inverse functions and their graphs;
(d) use logarithms to transform appropriate relationships to linear form.

Differentiation

(a) understand the idea of the gradient of a curve and use the notations $f'(x)$, $f''(x)$, $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$;
(b) understand the idea of 'differentiation from first principles' and be able to carry it out for x^n, where n is a positive or negative integer or $\pm\frac{1}{2}$;
(c) use the derivatives of x^n (for any rational n), e^x, $\ln x$, $\sin x$, $\cos x$, $\tan x$ and constant multiples, sums and differences;
(d) apply differentiation to gradients, tangents and normals, increasing and decreasing functions and rates of change;
(e) locate stationary points and determine by calculation whether the points are local maximum or minimum points (the ability to distinguish between maximum points and minimum points is required, but identification of points of inflexion is not included);
(f) use the chain rule for differentiation;
(g) differentiate products and quotients;
(h) determine the first and second derivatives of a function defined either parametrically or implicitly;
(i) sketch graphs of functions defined parametrically or implicitly, and find the cartesian equation of a function defined parametrically.

Integration

(a) understand integration as the reverse process of differentiation and integrate x^n (for all rational n), e^x, sin x, cos x and $\sec^2 x$, together with constant multiples, sums and differences;

(b) solve problems involving the evaluation of a constant of integration;

(c) use definite integration to find the area of a region bounded by a curve and lines parallel to the axes or between two curves;

(d) use trigonometric relationships (such as double angle formulae) to integrate functions such as $\cos^2 x$;

(e) integrate rational functions by means of decomposition into partial fractions;

(f) recognise and integrate an integrand of the form $\frac{kf'(x)}{f(x)}$;

(g) use integration by parts to integrate suitable products;

(h) use substitution to simplify and evaluate definite or indefinite integrals;

(i) calculate a volume of revolution about the x- or y-axis, to include the volume of a solid generated by rotating a region between two curves.

Vector Geometry

(a) use standard notations for vectors, and be familiar with the terms 'position vector' and 'displacement vector';

(b) carry out addition and subtraction of vectors and multiplication of a vector by a scalar, and interpret these operations in geometric terms;

(c) calculate the magnitude of a vector;

(d) calculate the scalar product of two vectors and use the scalar product to determine the angle between two vectors and to solve problems involving perpendicular vectors;

(e) understand and use the vector form of a straight line $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$; understand and use the equivalent cartesian form of a straight line;

(f) determine whether two lines are parallel, intersect or are skew;

(g) find the angle between two lines and the point of intersection of two lines when it exists.

Differential Equations

(a) formulate a simple statement involving a rate of change as a differential equation, introducing, if necessary, a constant of proportionality;

(b) find by integration a general form of solution for a first order differential equation in which the variables are separable;

(c) use an initial condition to find a particular solution;

(d) interpret the solution of a differential equation in the context of a problem being modelled by the equation.

Complex Numbers

(a) understand the definition of a complex number, and the meaning of the terms real part, imaginary part, modulus, argument, conjugate, and use the fact that two complex numbers are equal if and only if both real and imaginary parts are equal;
(b) carry out operations of addition, subtraction, multiplication and division of two complex numbers expressed in cartesian form $x + iy$;
(c) find the complex roots of a quadratic equation with real coefficients, or of a cubic equation with real coefficients where one real factor can be identified by the factor theorem;
(d) recognize that complex roots of polynomial equations occur in conjugate pairs when the coefficients are real;
(e) represent complex numbers geometrically by means of an Argand diagram.

Numerical Methods

(a) investigate the location of roots of $f(x) = 0$ by a change of sign of $y = f(x)$ over an interval;
(b) implement the direct iteration method for the numerical evaluation of an approximation to a root of $x = F(x)$, understand and be able to construct staircase and cobweb diagrams and recognise informally the relationship between the magnitude of the derivative of F at the root and the convergence or divergence of the iterative scheme;
(c) implement the Newton-Raphson iteration method for the numerical evaluation of an approximation to a root of $f(x) = 0$ and understand the geometric derivation of this method;
(d) understand the concept of rate of convergence of an iterative scheme, and in particular how this concept is realised for the schemes in (b) and (c).

Paper 1: PROBABILITY CONTENT

Candidates should be able to:

Analysis of Data

(a) use and interpret different measures of central tendency (mean, median, mode) and variation (range, interquartile range, standard deviation), e.g. in comparing and contrasting sets of data;
(b) calculate the mean, standard deviation and variance from raw data or summary statistics;
(c) identify outliers (using the '1.5 × IQR' criterion) and describe whether a set of data has positive or negative skew;
(d) understand the concepts of dependent and independent variables, linear correlation and regression lines for bivariate data;
(e) use the product-moment correlation coefficient as a measure of correlation, and use covariance and variance in the construction of regression lines.

Probability Laws

(a) evaluate probabilities in simple cases by calculation using permutations and combinations;
(b) understand the meaning of exclusive and independent events, and calculate and use conditional probabilities in simple cases, e.g. situations that can be represented by means of a tree diagram;
(c) use the notation $P(A)$ for the probability of the event A, and the notations $P(A \cup B)$, $P(A \cap B)$, $P(A|B)$ relating to probabilities involving two events;
(d) understand and use the result $P(A \cup B) = P(A) + P(B) - P(A \cap B)$;
(e) use the result $P(A \cap B) = P(A)\,P(B|A) = P(B)P(A|B)$ to solve problems involving conditional probability.

Permutations and Combinations

(a) solve problems about selections, e.g. finding the number of ways in which a team of 3 men and 2 women can be selected from a group of 6 men and 5 women;
(b) solve problems about arrangements of objects in a line, including those involving
  (i) repetition (e.g. the number of ways of arranging the letters of the word 'STATISTICS'),
  (ii) restriction (e.g. the number of ways several people can stand in a line if 2 particular people must, or must not, stand next to each other).

Discrete Random Variables

(a) use formulae for probabilities for the binomial and geometric distributions, model given situations by one of these as appropriate, and recognise the notations $B(n, p)$ and $Geo(p)$;
(b) use tables of cumulative binomial probabilities;
(c) construct a probability distribution table relating to a given situation involving a discrete random variable X, and calculate the expectation, variance and standard deviation of X;
(d) use formulae for the expectation and variance of the binomial and geometric distributions.

The Normal Distribution

(a) understand the use of a normal distribution to model a continuous random variable, and use normal distribution tables;
(b) solve problems concerning a variable X, where $X \sim N(\mu, \sigma^2)$, including
  (i) using given values of μ and σ to find the value of $P(X < x)$, or a related probability, or conversely to find the relevant value of x;
  (ii) finding the values of μ and σ from given probabilities.

Paper 2: MECHANICS CONTENT

Candidates should be able to:

Kinematics of Motion in a Straight Line

(a) recognise distance and speed as scalar quantities and displacement, velocity and acceleration as vector quantities;
(b) sketch and interpret displacement-time and velocity-time graphs, and use the facts that
  (i) the area under a v-t graph represents displacement,
  (ii) the gradient of an x-t graph represents velocity,
  (iii) the gradient of a v-t graph represents acceleration;
(c) use differentiation and integration with respect to time to solve simple problems concerning displacement, velocity and acceleration;
(d) use the formulae for motion with constant acceleration.

Force and Equilibrium

(a) understand the vector nature of force and use directed line segments to represent forces;
(b) calculate the resultant of two or more forces acting at a point and use vector addition to solve problems involving resultants and components of forces;
(c) find and use perpendicular components of a force, e.g. in finding the resultant of a system of forces, or to calculate the magnitude and direction of a force;
(d) identify the forces acting in a given situation, including weight, tension, normal reaction and friction;
(e) use the principle that a particle is in equilibrium if and only if the vector sum of the forces acting is zero, or equivalently if and only if the sum of the resolved parts in any given direction is zero;
(f) use the model of a 'smooth' contact and understand the limitations of the model.

Friction

(a) represent the contact force between two rough surfaces by two components, the 'normal force' and the 'frictional force';
(b) recall the definition of coefficient of friction;
(c) understand the concept of limiting friction and the relation $F \leq \mu R$;
(d) use the friction law in problems that involve equilibrium of a particle or the use of Newton's laws.

Newton's Laws of Motion

(a) apply Newton's laws of motion to the linear motion of bodies of constant mass moving under the action of constant forces;
(b) model the motion of a body moving vertically or on an inclined plane as motion with constant acceleration, and understand any limitations of this model;
(c) solve simple problems which may be modelled as the motion of two connected particles, for example a car towing a caravan, or two objects connected by a light inextensible string passing over a fixed smooth peg or light pulley;
(d) use Newton's third law, for example to calculate the reaction on a particle in an accelerating lift.

Linear Momentum and Impulse

(a) understand the vector nature of linear momentum of a particle or system of particles moving in a straight line;

(b) understand the concept of the impulse of a force causing a change in linear momentum, either where a constant force acts for a finite time or where an instantaneous collision occurs;

(c) understand and use conservation of linear momentum in simple applications involving the direct impact of two bodies moving in the same straight line, including the case where the bodies coalesce;

(d) understand and use Newton's law of restitution in the course of solving one-dimensional problems that may be modelled as the direct impact of two spheres or as the direct impact of a sphere with a fixed plane surface.

Motion of a Projectile

(a) model the motion of a projectile as a particle moving with constant acceleration and understand any limitations of this model;

(b) use horizontal and vertical equations of motion to solve problems on the motion of projectiles, including finding the magnitude and direction of the velocity at a given time or position, the range on a horizontal plane and the greatest height reached;

(c) derive and use the cartesian equation of the trajectory of a projectile, including problems in which the initial speed and/or angle of projection may be unknown.

Mathematical Formulae and Statistical Tables

PURE MATHEMATICS

Mensuration

Surface area of sphere = $4\pi r^2$

Area of curved surface of cone = $\pi r \times$ slant height

Trigonometry

$a^2 = b^2 + c^2 - 2bc \cos A$

Arithmetic series

$u_n = a + (n-1)d$

$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$

Geometric series

$u_n = ar^{n-1}$

$S_n = \frac{a(1 - r^n)}{1 - r}$

$S_\infty = \frac{a}{1 - r}$ for $|r| < 1$

Summations

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$$

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

Binomial Series

$$\binom{n}{r} + \binom{n}{r+1} = \binom{n+1}{r+1}$$

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n, \ (n \in \mathbb{N}), \text{ where } \binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1.2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1.2\ldots r}x^r + \ldots \ (|x| < 1, \ n \in \mathbb{R})$$

Logarithms and exponentials

$e^{x \ln a} = a^x$

Complex Numbers

$\{r(\cos\theta + \mathrm{i}\sin\theta)\}^n = r^n(\cos n\theta + \mathrm{i}\sin n\theta)$

$e^{\mathrm{i}\theta} = \cos\theta + \mathrm{i}\sin\theta$

The roots of $z^n = 1$ are given by $z = e^{\frac{2\pi k\,\mathrm{i}}{n}}$, for $k = 0, 1, 2, \ldots, n-1$

Maclaurin's Series

$$f(x) = f(0) + x\,f'(0) + \frac{x^2}{2!}f''(0) + \ldots + \frac{x^r}{r!}f^{(r)}(0) + \ldots$$

$$e^x = \exp(x) = 1 + x + \frac{x^2}{2!} + \ldots + \frac{x^r}{r!} + \ldots \quad \text{for all } x$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + (-1)^{r+1}\frac{x^r}{r} + \ldots \quad (-1 < x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + (-1)^r\frac{x^{2r+1}}{(2r+1)!} + \ldots \quad \text{for all } x$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + (-1)^r\frac{x^{2r}}{(2r)!} + \ldots \quad \text{for all } x$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \ldots + (-1)^r\frac{x^{2r+1}}{2r+1} + \ldots \quad (-1 \leq x \leq 1)$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots + \frac{x^{2r+1}}{(2r+1)!} + \ldots \quad \text{for all } x$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots + \frac{x^{2r}}{(2r)!} + \ldots \quad \text{for all } x$$

$$\tanh^{-1} x = x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots + \frac{x^{2r+1}}{2r+1} + \ldots \quad (-1 < x < 1)$$

Hyperbolic Functions

$$\cosh^2 x - \sinh^2 x = 1$$

$$\sinh 2x = 2\sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

$$\cosh^{-1} x = \ln\{x + \sqrt{x^2 - 1}\} \quad (x \geq 1)$$

$$\sinh^{-1} x = \ln\{x + \sqrt{x^2 + 1}\}$$

$$\tanh^{-1} x = \tfrac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \quad (|x| < 1)$$

Coordinate Geometry

The perpendicular distance from (h, k) to $ax + by + c = 0$ is $\dfrac{|ah + bk + c|}{\sqrt{a^2 + b^2}}$

The acute angle between lines with gradients m_1 and m_2 is $\tan^{-1}\left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$

Trigonometric Identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad (A \pm B \neq (k + \tfrac{1}{2})\pi)$$

For $t = \tan\frac{1}{2}A$: $\sin A = \dfrac{2t}{1+t^2}$, $\cos A = \dfrac{1-t^2}{1+t^2}$

$$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$$

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$

Vectors

The resolved part of $\mathbf{a}$ in the direction of $\mathbf{b}$ is $\dfrac{\mathbf{a}.\mathbf{b}}{|\mathbf{b}|}$

The point dividing AB in the ratio $\lambda : \mu$ is $\dfrac{\mu\mathbf{a} + \lambda\mathbf{b}}{\lambda + \mu}$

Vector product: $\mathbf{a} \times \mathbf{b} = |\mathbf{a}||\mathbf{b}| \sin\theta\, \hat{\mathbf{n}} = \begin{vmatrix} \mathbf{i} & a_1 & b_1 \\ \mathbf{j} & a_2 & b_2 \\ \mathbf{k} & a_3 & b_3 \end{vmatrix} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$

If A is the point with position vector $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and the direction vector $\mathbf{b}$ is given by $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$, then the straight line through A with direction vector $\mathbf{b}$ has cartesian equation

$$\frac{x - a_1}{b_1} = \frac{y - a_2}{b_2} = \frac{z - a_3}{b_3} \ (= \lambda)$$

The plane through A with normal vector $\mathbf{n} = n_1\mathbf{i} + n_2\mathbf{j} + n_3\mathbf{k}$ has cartesian equation $n_1x + n_2y + n_3z + d = 0$ where $d = -\mathbf{a}.\mathbf{n}$

The plane through non-collinear points A, B and C has vector equation $\mathbf{r} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a}) + \mu(\mathbf{c} - \mathbf{a}) = (1 - \lambda - \mu)\mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$

The plane through the point with position vector $\mathbf{a}$ and parallel to $\mathbf{b}$ and $\mathbf{c}$ has equation $\mathbf{r} = \mathbf{a} + s\mathbf{b} + t\mathbf{c}$

The perpendicular distance of (α, β, γ) from $n_1x + n_2y + n_3z + d = 0$ is $\dfrac{|n_1\alpha + n_2\beta + n_3\gamma + d|}{\sqrt{n_1^2 + n_2^2 + n_3^2}}$

Matrix Transformations

Anticlockwise rotation through θ about O: $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

Reflection in the line $y = (\tan\theta)x$: $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sin^{-1} x$	$\dfrac{1}{\sqrt{1 - x^2}}$
$\cos^{-1} x$	$-\dfrac{1}{\sqrt{1 - x^2}}$
$\tan^{-1} x$	$\dfrac{1}{1 + x^2}$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\operatorname{sech}^2 x$
$\sinh^{-1} x$	$\dfrac{1}{\sqrt{1 + x^2}}$
$\cosh^{-1} x$	$\dfrac{1}{\sqrt{x^2 - 1}}$
$\tanh^{-1} x$	$\dfrac{1}{1 - x^2}$

Integration *(+ constant; a > 0 where relevant)*

f(x)	$\int f(x)\,dx$
$\sec^2 kx$	$\frac{1}{k}\tan kx$
$\tan x$	$\ln\lvert\sec x\rvert$
$\cot x$	$\ln\lvert\sin x\rvert$
$\operatorname{cosec} x$	$-\ln\lvert\operatorname{cosec} x + \cot x\rvert = \ln\lvert\tan(\frac{1}{2}x)\rvert$
$\sec x$	$\ln\lvert\sec x + \tan x\rvert = \ln\lvert\tan(\frac{1}{2}x + \frac{1}{4}\pi)\rvert$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\ln\cosh x$
$\frac{1}{\sqrt{a^2 - x^2}}$	$\sin^{-1}\left(\frac{x}{a}\right)$ $(\lvert x\rvert < a)$
$\frac{1}{a^2 + x^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$
$\frac{1}{\sqrt{x^2 - a^2}}$	$\cosh^{-1}\left(\frac{x}{a}\right) = \ln\left\{x + \sqrt{x^2 - a^2}\right\}$ $(x > a)$
$\frac{1}{\sqrt{a^2 + x^2}}$	$\sinh^{-1}\left(\frac{x}{a}\right) = \ln\left\{x + \sqrt{x^2 + a^2}\right\}$
$\frac{1}{a^2 - x^2}$	$\frac{1}{2a}\ln\left\lvert\frac{a+x}{a-x}\right\rvert = \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right)$ $(\lvert x\rvert < a)$
$\frac{1}{x^2 - a^2}$	$\frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

Area of a sector

$A = \frac{1}{2}\int r^2\,d\theta$ (polar coordinates)

$$A = \frac{1}{2}\int\left(x\frac{dy}{dt} - y\frac{dx}{dt}\right)dt$$ (parametric form)

Arc Length

$$s = \int\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$ (cartesian coordinates)

$$s = \int\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$ (parametric form)

$$s = \int\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\,d\theta$$ (polar form)

Surface area of revolution

$S_x = 2\pi \int y \, ds$

$S_y = 2\pi \int x \, ds$

Numerical Solution of Equations

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

MECHANICS

Motion in a circle

Transverse velocity: $v = r\dot{\theta}$

Transverse acceleration: $\dot{v} = r\ddot{\theta}$

Radial acceleration: $-r\dot{\theta}^2 = -\dfrac{v^2}{r}$

PROBABILITY

Probability

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A)\, P(B \mid A)$

$$P(A \mid B) = \frac{P(B \mid A)\, P(A)}{P(B \mid A)\, P(A) + P(B \mid A')\, P(A')}$$

Bayes' Theorem: $P(A_j \mid B) = \dfrac{P(A_j)\, P(B \mid A_j)}{\Sigma P(A_i)\, P(B \mid A_i)}$

Discrete distributions

For a discrete random variable X taking values x_i with probabilities p_i

Expectation (mean): $E(X) = \mu = \Sigma\, x_i p_i$

Variance: $\mathrm{Var}(X) = \sigma^2 = \Sigma (x_i - \mu)^2\, p_i = \Sigma x_i^2\, p_i - \mu^2$

For a function $g(X)$: $E(g(X)) = \Sigma\, g(x_i)\, p_i$

The probability generating function of X is $G_X(t) = E(t^X)$, and $E(X) = G_X'(1)$,

$\mathrm{Var}(X) = G_X''(1) + G_X'(1) - \{G_X'(1)\}^2$

For $Z = X + Y$, where X and Y are independent: $G_Z(t) = G_X(t)\, G_Y(t)$

Standard discrete distributions

Distribution of X	$P(X = x)$	Mean	Variance	P.G.F.
Binomial $B(n, p)$	$\binom{n}{x} p^x (1-p)^{n-x}$	np	$np(1-p)$	$(1 - p + pt)^n$
Poisson $Po(\lambda)$	$e^{-\lambda} \dfrac{\lambda^x}{x!}$	λ	λ	$e^{\lambda(t-1)}$
Geometric $Geo(p)$ on 1, 2, ...	$p(1-p)^{x-1}$	$\dfrac{1}{p}$	$\dfrac{1-p}{p^2}$	$\dfrac{pt}{1-(1-p)t}$

Continuous distributions

For a continuous random variable X having probability density function f

Expectation (mean): $\mathrm{E}(X) = \mu = \int x\, \mathrm{f}(x)\, \mathrm{d}x$

Variance: $\mathrm{Var}(X) = \sigma^2 = \int (x-\mu)^2\, \mathrm{f}(x)\, \mathrm{d}x = \int x^2\, \mathrm{f}(x)\, \mathrm{d}x - \mu^2$

For a function g(X) : $\mathrm{E}(\mathrm{g}(X)) = \int \mathrm{g}(x)\, \mathrm{f}(x)\, \mathrm{d}x$

Cumulative distribution function: $\mathrm{F}(x) = \mathrm{P}(X \leq x) = \int_{-\infty}^{x} \mathrm{f}(t)\, \mathrm{d}t$

The moment generating function of X is $\mathrm{M}_X(t) = \mathrm{E}(\mathrm{e}^{tX})$ and $\mathrm{E}(X) = \mathrm{M}_X'(0)$, $\mathrm{E}(X^n) = \mathrm{M}_X^{(n)}(0)$, $\mathrm{Var}(X) = \mathrm{M}_X''(0) - \{\mathrm{M}_X'(0)\}^2$

For $Z = X + Y$, where X and Y are independent: $\mathrm{M}_Z(t) = \mathrm{M}_X(t)\, \mathrm{M}_Y(t)$

Standard continuous distributions

Distribution of X	P.D.F.	Mean	Variance	M.G.F
Uniform (Rectangular) on $[a, b]$	$\frac{1}{b-a}$	$\frac{1}{2}(a+b)$	$\frac{1}{12}(b-a)^2$	$\frac{\mathrm{e}^{bt} - \mathrm{e}^{at}}{(b-a)t}$
Exponential	$\lambda\, \mathrm{e}^{-\lambda x}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	$\frac{\lambda}{\lambda - t}$
Normal $\mathrm{N}(\mu, \sigma^2)$	$\frac{1}{\sigma\sqrt{2\pi}}\, \mathrm{e}^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$	μ	σ^2	$\mathrm{e}^{\mu t + \frac{1}{2}\sigma^2 t^2}$

Expectation algebra

For independent random variables X and Y

$\mathrm{E}(XY) = \mathrm{E}(X)\, \mathrm{E}(Y)$, $\mathrm{Var}\,(aX \pm bY) = a^2\, \mathrm{Var}(X) + b^2\, \mathrm{Var}\,(Y)$

Sampling Distributions

For a random sample $X_1, X_2, \ldots, X_n$ of n independent observations from a distribution having mean μ and variance σ^2

$\bar{X}$ is an unbiased estimator of μ, with $\mathrm{Var}(\bar{X}) = \frac{\sigma^2}{n}$

S^2 is an unbiased estimator of σ^2, where $S^2 = \frac{\Sigma (X_i - \bar{X})^2}{n-1}$

For a random sample of n observations from $\mathrm{N}(\mu, \sigma^2)$

$$\frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim \mathrm{N}(0,1$$

$\frac{\bar{X} - \mu}{S / \sqrt{n}} \sim t_{n-1}$ (also valid in matched-pairs situations)

If X is the observed number of successes in n independent Bernoulli trials in each of which the probability of success is p, and $Y = \frac{X}{n}$, then $\mathrm{E}(Y) = p$ and $\mathrm{Var}(Y) = \frac{p(1-p)}{n}$

For a random sample of n_x observations from $N(\mu_x, \sigma_x^2)$ and, independently, a random sample of n_y observations from $N(\mu_y, \sigma_y^2)$

$$\frac{(\overline{X}-\overline{Y})-(\mu_x-\mu_y)}{\sqrt{\frac{\sigma_x^2}{n_x}+\frac{\sigma_y^2}{n_y}}} \sim N(0,$$

If $\sigma_x^2 = \sigma_y^2 = \sigma^2$ (unknown) then $\frac{(\overline{X}-\overline{Y})-(\mu_x-\mu_y)}{\sqrt{S_p^2\left(\frac{1}{n_x}+\frac{1}{n_y}\right)}} \sim t_{n_x+n_y-2}$, where $S_p^2 = \frac{(n_x-1)S_x^2+(n_y-1)S_y^2}{n_x+n_y-2}$

Correlation and Regression

For a set of n pairs of values (x_i, y_i)

$$S_{xx} = \Sigma(x_i-\overline{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}$$

$$S_{yy} = \Sigma(y_i-\overline{y})^2 = \Sigma y_i^2 - \frac{(\Sigma y_i)^2}{n}$$

$$S_{xy} = \Sigma(x_i-\overline{x})(y_i-\overline{y}) = \Sigma x_i y_i - \frac{(\Sigma x_i)(\Sigma y_i)}{n}$$

The product-moment correlation coefficient is

$$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{\Sigma(x_i-\overline{x})(y_i-\overline{y})}{\sqrt{\{\Sigma(x_i-\overline{x})^2\}\{\Sigma(y_i-\overline{y})^2\}}} = \frac{\Sigma x_i y_i - \frac{(\Sigma x_i)(\Sigma y_i)}{n}}{\sqrt{\left(\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}\right)\left(\Sigma y_i^2 - \frac{(\Sigma y_i)^2}{n}\right)}}$$

The regression coefficient of y on x is $b = \frac{S_{xy}}{S_{xx}} = \frac{\Sigma(x_i-\overline{x})(y_i-\overline{y})}{\Sigma(x_i-\overline{x})^2}$

Least squares regression line of y on x is $y = a + bx$ where $a = \overline{y} - b\overline{x}$

CUMULATIVE BINOMIAL PROBABILITIES

$n = 5$																								
p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
$x = 0$	0.7738	0.5905	0.4437	0.4019	0.3277	0.2373	0.1681	0.1317	0.1160	0.0778	0.0503	0.0313	0.0185	0.0102	0.0053	0.0041	0.0024	0.0010	0.0003	0.0001	0.0001	0.0000	0.0000
1	0.9774	0.9185	0.8352	0.8038	0.7373	0.6328	0.5282	0.4609	0.4284	0.3370	0.2562	0.1875	0.1312	0.0870	0.0540	0.0453	0.0308	0.0156	0.0067	0.0033	0.0022	0.0005	0.0000
2	0.9988	0.9914	0.9734	0.9645	0.9421	0.8965	0.8369	0.7901	0.7648	0.6826	0.5931	0.5000	0.4069	0.3174	0.2352	0.2099	0.1631	0.1035	0.0579	0.0355	0.0266	0.0086	0.0012
3	1.0000	0.9995	0.9978	0.9967	0.9933	0.9844	0.9692	0.9547	0.9460	0.9130	0.8688	0.8125	0.7438	0.6630	0.5716	0.5391	0.4718	0.3672	0.2627	0.1962	0.1648	0.0815	0.0226
4	1.0000	1.0000	0.9999	0.9999	0.9997	0.9990	0.9976	0.9959	0.9947	0.9898	0.9815	0.9688	0.9497	0.9222	0.8840	0.8683	0.8319	0.7627	0.6723	0.5981	0.5563	0.4095	0.2262
5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$n = 6$																								
p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
$x = 0$	0.7351	0.5314	0.3771	0.3349	0.2621	0.1780	0.1176	0.0878	0.0754	0.0467	0.0277	0.0156	0.0083	0.0041	0.0018	0.0014	0.0007	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000
1	0.9672	0.8857	0.7765	0.7368	0.6554	0.5339	0.4202	0.3512	0.3191	0.2333	0.1636	0.1094	0.0692	0.0410	0.0223	0.0178	0.0109	0.0046	0.0016	0.0007	0.0004	0.0001	0.0000
2	0.9978	0.9842	0.9527	0.9377	0.9011	0.8306	0.7443	0.6804	0.6471	0.5443	0.4415	0.3438	0.2553	0.1792	0.1174	0.1001	0.0705	0.0376	0.0170	0.0087	0.0059	0.0013	0.0001
3	0.9999	0.9987	0.9941	0.9913	0.9830	0.9624	0.9295	0.8999	0.8826	0.8208	0.7447	0.6563	0.5585	0.4557	0.3529	0.3196	0.2557	0.1694	0.0989	0.0623	0.0473	0.0159	0.0022
4	1.0000	0.9999	0.9996	0.9993	0.9984	0.9954	0.9891	0.9822	0.9777	0.9590	0.9308	0.8906	0.8364	0.7667	0.6809	0.6488	0.5798	0.4661	0.3446	0.2632	0.2235	0.1143	0.0328
5	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9993	0.9986	0.9982	0.9959	0.9917	0.9844	0.9723	0.9533	0.9246	0.9122	0.8824	0.8220	0.7379	0.6651	0.6229	0.4686	0.2649
6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$n = 7$																								
p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
$x = 0$	0.6983	0.4783	0.3206	0.2791	0.2097	0.1335	0.0824	0.0585	0.0490	0.0280	0.0152	0.0078	0.0037	0.0016	0.0006	0.0005	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.9556	0.8503	0.7166	0.6698	0.5767	0.4449	0.3294	0.2634	0.2338	0.1586	0.1024	0.0625	0.0357	0.0188	0.0090	0.0069	0.0038	0.0013	0.0004	0.0001	0.0001	0.0000	0.0000
2	0.9962	0.9743	0.9262	0.9042	0.8520	0.7564	0.6471	0.5706	0.5323	0.4199	0.3164	0.2266	0.1529	0.0963	0.0556	0.0453	0.0288	0.0129	0.0047	0.0020	0.0012	0.0002	0.0000
3	0.9998	0.9973	0.9879	0.9824	0.9667	0.9294	0.8740	0.8267	0.8002	0.7102	0.6083	0.5000	0.3917	0.2898	0.1998	0.1733	0.1260	0.0706	0.0333	0.0176	0.0121	0.0027	0.0002
4	1.0000	0.9998	0.9988	0.9980	0.9953	0.9871	0.9712	0.9547	0.9444	0.9037	0.8471	0.7734	0.6836	0.5801	0.4677	0.4294	0.3529	0.2436	0.1480	0.0958	0.0738	0.0257	0.0038
5	1.0000	1.0000	0.9999	0.9999	0.9996	0.9987	0.9962	0.9931	0.9910	0.9812	0.9643	0.9375	0.8976	0.8414	0.7662	0.7366	0.6706	0.5551	0.4233	0.3302	0.2834	0.1497	0.0444
6	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9995	0.9994	0.9984	0.9963	0.9922	0.9848	0.9720	0.9510	0.9415	0.9176	0.8665	0.7903	0.7209	0.6794	0.5217	0.3017
7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$n = 8$																								
p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
$x = 0$	0.6634	0.4305	0.2725	0.2326	0.1678	0.1001	0.0576	0.0390	0.0319	0.0168	0.0084	0.0039	0.0017	0.0007	0.0002	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.9428	0.8131	0.6572	0.6047	0.5033	0.3671	0.2553	0.1951	0.1691	0.1064	0.0632	0.0352	0.0181	0.0085	0.0036	0.0026	0.0013	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000
2	0.9942	0.9619	0.8948	0.8652	0.7969	0.6785	0.5518	0.4682	0.4278	0.3154	0.2201	0.1445	0.0885	0.0498	0.0253	0.0197	0.0113	0.0042	0.0012	0.0004	0.0002	0.0000	0.0000
3	0.9996	0.9950	0.9786	0.9693	0.9437	0.8862	0.8059	0.7414	0.7064	0.5941	0.4770	0.3633	0.2604	0.1737	0.1061	0.0879	0.0580	0.0273	0.0104	0.0046	0.0029	0.0004	0.0000
4	1.0000	0.9996	0.9971	0.9954	0.9896	0.9727	0.9420	0.9121	0.8939	0.8263	0.7396	0.6367	0.5230	0.4059	0.2936	0.2586	0.1941	0.1138	0.0563	0.0307	0.0214	0.0050	0.0004
5	1.0000	1.0000	0.9998	0.9996	0.9988	0.9958	0.9887	0.9803	0.9747	0.9502	0.9115	0.8555	0.7799	0.6846	0.5722	0.5318	0.4482	0.3215	0.2031	0.1348	0.1052	0.0381	0.0058
6	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9987	0.9974	0.9964	0.9915	0.9819	0.9648	0.9368	0.8936	0.8309	0.8049	0.7447	0.6329	0.4967	0.3953	0.3428	0.1869	0.0572
7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9998	0.9993	0.9983	0.9961	0.9916	0.9832	0.9681	0.9610	0.9424	0.8999	0.8322	0.7674	0.7275	0.5695	0.3366
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

CUMULATIVE BINOMIAL PROBABILITIES

$n = 9$																								
p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
$x = 0$	0.6302	0.3874	0.2316	0.1938	0.1342	0.0751	0.0404	0.0260	0.0207	0.0101	0.0046	0.0020	0.0008	0.0003	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.9288	0.7748	0.5995	0.5427	0.4362	0.3003	0.1960	0.1431	0.1211	0.0705	0.0385	0.0195	0.0091	0.0038	0.0014	0.0010	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.9916	0.9470	0.8591	0.8217	0.7382	0.6007	0.4628	0.3772	0.3373	0.2318	0.1495	0.0898	0.0498	0.0250	0.0112	0.0083	0.0043	0.0013	0.0003	0.0001	0.0000	0.0000	0.0000
3	0.9994	0.9917	0.9661	0.9520	0.9144	0.8343	0.7297	0.6503	0.6089	0.4826	0.3614	0.2539	0.1658	0.0994	0.0536	0.0424	0.0253	0.0100	0.0031	0.0011	0.0006	0.0001	0.0000
4	1.0000	0.9991	0.9944	0.9910	0.9804	0.9511	0.9012	0.8552	0.8283	0.7334	0.6214	0.5000	0.3786	0.2666	0.1717	0.1448	0.0988	0.0489	0.0196	0.0090	0.0056	0.0009	0.0000
5	1.0000	0.9999	0.9994	0.9989	0.9969	0.9900	0.9747	0.9576	0.9464	0.9006	0.8342	0.7461	0.6386	0.5174	0.3911	0.3497	0.2703	0.1657	0.0856	0.0480	0.0339	0.0083	0.0006
6	1.0000	1.0000	1.0000	0.9999	0.9997	0.9987	0.9957	0.9917	0.9888	0.9750	0.9502	0.9102	0.8505	0.7682	0.6627	0.6228	0.5372	0.3993	0.2618	0.1783	0.1409	0.0530	0.0084
7	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9990	0.9986	0.9962	0.9909	0.9805	0.9615	0.9295	0.8789	0.8569	0.8040	0.6997	0.5638	0.4573	0.4005	0.2252	0.0712
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997	0.9992	0.9980	0.9954	0.9899	0.9793	0.9740	0.9596	0.9249	0.8658	0.8062	0.7684	0.6126	0.3698
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$n = 10$																								
p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
$x = 0$	0.5987	0.3487	0.1969	0.1615	0.1074	0.0563	0.0282	0.0173	0.0135	0.0060	0.0025	0.0010	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.9139	0.7361	0.5443	0.4845	0.3758	0.2440	0.1493	0.1040	0.0860	0.0464	0.0233	0.0107	0.0045	0.0017	0.0005	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.9885	0.9298	0.8202	0.7752	0.6778	0.5256	0.3828	0.2991	0.2616	0.1673	0.0996	0.0547	0.0274	0.0123	0.0048	0.0034	0.0016	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000
3	0.9990	0.9872	0.9500	0.9303	0.8791	0.7759	0.6496	0.5593	0.5138	0.3823	0.2660	0.1719	0.1020	0.0548	0.0260	0.0197	0.0106	0.0035	0.0009	0.0003	0.0001	0.0000	0.0000
4	0.9999	0.9984	0.9901	0.9845	0.9672	0.9219	0.8497	0.7869	0.7515	0.6331	0.5044	0.3770	0.2616	0.1662	0.0949	0.0766	0.0473	0.0197	0.0064	0.0024	0.0014	0.0001	0.0000
5	1.0000	0.9999	0.9986	0.9976	0.9936	0.9803	0.9527	0.9234	0.9051	0.8338	0.7384	0.6230	0.4956	0.3669	0.2485	0.2131	0.1503	0.0781	0.0328	0.0155	0.0099	0.0016	0.0001
6	1.0000	1.0000	0.9999	0.9997	0.9991	0.9965	0.9894	0.9803	0.9740	0.9452	0.8980	0.8281	0.7340	0.6177	0.4862	0.4407	0.3504	0.2241	0.1209	0.0697	0.0500	0.0128	0.0010
7	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9984	0.9966	0.9952	0.9877	0.9726	0.9453	0.9004	0.8327	0.7384	0.7009	0.6172	0.4744	0.3222	0.2248	0.1798	0.0702	0.0115
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9995	0.9983	0.9955	0.9893	0.9767	0.9536	0.9140	0.8960	0.8507	0.7560	0.6242	0.5155	0.4557	0.2639	0.0861
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9990	0.9975	0.9940	0.9865	0.9827	0.9718	0.9437	0.8926	0.8385	0.8031	0.6513	0.4013
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$n = 12$																								
p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
$x = 0$	0.5404	0.2824	0.1422	0.1122	0.0687	0.0317	0.0138	0.0077	0.0057	0.0022	0.0008	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.8816	0.6590	0.4435	0.3813	0.2749	0.1584	0.0850	0.0540	0.0424	0.0196	0.0083	0.0032	0.0011	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.9804	0.8891	0.7358	0.6774	0.5583	0.3907	0.2528	0.1811	0.1513	0.0834	0.0421	0.0193	0.0079	0.0028	0.0008	0.0005	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.9978	0.9744	0.9078	0.8748	0.7946	0.6488	0.4925	0.3931	0.3467	0.2253	0.1345	0.0730	0.0356	0.0153	0.0056	0.0039	0.0017	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000
4	0.9998	0.9957	0.9761	0.9636	0.9274	0.8424	0.7237	0.6315	0.5833	0.4382	0.3044	0.1938	0.1117	0.0573	0.0255	0.0188	0.0095	0.0028	0.0006	0.0002	0.0001	0.0000	0.0000
5	1.0000	0.9995	0.9954	0.9921	0.9806	0.9456	0.8822	0.8223	0.7873	0.6652	0.5269	0.3872	0.2607	0.1582	0.0846	0.0664	0.0386	0.0143	0.0039	0.0013	0.0007	0.0001	0.0000
6	1.0000	0.9999	0.9993	0.9987	0.9961	0.9857	0.9614	0.9336	0.9154	0.8418	0.7393	0.6128	0.4731	0.3348	0.2127	0.1777	0.1178	0.0544	0.0194	0.0079	0.0046	0.0005	0.0000
7	1.0000	1.0000	0.9999	0.9998	0.9994	0.9972	0.9905	0.9812	0.9745	0.9427	0.8883	0.8062	0.6956	0.5618	0.4167	0.3685	0.2763	0.1576	0.0726	0.0364	0.0239	0.0043	0.0002
8	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9983	0.9961	0.9944	0.9847	0.9644	0.9270	0.8655	0.7747	0.6533	0.6069	0.5075	0.3512	0.2054	0.1252	0.0922	0.0256	0.0022
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9995	0.9992	0.9972	0.9921	0.9807	0.9579	0.9166	0.8487	0.8189	0.7472	0.6093	0.4417	0.3226	0.2642	0.1109	0.0196
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9989	0.9968	0.9917	0.9804	0.9576	0.9460	0.9150	0.8416	0.7251	0.6187	0.5565	0.3410	0.1184
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9992	0.9978	0.9943	0.9923	0.9862	0.9683	0.9313	0.8878	0.8578	0.7176	0.4596
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

CUMULATIVE BINOMIAL PROBABILITIES

$n = 14$																							
p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
$x = 0$	0.4877	0.2288	0.1028	0.0779	0.0440	0.0178	0.0068	0.0034	0.0024	0.0008	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.8470	0.5846	0.3567	0.2960	0.1979	0.1010	0.0475	0.0274	0.0205	0.0081	0.0029	0.0009	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.9699	0.8416	0.6479	0.5795	0.4481	0.2811	0.1608	0.1053	0.0839	0.0398	0.0170	0.0065	0.0022	0.0006	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.9958	0.9559	0.8535	0.8063	0.6982	0.5213	0.3552	0.2612	0.2205	0.1243	0.0632	0.0287	0.0114	0.0039	0.0011	0.0007	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.9996	0.9908	0.9533	0.9310	0.8702	0.7415	0.5842	0.4755	0.4227	0.2793	0.1672	0.0898	0.0426	0.0175	0.0060	0.0040	0.0017	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000
5	1.0000	0.9985	0.9885	0.9809	0.9561	0.8883	0.7805	0.6898	0.6405	0.4859	0.3373	0.2120	0.1189	0.0583	0.0243	0.0174	0.0083	0.0022	0.0004	0.0001	0.0000	0.0000	0.0000
6	1.0000	0.9998	0.9978	0.9959	0.9884	0.9617	0.9067	0.8505	0.8164	0.6925	0.5461	0.3953	0.2586	0.1501	0.0753	0.0576	0.0315	0.0103	0.0024	0.0007	0.0003	0.0000	0.0000
7	1.0000	1.0000	0.9997	0.9993	0.9976	0.9897	0.9685	0.9424	0.9247	0.8499	0.7414	0.6047	0.4539	0.3075	0.1836	0.1495	0.0933	0.0383	0.0116	0.0041	0.0022	0.0002	0.0000
8	1.0000	1.0000	1.0000	0.9999	0.9996	0.9978	0.9917	0.9826	0.9757	0.9417	0.8811	0.7880	0.6627	0.5141	0.3595	0.3102	0.2195	0.1117	0.0439	0.0191	0.0115	0.0015	0.0000
9	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9983	0.9960	0.9940	0.9825	0.9574	0.9102	0.8328	0.7207	0.5773	0.5245	0.4158	0.2585	0.1298	0.0690	0.0467	0.0092	0.0004
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9993	0.9989	0.9961	0.9886	0.9713	0.9368	0.8757	0.7795	0.7388	0.6448	0.4787	0.3018	0.1937	0.1465	0.0441	0.0042
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9994	0.9978	0.9935	0.9830	0.9602	0.9161	0.8947	0.8392	0.7189	0.5519	0.4205	0.3521	0.1584	0.0301
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9991	0.9971	0.9919	0.9795	0.9726	0.9525	0.8990	0.8021	0.7040	0.6433	0.4154	0.1530
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9992	0.9976	0.9966	0.9932	0.9822	0.9560	0.9221	0.8972	0.7712	0.5123
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$n = 16$																							
p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
$x = 0$	0.4401	0.1853	0.0743	0.0541	0.0281	0.0100	0.0033	0.0015	0.0010	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.8108	0.5147	0.2839	0.2272	0.1407	0.0635	0.0261	0.0137	0.0098	0.0033	0.0010	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.9571	0.7892	0.5614	0.4868	0.3518	0.1971	0.0994	0.0594	0.0451	0.0183	0.0066	0.0021	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.9930	0.9316	0.7899	0.7291	0.5981	0.4050	0.2459	0.1659	0.1339	0.0651	0.0281	0.0106	0.0035	0.0009	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.9991	0.9830	0.9209	0.8866	0.7982	0.6302	0.4499	0.3391	0.2892	0.1666	0.0853	0.0384	0.0149	0.0049	0.0013	0.0008	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.9999	0.9967	0.9765	0.9622	0.9183	0.8103	0.6598	0.5469	0.4900	0.3288	0.1976	0.1051	0.0486	0.0191	0.0062	0.0040	0.0016	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000
6	1.0000	0.9995	0.9944	0.9899	0.9733	0.9204	0.8247	0.7374	0.6881	0.5272	0.3660	0.2272	0.1241	0.0583	0.0229	0.0159	0.0071	0.0016	0.0002	0.0000	0.0000	0.0000	0.0000
7	1.0000	0.9999	0.9989	0.9979	0.9930	0.9729	0.9256	0.8735	0.8406	0.7161	0.5629	0.4018	0.2559	0.1423	0.0671	0.0500	0.0257	0.0075	0.0015	0.0004	0.0002	0.0000	0.0000
8	1.0000	1.0000	0.9998	0.9996	0.9985	0.9925	0.9743	0.9500	0.9329	0.8577	0.7441	0.5982	0.4371	0.2839	0.1594	0.1265	0.0744	0.0271	0.0070	0.0021	0.0011	0.0001	0.0000
9	1.0000	1.0000	1.0000	1.0000	0.9998	0.9984	0.9929	0.9841	0.9771	0.9417	0.8759	0.7728	0.6340	0.4728	0.3119	0.2626	0.1753	0.0796	0.0267	0.0101	0.0056	0.0005	0.0000
10	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9984	0.9960	0.9938	0.9809	0.9514	0.8949	0.8024	0.6712	0.5100	0.4531	0.3402	0.1897	0.0817	0.0378	0.0235	0.0033	0.0001
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9992	0.9987	0.9951	0.9851	0.9616	0.9147	0.8334	0.7108	0.6609	0.5501	0.3698	0.2018	0.1134	0.0791	0.0170	0.0009
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9991	0.9965	0.9894	0.9719	0.9349	0.8661	0.8341	0.7541	0.5950	0.4019	0.2709	0.2101	0.0684	0.0070
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9979	0.9934	0.9817	0.9549	0.9406	0.9006	0.8029	0.6482	0.5132	0.4386	0.2108	0.0429
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9990	0.9967	0.9902	0.9863	0.9739	0.9365	0.8593	0.7728	0.7161	0.4853	0.1892
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9990	0.9985	0.9967	0.9900	0.9719	0.9459	0.9257	0.8147	0.5599
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

CUMULATIVE BINOMIAL PROBABILITIES

$n = 18$																							
p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
$x = 0$	0.3972	0.1501	0.0536	0.0376	0.0180	0.0056	0.0016	0.0007	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.7735	0.4503	0.2241	0.1728	0.0991	0.0395	0.0142	0.0068	0.0046	0.0013	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.9419	0.7338	0.4797	0.4027	0.2713	0.1353	0.0600	0.0326	0.0236	0.0082	0.0025	0.0007	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.9891	0.9018	0.7202	0.6479	0.5010	0.3057	0.1646	0.1017	0.0783	0.0328	0.0120	0.0038	0.0010	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.9985	0.9718	0.8794	0.8318	0.7164	0.5187	0.3327	0.2311	0.1886	0.0942	0.0411	0.0154	0.0049	0.0013	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.9998	0.9936	0.9581	0.9347	0.8671	0.7175	0.5344	0.4122	0.3550	0.2088	0.1077	0.0481	0.0183	0.0058	0.0014	0.0009	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
6	1.0000	0.9988	0.9882	0.9794	0.9487	0.8610	0.7217	0.6085	0.5491	0.3743	0.2258	0.1189	0.0537	0.0203	0.0062	0.0039	0.0014	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
7	1.0000	0.9998	0.9973	0.9947	0.9837	0.9431	0.8593	0.7767	0.7283	0.5634	0.3915	0.2403	0.1280	0.0576	0.0212	0.0144	0.0061	0.0012	0.0002	0.0000	0.0000	0.0000	0.0000
8	1.0000	1.0000	0.9995	0.9989	0.9957	0.9807	0.9404	0.8924	0.8609	0.7368	0.5778	0.4073	0.2527	0.1347	0.0597	0.0433	0.0210	0.0054	0.0009	0.0002	0.0001	0.0000	0.0000
9	1.0000	1.0000	0.9999	0.9998	0.9991	0.9946	0.9790	0.9567	0.9403	0.8653	0.7473	0.5927	0.4222	0.2632	0.1391	0.1076	0.0596	0.0193	0.0043	0.0011	0.0005	0.0000	0.0000
10	1.0000	1.0000	1.0000	1.0000	0.9998	0.9988	0.9939	0.9856	0.9788	0.9424	0.8720	0.7597	0.6085	0.4366	0.2717	0.2233	0.1407	0.0569	0.0163	0.0053	0.0027	0.0002	0.0000
11	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9986	0.9961	0.9938	0.9797	0.9463	0.8811	0.7742	0.6257	0.4509	0.3915	0.2783	0.1390	0.0513	0.0206	0.0118	0.0012	0.0000
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9991	0.9986	0.9942	0.9817	0.9519	0.8923	0.7912	0.6450	0.5878	0.4656	0.2825	0.1329	0.0653	0.0419	0.0064	0.0002
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9987	0.9951	0.9846	0.9589	0.9058	0.8114	0.7689	0.6673	0.4813	0.2836	0.1682	0.1206	0.0282	0.0015
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9990	0.9962	0.9880	0.9672	0.9217	0.8983	0.8354	0.6943	0.4990	0.3521	0.2798	0.0982	0.0109
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9993	0.9975	0.9918	0.9764	0.9674	0.9400	0.8647	0.7287	0.5973	0.5203	0.2662	0.0581
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9987	0.9954	0.9932	0.9858	0.9605	0.9009	0.8272	0.7759	0.5497	0.2265
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9993	0.9984	0.9944	0.9820	0.9624	0.9464	0.8499	0.6028
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$n = 20$																							
p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
$x = 0$	0.3585	0.1216	0.0388	0.0261	0.0115	0.0032	0.0008	0.0003	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.7358	0.3917	0.1756	0.1304	0.0692	0.0243	0.0076	0.0033	0.0021	0.0005	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.9245	0.6769	0.4049	0.3287	0.2061	0.0913	0.0355	0.0176	0.0121	0.0036	0.0009	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.9841	0.8670	0.6477	0.5665	0.4114	0.2252	0.1071	0.0604	0.0444	0.0160	0.0049	0.0013	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.9974	0.9568	0.8298	0.7687	0.6296	0.4148	0.2375	0.1515	0.1182	0.0510	0.0189	0.0059	0.0015	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.9997	0.9887	0.9327	0.8982	0.8042	0.6172	0.4164	0.2972	0.2454	0.1256	0.0553	0.0207	0.0064	0.0016	0.0003	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
6	1.0000	0.9976	0.9781	0.9629	0.9133	0.7858	0.6080	0.4793	0.4166	0.2500	0.1299	0.0577	0.0214	0.0065	0.0015	0.0009	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
7	1.0000	0.9996	0.9941	0.9887	0.9679	0.8982	0.7723	0.6615	0.6010	0.4159	0.2520	0.1316	0.0580	0.0210	0.0060	0.0037	0.0013	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
8	1.0000	0.9999	0.9987	0.9972	0.9900	0.9591	0.8867	0.8095	0.7624	0.5956	0.4143	0.2517	0.1308	0.0565	0.0196	0.0130	0.0051	0.0009	0.0001	0.0000	0.0000	0.0000	0.0000
9	1.0000	1.0000	0.9998	0.9994	0.9974	0.9861	0.9520	0.9081	0.8782	0.7553	0.5914	0.4119	0.2493	0.1275	0.0532	0.0376	0.0171	0.0039	0.0006	0.0001	0.0000	0.0000	0.0000
10	1.0000	1.0000	1.0000	0.9999	0.9994	0.9961	0.9829	0.9624	0.9468	0.8725	0.7507	0.5881	0.4086	0.2447	0.1218	0.0919	0.0480	0.0139	0.0026	0.0006	0.0002	0.0000	0.0000
11	1.0000	1.0000	1.0000	1.0000	0.9999	0.9991	0.9949	0.9870	0.9804	0.9435	0.8692	0.7483	0.5857	0.4044	0.2376	0.1905	0.1133	0.0409	0.0100	0.0028	0.0013	0.0001	0.0000
12	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9987	0.9963	0.9940	0.9790	0.9420	0.8684	0.7480	0.5841	0.3990	0.3385	0.2277	0.1018	0.0321	0.0113	0.0059	0.0004	0.0000
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9991	0.9985	0.9935	0.9786	0.9423	0.8701	0.7500	0.5834	0.5207	0.3920	0.2142	0.0867	0.0371	0.0219	0.0024	0.0000
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9997	0.9984	0.9936	0.9793	0.9447	0.8744	0.7546	0.7028	0.5836	0.3828	0.1958	0.1018	0.0673	0.0113	0.0003
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9985	0.9941	0.9811	0.9490	0.8818	0.8485	0.7625	0.5852	0.3704	0.2313	0.1702	0.0432	0.0026
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9987	0.9951	0.9840	0.9556	0.9396	0.8929	0.7748	0.5886	0.4335	0.3523	0.1330	0.0159
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9991	0.9964	0.9879	0.9824	0.9645	0.9087	0.7939	0.6713	0.5951	0.3231	0.0755
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9979	0.9967	0.9924	0.9757	0.9308	0.8696	0.8244	0.6083	0.2642
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9997	0.9992	0.9968	0.9885	0.9739	0.9612	0.8784	0.6415
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

CUMULATIVE BINOMIAL PROBABILITIES

$n = 25$																								
p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
$x = 0$	0.2774	0.0718	0.0172	0.0105	0.0038	0.0008	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.6424	0.2712	0.0931	0.0629	0.0274	0.0070	0.0016	0.0005	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.8729	0.5371	0.2537	0.1887	0.0982	0.0321	0.0090	0.0035	0.0021	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.9659	0.7636	0.4711	0.3816	0.2340	0.0962	0.0332	0.0149	0.0097	0.0024	0.0005	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.9928	0.9020	0.6821	0.5937	0.4207	0.2137	0.0905	0.0462	0.0320	0.0095	0.0023	0.0005	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.9988	0.9666	0.8385	0.7720	0.6167	0.3783	0.1935	0.1120	0.0826	0.0294	0.0086	0.0020	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
6	0.9998	0.9905	0.9305	0.8908	0.7800	0.5611	0.3407	0.2215	0.1734	0.0736	0.0258	0.0073	0.0016	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
7	1.0000	0.9977	0.9745	0.9553	0.8909	0.7265	0.5118	0.3703	0.3061	0.1536	0.0639	0.0216	0.0058	0.0012	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
8	1.0000	0.9995	0.9920	0.9843	0.9532	0.8506	0.6769	0.5376	0.4668	0.2735	0.1340	0.0539	0.0174	0.0043	0.0008	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
9	1.0000	0.9999	0.9979	0.9953	0.9827	0.9287	0.8106	0.6956	0.6303	0.4246	0.2424	0.1148	0.0440	0.0132	0.0029	0.0016	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
10	1.0000	1.0000	0.9995	0.9988	0.9944	0.9703	0.9022	0.8220	0.7712	0.5858	0.3843	0.2122	0.0960	0.0344	0.0093	0.0056	0.0018	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
11	1.0000	1.0000	0.9999	0.9997	0.9985	0.9893	0.9558	0.9082	0.8746	0.7323	0.5426	0.3450	0.1827	0.0778	0.0255	0.0164	0.0060	0.0009	0.0001	0.0000	0.0000	0.0000	0.0000
12	1.0000	1.0000	1.0000	0.9999	0.9996	0.9966	0.9825	0.9585	0.9396	0.8462	0.6937	0.5000	0.3063	0.1538	0.0604	0.0415	0.0175	0.0034	0.0004	0.0001	0.0000	0.0000	0.0000
13	1.0000	1.0000	1.0000	1.0000	0.9999	0.9991	0.9940	0.9836	0.9745	0.9222	0.8173	0.6550	0.4574	0.2677	0.1254	0.0918	0.0442	0.0107	0.0015	0.0003	0.0001	0.0000	0.0000
14	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9982	0.9944	0.9907	0.9656	0.9040	0.7878	0.6157	0.4142	0.2288	0.1780	0.0978	0.0297	0.0056	0.0012	0.0005	0.0000	0.0000
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9995	0.9984	0.9971	0.9868	0.9560	0.8852	0.7576	0.5754	0.3697	0.3044	0.1894	0.0713	0.0173	0.0047	0.0021	0.0001	0.0000
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9992	0.9957	0.9826	0.9461	0.8660	0.7265	0.5332	0.4624	0.3231	0.1494	0.0468	0.0157	0.0080	0.0005	0.0000
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9988	0.9942	0.9784	0.9361	0.8464	0.6939	0.6297	0.4882	0.2735	0.1091	0.0447	0.0255	0.0023	0.0000
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9984	0.9927	0.9742	0.9264	0.8266	0.7785	0.6593	0.4389	0.2200	0.1092	0.0695	0.0095	0.0002
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9980	0.9914	0.9706	0.9174	0.8880	0.8065	0.6217	0.3833	0.2280	0.1615	0.0334	0.0012
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9977	0.9905	0.9680	0.9538	0.9095	0.7863	0.5793	0.4063	0.3179	0.0980	0.0072
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9976	0.9903	0.9851	0.9668	0.9038	0.7660	0.6184	0.5289	0.2364	0.0341
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9979	0.9965	0.9910	0.9679	0.9018	0.8113	0.7463	0.4629	0.1271
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9995	0.9984	0.9930	0.9726	0.9371	0.9069	0.7288	0.3576
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9992	0.9962	0.9895	0.9828	0.9282	0.7226
25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

CUMULATIVE BINOMIAL PROBABILITIES

$n = 30$																								
p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
$x = 0$	0.2146	0.0424	0.0076	0.0042	0.0012	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.5535	0.1837	0.0480	0.0295	0.0105	0.0020	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.8122	0.4114	0.1514	0.1028	0.0442	0.0106	0.0021	0.0007	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.9392	0.6474	0.3217	0.2396	0.1227	0.0374	0.0093	0.0033	0.0019	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.9844	0.8245	0.5245	0.4243	0.2552	0.0979	0.0302	0.0122	0.0075	0.0015	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.9967	0.9268	0.7106	0.6164	0.4275	0.2026	0.0766	0.0355	0.0233	0.0057	0.0011	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
6	0.9994	0.9742	0.8474	0.7765	0.6070	0.3481	0.1595	0.0838	0.0586	0.0172	0.0040	0.0007	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
7	0.9999	0.9922	0.9302	0.8863	0.7608	0.5143	0.2814	0.1668	0.1238	0.0435	0.0121	0.0026	0.0004	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
8	1.0000	0.9980	0.9722	0.9494	0.8713	0.6736	0.4315	0.2860	0.2247	0.0940	0.0312	0.0081	0.0016	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
9	1.0000	0.9995	0.9903	0.9803	0.9389	0.8034	0.5888	0.4317	0.3575	0.1763	0.0694	0.0214	0.0050	0.0009	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
10	1.0000	0.9999	0.9971	0.9933	0.9744	0.8943	0.7304	0.5848	0.5078	0.2915	0.1350	0.0494	0.0138	0.0029	0.0004	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
11	1.0000	1.0000	0.9992	0.9980	0.9905	0.9493	0.8407	0.7239	0.6548	0.4311	0.2327	0.1002	0.0334	0.0083	0.0014	0.0007	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
12	1.0000	1.0000	0.9998	0.9995	0.9969	0.9784	0.9155	0.8340	0.7802	0.5785	0.3592	0.1808	0.0714	0.0212	0.0045	0.0025	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
13	1.0000	1.0000	1.0000	0.9999	0.9991	0.9918	0.9599	0.9102	0.8737	0.7145	0.5025	0.2923	0.1356	0.0481	0.0124	0.0072	0.0021	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
14	1.0000	1.0000	1.0000	1.0000	0.9998	0.9973	0.9831	0.9565	0.9348	0.8246	0.6448	0.4278	0.2309	0.0971	0.0301	0.0188	0.0064	0.0008	0.0001	0.0000	0.0000	0.0000	0.0000
15	1.0000	1.0000	1.0000	1.0000	0.9999	0.9992	0.9936	0.9812	0.9699	0.9029	0.7691	0.5722	0.3552	0.1754	0.0652	0.0435	0.0169	0.0027	0.0002	0.0000	0.0000	0.0000	0.0000
16	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9979	0.9928	0.9876	0.9519	0.8644	0.7077	0.4975	0.2855	0.1263	0.0898	0.0401	0.0082	0.0009	0.0001	0.0000	0.0000	0.0000
17	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9975	0.9955	0.9788	0.9286	0.8192	0.6408	0.4215	0.2198	0.1660	0.0845	0.0216	0.0031	0.0005	0.0002	0.0000	0.0000
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9993	0.9986	0.9917	0.9666	0.8998	0.7673	0.5689	0.3452	0.2761	0.1593	0.0507	0.0095	0.0020	0.0008	0.0000	0.0000
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9996	0.9971	0.9862	0.9506	0.8650	0.7085	0.4922	0.4152	0.2696	0.1057	0.0256	0.0067	0.0029	0.0001	0.0000
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9991	0.9950	0.9786	0.9306	0.8237	0.6425	0.5683	0.4112	0.1966	0.0611	0.0197	0.0097	0.0005	0.0000
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9984	0.9919	0.9688	0.9060	0.7753	0.7140	0.5685	0.3264	0.1287	0.0506	0.0278	0.0020	0.0000
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9996	0.9974	0.9879	0.9565	0.8762	0.8332	0.7186	0.4857	0.2392	0.1137	0.0698	0.0078	0.0001
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9993	0.9960	0.9828	0.9414	0.9162	0.8405	0.6519	0.3930	0.2235	0.1526	0.0258	0.0006
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9989	0.9943	0.9767	0.9645	0.9234	0.7974	0.5725	0.3836	0.2894	0.0732	0.0033
25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9985	0.9925	0.9878	0.9698	0.9021	0.7448	0.5757	0.4755	0.1755	0.0156
26	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9981	0.9967	0.9907	0.9626	0.8773	0.7604	0.6783	0.3526	0.0608
27	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9993	0.9979	0.9894	0.9558	0.8972	0.8486	0.5886	0.1878
28	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9980	0.9895	0.9705	0.9520	0.8163	0.4465
29	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9988	0.9958	0.9924	0.9576	0.7854
30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

CUMULATIVE POISSON PROBABILITIES

λ	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
$x = 0$	0.9900	0.9802	0.9704	0.9608	0.9512	0.9418	0.9324	0.9231	0.9139
1	1.0000	0.9998	0.9996	0.9992	0.9988	0.9983	0.9977	0.9970	0.9962
2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999
3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

λ	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
$x = 0$	0.9048	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493	0.4066
1	0.9953	0.9825	0.9631	0.9384	0.9098	0.8781	0.8442	0.8088	0.7725
2	0.9998	0.9989	0.9964	0.9921	0.9856	0.9769	0.9659	0.9526	0.9371
3	1.0000	0.9999	0.9997	0.9992	0.9982	0.9966	0.9942	0.9909	0.9865
4	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9986	0.9977
5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9997
6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

λ	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90
$x = 0$	0.3679	0.3329	0.3012	0.2725	0.2466	0.2231	0.2019	0.1827	0.1653	0.1496
1	0.7358	0.6990	0.6626	0.6268	0.5918	0.5578	0.5249	0.4932	0.4628	0.4337
2	0.9197	0.9004	0.8795	0.8571	0.8335	0.8088	0.7834	0.7572	0.7306	0.7037
3	0.9810	0.9743	0.9662	0.9569	0.9463	0.9344	0.9212	0.9068	0.8913	0.8747
4	0.9963	0.9946	0.9923	0.9893	0.9857	0.9814	0.9763	0.9704	0.9636	0.9559
5	0.9994	0.9990	0.9985	0.9978	0.9968	0.9955	0.9940	0.9920	0.9896	0.9868
6	0.9999	0.9999	0.9997	0.9996	0.9994	0.9991	0.9987	0.9981	0.9974	0.9966
7	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9997	0.9996	0.9994	0.9992
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

λ	2.00	2.10	2.20	2.30	2.40	2.50	2.60	2.70	2.80	2.90
$x = 0$	0.1353	0.1225	0.1108	0.1003	0.0907	0.0821	0.0743	0.0672	0.0608	0.0550
1	0.4060	0.3796	0.3546	0.3309	0.3084	0.2873	0.2674	0.2487	0.2311	0.2146
2	0.6767	0.6496	0.6227	0.5960	0.5697	0.5438	0.5184	0.4936	0.4695	0.4460
3	0.8571	0.8386	0.8194	0.7993	0.7787	0.7576	0.7360	0.7141	0.6919	0.6696
4	0.9473	0.9379	0.9275	0.9162	0.9041	0.8912	0.8774	0.8629	0.8477	0.8318
5	0.9834	0.9796	0.9751	0.9700	0.9643	0.9580	0.9510	0.9433	0.9349	0.9258
6	0.9955	0.9941	0.9925	0.9906	0.9884	0.9858	0.9828	0.9794	0.9756	0.9713
7	0.9989	0.9985	0.9980	0.9974	0.9967	0.9958	0.9947	0.9934	0.9919	0.9901
8	0.9998	0.9997	0.9995	0.9994	0.9991	0.9989	0.9985	0.9981	0.9976	0.9969
9	1.0000	0.9999	0.9999	0.9999	0.9998	0.9997	0.9996	0.9995	0.9993	0.9991
10	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9998	0.9998
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

λ	3.00	3.10	3.20	3.30	3.40	3.50	3.60	3.70	3.80	3.90
$x = 0$	0.0498	0.0450	0.0408	0.0369	0.0334	0.0302	0.0273	0.0247	0.0224	0.0202
1	0.1991	0.1847	0.1712	0.1586	0.1468	0.1359	0.1257	0.1162	0.1074	0.0992
2	0.4232	0.4012	0.3799	0.3594	0.3397	0.3208	0.3027	0.2854	0.2689	0.2531
3	0.6472	0.6248	0.6025	0.5803	0.5584	0.5366	0.5152	0.4942	0.4735	0.4532
4	0.8153	0.7982	0.7806	0.7626	0.7442	0.7254	0.7064	0.6872	0.6678	0.6484
5	0.9161	0.9057	0.8946	0.8829	0.8705	0.8576	0.8441	0.8301	0.8156	0.8006
6	0.9665	0.9612	0.9554	0.9490	0.9421	0.9347	0.9267	0.9182	0.9091	0.8995
7	0.9881	0.9858	0.9832	0.9802	0.9769	0.9733	0.9692	0.9648	0.9599	0.9546
8	0.9962	0.9953	0.9943	0.9931	0.9917	0.9901	0.9883	0.9863	0.9840	0.9815
9	0.9989	0.9986	0.9982	0.9978	0.9973	0.9967	0.9960	0.9952	0.9942	0.9931
10	0.9997	0.9996	0.9995	0.9994	0.9992	0.9990	0.9987	0.9984	0.9981	0.9977
11	0.9999	0.9999	0.9999	0.9998	0.9998	0.9997	0.9996	0.9995	0.9994	0.9993
12	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

CUMULATIVE POISSON PROBABILITIES

λ	4.00	4.10	4.20	4.30	4.40	4.50	4.60	4.70	4.80	4.90
$x = 0$	0.0183	0.0166	0.0150	0.0136	0.0123	0.0111	0.0101	0.0091	0.0082	0.0074
1	0.0916	0.0845	0.0780	0.0719	0.0663	0.0611	0.0563	0.0518	0.0477	0.0439
2	0.2381	0.2238	0.2102	0.1974	0.1851	0.1736	0.1626	0.1523	0.1425	0.1333
3	0.4335	0.4142	0.3954	0.3772	0.3594	0.3423	0.3257	0.3097	0.2942	0.2793
4	0.6288	0.6093	0.5898	0.5704	0.5512	0.5321	0.5132	0.4946	0.4763	0.4582
5	0.7851	0.7693	0.7531	0.7367	0.7199	0.7029	0.6858	0.6684	0.6510	0.6335
6	0.8893	0.8786	0.8675	0.8558	0.8436	0.8311	0.8180	0.8046	0.7908	0.7767
7	0.9489	0.9427	0.9361	0.9290	0.9214	0.9134	0.9049	0.8960	0.8867	0.8769
8	0.9786	0.9755	0.9721	0.9683	0.9642	0.9597	0.9549	0.9497	0.9442	0.9382
9	0.9919	0.9905	0.9889	0.9871	0.9851	0.9829	0.9805	0.9778	0.9749	0.9717
10	0.9972	0.9966	0.9959	0.9952	0.9943	0.9933	0.9922	0.9910	0.9896	0.9880
11	0.9991	0.9989	0.9986	0.9983	0.9980	0.9976	0.9971	0.9966	0.9960	0.9953
12	0.9997	0.9997	0.9996	0.9995	0.9993	0.9992	0.9990	0.9988	0.9986	0.9983
13	0.9999	0.9999	0.9999	0.9998	0.9998	0.9997	0.9997	0.9996	0.9995	0.9994
14	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

λ	5.00	5.50	6.00	6.50	7.00	7.50	8.00	8.50	9.00	9.50
$x = 0$	0.0067	0.0041	0.0025	0.0015	0.0009	0.0006	0.0003	0.0002	0.0001	0.0001
1	0.0404	0.0266	0.0174	0.0113	0.0073	0.0047	0.0030	0.0019	0.0012	0.0008
2	0.1247	0.0884	0.0620	0.0430	0.0296	0.0203	0.0138	0.0093	0.0062	0.0042
3	0.2650	0.2017	0.1512	0.1118	0.0818	0.0591	0.0424	0.0301	0.0212	0.0149
4	0.4405	0.3575	0.2851	0.2237	0.1730	0.1321	0.0996	0.0744	0.0550	0.0403
5	0.6160	0.5289	0.4457	0.3690	0.3007	0.2414	0.1912	0.1496	0.1157	0.0885
6	0.7622	0.6860	0.6063	0.5265	0.4497	0.3782	0.3134	0.2562	0.2068	0.1649
7	0.8666	0.8095	0.7440	0.6728	0.5987	0.5246	0.4530	0.3856	0.3239	0.2687
8	0.9319	0.8944	0.8472	0.7916	0.7291	0.6620	0.5925	0.5231	0.4557	0.3918
9	0.9682	0.9462	0.9161	0.8774	0.8305	0.7764	0.7166	0.6530	0.5874	0.5218
10	0.9863	0.9747	0.9574	0.9332	0.9015	0.8622	0.8159	0.7634	0.7060	0.6453
11	0.9945	0.9890	0.9799	0.9661	0.9467	0.9208	0.8881	0.8487	0.8030	0.7520
12	0.9980	0.9955	0.9912	0.9840	0.9730	0.9573	0.9362	0.9091	0.8758	0.8364
13	0.9993	0.9983	0.9964	0.9929	0.9872	0.9784	0.9658	0.9486	0.9261	0.8981
14	0.9998	0.9994	0.9986	0.9970	0.9943	0.9897	0.9827	0.9726	0.9585	0.9400
15	0.9999	0.9998	0.9995	0.9988	0.9976	0.9954	0.9918	0.9862	0.9780	0.9665
16	1.0000	0.9999	0.9998	0.9996	0.9990	0.9980	0.9963	0.9934	0.9889	0.9823
17	1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9984	0.9970	0.9947	0.9911
18	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997	0.9993	0.9987	0.9976	0.9957
19	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9995	0.9989	0.9980
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9991
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

CUMULATIVE POISSON PROBABILITIES

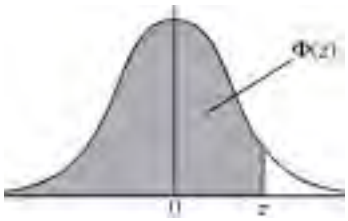
λ	10.00	11.00	12.00	13.00	14.00	15.00	16.00	17.00	18.00	19.00
$x = 0$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0005	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0028	0.0012	0.0005	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.0103	0.0049	0.0023	0.0011	0.0005	0.0002	0.0001	0.0000	0.0000	0.0000
4	0.0293	0.0151	0.0076	0.0037	0.0018	0.0009	0.0004	0.0002	0.0001	0.0000
5	0.0671	0.0375	0.0203	0.0107	0.0055	0.0028	0.0014	0.0007	0.0003	0.0002
6	0.1301	0.0786	0.0458	0.0259	0.0142	0.0076	0.0040	0.0021	0.0010	0.0005
7	0.2202	0.1432	0.0895	0.0540	0.0316	0.0180	0.0100	0.0054	0.0029	0.0015
8	0.3328	0.2320	0.1550	0.0998	0.0621	0.0374	0.0220	0.0126	0.0071	0.0039
9	0.4579	0.3405	0.2424	0.1658	0.1094	0.0699	0.0433	0.0261	0.0154	0.0089
10	0.5830	0.4599	0.3472	0.2517	0.1757	0.1185	0.0774	0.0491	0.0304	0.0183
11	0.6968	0.5793	0.4616	0.3532	0.2600	0.1848	0.1270	0.0847	0.0549	0.0347
12	0.7916	0.6887	0.5760	0.4631	0.3585	0.2676	0.1931	0.1350	0.0917	0.0606
13	0.8645	0.7813	0.6815	0.5730	0.4644	0.3632	0.2745	0.2009	0.1426	0.0984
14	0.9165	0.8540	0.7720	0.6751	0.5704	0.4657	0.3675	0.2808	0.2081	0.1497
15	0.9513	0.9074	0.8444	0.7636	0.6694	0.5681	0.4667	0.3715	0.2867	0.2148
16	0.9730	0.9441	0.8987	0.8355	0.7559	0.6641	0.5660	0.4677	0.3751	0.2920
17	0.9857	0.9678	0.9370	0.8905	0.8272	0.7489	0.6593	0.5640	0.4686	0.3784
18	0.9928	0.9823	0.9626	0.9302	0.8826	0.8195	0.7423	0.6550	0.5622	0.4695
19	0.9965	0.9907	0.9787	0.9573	0.9235	0.8752	0.8122	0.7363	0.6509	0.5606
20	0.9984	0.9953	0.9884	0.9750	0.9521	0.9170	0.8682	0.8055	0.7307	0.6472
21	0.9993	0.9977	0.9939	0.9859	0.9712	0.9469	0.9108	0.8615	0.7991	0.7255
22	0.9997	0.9990	0.9970	0.9924	0.9833	0.9673	0.9418	0.9047	0.8551	0.7931
23	0.9999	0.9995	0.9985	0.9960	0.9907	0.9805	0.9633	0.9367	0.8989	0.8490
24	1.0000	0.9998	0.9993	0.9980	0.9950	0.9888	0.9777	0.9594	0.9317	0.8933
25	1.0000	0.9999	0.9997	0.9990	0.9974	0.9938	0.9869	0.9748	0.9554	0.9269
26	1.0000	1.0000	0.9999	0.9995	0.9987	0.9967	0.9925	0.9848	0.9718	0.9514
27	1.0000	1.0000	0.9999	0.9998	0.9994	0.9983	0.9959	0.9912	0.9827	0.9687
28	1.0000	1.0000	1.0000	0.9999	0.9997	0.9991	0.9978	0.9950	0.9897	0.9805
29	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9989	0.9973	0.9941	0.9882
30	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9994	0.9986	0.9967	0.9930
31	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9993	0.9982	0.9960
32	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9990	0.9978
33	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9995	0.9988
34	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9994
35	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997
36	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998
37	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
38	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

THE NORMAL DISTRIBUTION FUNCTION

If Z has a normal distribution with mean 0 and variance 1 then, for each value of z, the table gives the value of $\Phi(z)$, where

$$\Phi(z) = \mathrm{P}(Z \leq z).$$

For negative values of z use $\Phi(-z) = 1 - \Phi(z)$.



z	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
											ADD								
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	4	8	12	16	20	24	28	32	36
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	4	8	12	16	20	24	28	32	36
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	4	8	12	15	19	23	27	31	35
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	4	7	11	15	19	22	26	30	34
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	4	7	11	14	18	22	25	29	32
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	3	7	10	14	17	20	24	27	31
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	3	7	10	13	16	19	23	26	29
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	3	6	9	12	15	18	21	24	27
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	3	5	8	11	14	16	19	22	25
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	3	5	8	10	13	15	18	20	23
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621	2	5	7	9	12	14	16	19	21
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830	2	4	6	8	10	12	14	16	18
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015	2	4	6	7	9	11	13	15	17
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177	2	3	5	6	8	10	11	13	14
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	1	3	4	6	7	8	10	11	13
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441	1	2	4	5	6	7	8	10	11
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545	1	2	3	4	5	6	7	8	9
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633	1	2	3	4	4	5	6	7	8
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706	1	1	2	3	4	4	5	6	6
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767	1	1	2	2	3	4	4	5	5
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817	0	1	1	2	2	3	3	4	4
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857	0	1	1	2	2	2	3	3	4
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890	0	1	1	1	2	2	2	3	3
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916	0	1	1	1	1	2	2	2	2
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936	0	0	1	1	1	1	1	2	2
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952	0	0	0	1	1	1	1	1	1
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964	0	0	0	0	1	1	1	1	1
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974	0	0	0	0	0	1	1	1	1
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981	0	0	0	0	0	0	0	1	1
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986	0	0	0	0	0	0	0	0	0

Critical values for the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that

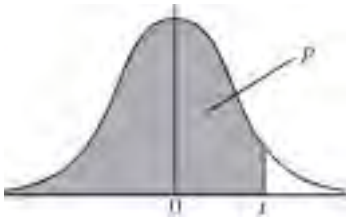
$$\mathrm{P}(Z \leq z) = p.$$

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

CRITICAL VALUES FOR THE *t*-DISTRIBUTION

If T has a t-distribution with ν degrees of freedom then, for each pair of values of p and ν, the table gives the value of t such that:

$\mathrm{P}(T \leq t) = p$.



p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
$\nu = 1$	1.000	3.078	6.314	12.71	31.82	63.66	127.3	318.3	636.6
2	0.816	1.886	2.920	4.303	6.965	9.925	14.09	22.33	31.60
3	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.21	12.92
4	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.894	6.869
6	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.768
24	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.689
28	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.660
30	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Mathematical Notation

Examinations for the syllabus in this booklet may use relevant notation from the following list.

1 Set Notation

$\in$	is an element of
$\notin$	is not an element of
$\{x_1, x_2, ...\}$	the set with elements $x_1, x_2, ...$
$\{x : ...\}$	the set of all x such that ...
$\mathrm{n}(A)$	the number of elements in set A
$\varnothing$	the empty set
$\mathscr{E}$	the universal set
A'	the complement of the set A
$\mathbb{N}$	the set of natural numbers, $\{1, 2, 3, ...\}$
$\mathbb{Z}$	the set of integers, $\{0, \pm 1, \pm 2, \pm 3, ...\}$
$\mathbb{Z}^+$	the set of positive integers, $\{1, 2, 3, ...\}$
$\mathbb{Z}_n$	the set of integers modulo n, $\{0, 1, 2, ..., n-1\}$
$\mathbb{Q}$	the set of rational numbers, $\left\{\frac{p}{q} : p \in \mathbb{Z}, q \in \mathbb{Z}^+\right\}$
$\mathbb{Q}^+$	the set of positive rational numbers, $\{x \in \mathbb{Q} : x > 0\}$
$\mathbb{Q}_0^+$	set of positive rational numbers and zero, $\{x \in \mathbb{Q} : x \geq 0\}$
$\mathbb{R}$	the set of real numbers
$\mathbb{R}^+$	the set of positive real numbers, $\{x \in \mathbb{R} : x > 0\}$
$\mathbb{R}_0^+$	the set of positive real numbers and zero, $\{x \in \mathbb{R} : x \geq 0\}$
$\mathbb{C}$	the set of complex numbers
(x, y)	the ordered pair x, y
$A \times B$	the cartesian product of sets A and B, i.e. $A \times B = \{(a, b) : a \in A, b \in B\}$
$\subseteq$	is a subset of
$\subset$	is a proper subset of
$\cup$	union
$\cap$	intersection
$[a, b]$	the closed interval $\{x \in \mathbb{R} : a \leq x \leq b\}$
$[a, b)$	the interval $\{x \in \mathbb{R} : a \leq x < b\}$
$(a, b]$	the interval $\{x \in \mathbb{R} : a < x \leq b\}$
(a, b)	the open interval $\{x \in \mathbb{R} : a < x < b\}$
$y \, R \, x$	y is related to x by the relation R
$y \sim x$	y is equivalent to x, in the context of some equivalence relation

2 Miscellaneous Symbols

$=$	is equal to
$\neq$	is not equal to
$\equiv$	is identical to or is congruent to
$\approx$	is approximately equal to
$\cong$	is isomorphic to
$\propto$	is proportional to
$<$	is less than
$\leq$	is less than or equal to, is not greater than
$>$	is greater than
$\geq$	is greater than or equal to, is not less than
∞	infinity
$p \wedge q$	p and q
$p \vee q$	p or q (or both)
$\sim p$	not p
$p \Rightarrow q$	p implies q (if p then q)
$p \Leftarrow q$	p is implied by q (if q then p)
$p \Leftrightarrow q$	p implies and is implied by q (p is equivalent to q)
$\exists$	there exists
$\forall$	for all

3 Operations

$a + b$	a plus b
$a - b$	a minus b
$a \times b$, ab, $a.b$	a multiplied by b
$a \div b$, $\frac{a}{b}$, a / b	a divided by b
$\sum_{i=1}^{n} a_i$	$a_1 + a_2 + \ldots + a_n$
$\prod_{i=1}^{n} a_i$	$a_1 \times a_2 \times \ldots \times a_n$
$\sqrt{a}$	the positive square root of a
$\lvert a \rvert$	the modulus of a
$n!$	n factorial
$\binom{n}{r}$	the binomial coefficient $\frac{n!}{r!\,(n-r)!}$ for $n \in \mathbb{Z}^+$ or $\frac{n(n-1)\ldots(n-r+1)}{r!}$ for $n \in \mathbb{Q}$

4 Functions

$\mathrm{f}(x)$	the value of the function f at x
$\mathrm{f} : A \to B$	f is a function under which each element of set A has an image in set B
$\mathrm{f} : x \mapsto y$	the function f maps the element x to the element y
f^{-1}	the inverse function of the function f
gf	the composite function of f and g which is defined by $\mathrm{gf}(x) = \mathrm{g}(\mathrm{f}(x))$
$\lim_{x \to a} \mathrm{f}(x)$	the limit of $\mathrm{f}(x)$ as x tends to a

Δx, δx	an increment of x
$\frac{dy}{dx}$	the derivative of y with respect to x
$\frac{d^n y}{dx^n}$	the nth derivative of y with respect to x
$f'(x)$, $f''(x)$, …, $f^{(n)}(x)$	the first, second, ..., nth derivatives of $f(x)$ with respect to x
$\int y\,dx$	the indefinite integral of y with respect to x
$\int_a^b y\,dx$	the definite integral of y with respect to x between the limits $x = a$ and $x = b$
$\frac{\partial V}{\partial x}$	the partial derivative of V with respect to x
$\dot{x}$, $\ddot{x}$, ...	the first, second, ... derivatives of x with respect to t

5 Exponential and Logarithmic Functions

e	base of natural logarithms
e^x, $\exp x$	exponential function of x
$\log_a x$	logarithm to the base a of x
$\ln x$, $\log_e x$	natural logarithm of x
$\lg x$, $\log_{10} x$	logarithm of x to base 10

6 Circular and Hyperbolic Functions

$\left.\begin{matrix}\sin,\ \cos,\ \tan,\\ \operatorname{cosec},\ \sec,\ \cot\end{matrix}\right\}$	the circular functions
$\left.\begin{matrix}\sin^{-1},\ \cos^{-1},\ \tan^{-1},\\ \operatorname{cosec}^{-1},\ \sec^{-1},\ \cot^{-1}\end{matrix}\right\}$	the inverse circular functions
$\left.\begin{matrix}\sinh,\ \cosh,\ \tanh,\\ \operatorname{cosech},\ \operatorname{sech},\ \coth\end{matrix}\right\}$	the hyperbolic functions
$\left.\begin{matrix}\sinh^{-1},\ \cosh^{-1},\ \tanh^{-1},\\ \operatorname{cosech}^{-1},\ \operatorname{sech}^{-1},\ \coth^{-1}\end{matrix}\right\}$	the inverse hyperbolic functions

7 Complex Numbers

i	square root of -1
z	a complex number, $z = x + \mathrm{i}\,y = r(\cos\theta + \mathrm{i}\sin\theta)$
$\mathrm{Re}\,z$	the real part of z, $\mathrm{Re}\,z = x$
$\mathrm{Im}\,z$	the imaginary part of z, $\mathrm{Im}\,z = y$
$\lvert z\rvert$	the modulus of z, $\lvert z\rvert = \sqrt{x^2 + y^2}$
$\arg z$	the argument of z, $\arg z = \theta$, $-\pi < \theta \leq \pi$
z^*	the complex conjugate of z, $x - \mathrm{i}\,y$

8 Matrices

$\mathbf{M}$	a matrix $\mathbf{M}$
$\mathbf{M}^{-1}$	the inverse of the matrix $\mathbf{M}$
$\mathbf{M}^{\mathrm{T}}$	the transpose of the matrix $\mathbf{M}$
$\det \mathbf{M}$ or $\lvert \mathbf{M} \rvert$	the determinant of the square matrix $\mathbf{M}$

9 Vectors

$\mathbf{a}$	the vector $\mathbf{a}$
$\overrightarrow{AB}$	the vector represented in magnitude and direction by the directed line segment AB
$\hat{\mathbf{a}}$	a unit vector in the direction of $\mathbf{a}$
$\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$	unit vectors in the directions of the cartesian coordinate axes
$\lvert\mathbf{a}\rvert$, a	the magnitude of $\mathbf{a}$
$\lvert\overrightarrow{AB}\rvert$, AB	the magnitude of $\overrightarrow{AB}$
$\mathbf{a}.\mathbf{b}$	the scalar product of $\mathbf{a}$ and $\mathbf{b}$
$\mathbf{a} \times \mathbf{b}$	the vector product of $\mathbf{a}$ and $\mathbf{b}$

10 Probability and Statistics

A, B, C, etc.	events
$A \cup B$	union of the events A and B
$A \cap B$	intersection of the events A and B
$\mathrm{P}(A)$	probability of the event A
A'	complement of the event A
$\mathrm{P}(A\lvert B)$	probability of the event A conditional on the event B
X, Y, R, etc.	random variables
x, y, r, etc.	values of the random variables X, Y, R, etc.
x_1, x_2, ...	observations
f_1, f_2, ...	frequencies with which the observations x_1, x_2, ... occur
$\mathrm{p}(x)$	probability function $\mathrm{P}(X = x)$ of the discrete random variable X
p_1, p_2, ...	probabilities of the values x_1, x_2, ... of the discrete random variable X
$\mathrm{f}(x)$, $\mathrm{g}(x)$, ...	the value of the probability density function of a continuous random variable X
$\mathrm{F}(x)$, $\mathrm{G}(x)$, ...	the value of the (cumulative) distribution function $\mathrm{P}(X \leq x)$ of a continuous random variable X
$\mathrm{E}(X)$	expectation of the random variable X
$\mathrm{E}(\mathrm{g}(X))$	expectation of $\mathrm{g}(X)$
$\mathrm{Var}(X)$	variance of the random variable X
$\mathrm{G}(t)$	probability generating function for a random variable which takes the values $0, 1, 2, \ldots$
$\mathrm{B}(n, p)$	binomial distribution with parameters n and p
$\mathrm{Geo}(p)$	geometric distribution with parameter p
$\mathrm{Po}(\lambda)$	Poisson distribution with parameter λ
$\mathrm{N}(\mu, \sigma^2)$	normal distribution with mean μ and variance σ^2
μ	population mean
σ^2	population variance
σ	population standard deviation
$\bar{x}$, m	sample mean
s^2, $\hat{\sigma}^2$	unbiased estimate of population variance from a sample, $s^2 = \frac{1}{n-1}\sum (x_i - \bar{x})^2$
ϕ	probability density function of the standardised normal variable with distribution $\mathrm{N}(0, 1)$
Φ	corresponding cumulative distribution function

Appendix 1: Grade Descriptors

The following grade descriptors indicate the level of attainment characteristic of the given grade. They give a general indication of the required standard at each specified grade. The descriptors should be interpreted in relation to the content outlined in the syllabus; they are not designed to define that content.

The grade awarded will depend in practice upon the extent to which the candidate has met the assessment objectives overall. Shortcomings in some aspects of the examination may be balanced by better performance in others.

Distinction (D2)

Candidates manipulate mathematical expressions and use graphs, sketches and diagrams, all with high accuracy and skill. They use mathematical language correctly and proceed logically and rigorously through extended arguments or proofs. When confronted with unstructured problems they can usually devise and implement an effective solution strategy. If errors are made in their calculations or logic, these are usually noticed and corrected.

Candidates recall or recognise almost all the mathematical facts, concepts and techniques that are needed, and select appropriate ones to use in a wide variety of contexts.

Candidates recall or recognise almost all the standard models that are needed, and select appropriate ones to represent a wide variety of situations in the real world. They comprehend or understand the meaning of almost all translations into mathematics of common realistic contexts. They correctly refer results from calculations using a mathematical model to the original situation; they give sensible interpretations of their results in context and usually make sensible comments or predictions. They make intelligent comments on any modelling assumptions. They often are able to solve problems in less familiar contexts.

Candidates make appropriate and efficient use of calculators and other permitted resources, and are aware of any limitations to their use. They present results to an appropriate degree of accuracy.

Merit (M2)

Candidates manipulate mathematical expressions and use graphs, sketches and diagrams, all with a reasonable level of accuracy and skill. They use mathematical language with some skill and often proceed logically through extended arguments or proofs. When confronted with unstructured problems they often devise and implement an effective solution strategy. They sometimes notice and correct errors in their calculations.

Candidates recall or recognise most of the mathematical facts, concepts and techniques that are needed, and usually select appropriate ones to use in a variety of contexts.

Candidates recall or recognise most of the standard models that are needed, and usually select appropriate ones to represent a variety of situations in the real world. They comprehend or understand the meaning of most translations into mathematics of common realistic contexts.

They often correctly refer results from calculations using a mathematical model to the original situation; they often give sensible interpretations of their results in context and sometimes make sensible comments or predictions. They sometimes make intelligent comments on any modelling assumptions. They sometimes are able to solve problems in less familiar contexts.

Candidates usually make appropriate and efficient use of calculators and other permitted resources, and are often aware of any limitations to their use. They usually present results to an appropriate degree of accuracy.

Pass (P2)

Candidates manipulate mathematical expressions and use graphs, sketches and diagrams, all with some accuracy and skill. They sometimes use mathematical language correctly and occasionally proceed logically through extended arguments or proofs.

Candidates recall or recognise some of the mathematical facts, concepts and techniques that are needed, and sometimes select appropriate ones to use in some contexts.

Candidates recall or recognise some of the standard models that are needed, and sometimes select appropriate ones to represent a variety of situations in the real world. They sometimes comprehend or understand the meaning of translations into mathematics of common realistic contexts. They sometimes correctly refer results from calculations using a mathematical model to the original situation; they sometimes interpret their results in context and attempt to make sensible comments or predictions. They try to solve problems in less familiar contexts.

Candidates usually make appropriate and efficient use of calculators and other permitted resources, and are often aware of any limitations to their use. They often present results to an appropriate degree of accuracy.

Appendix 2: Additional Information

Guided Learning Hours

It is intended that each Principal Subject should be delivered through 380 hours of guided learning. This is a notional measure of the substance of the qualification. It includes an estimate of the time that might be allocated to direct teaching or instruction, together with other structured learning time such as directed assignments or supported individual study and practice. It excludes learner-initiated private study.

Certification Title

This qualification is shown on the certificate as:

- Cambridge International Level 3 Pre-U Certificate in **Mathematics (Principal)**

The qualification is accredited at Level 3 of the UK National Qualifications Framework and provides a solid grounding for candidates to pursue a variety of progression pathways.

Entries

For Entry information please refer to the *Pre-U E3 booklet*.

Grading and Reporting

The Cambridge International Level 3 Pre-U Certificates in the Principal Subjects are qualifications in their own right. They are acceptable as an alternative to A Level (or other Level 3 qualifications) for entry into Higher Education or employment. Each individual Principal Subject is graded separately on a scale of nine grades: Distinction 1, Distinction 2, Distinction 3, Merit 1, Merit 2, Merit 3, Pass 1, Pass 2, Pass 3.

Subjects can also be combined with two core components to meet the requirements for eligibility for the Cambridge International Level 3 Pre-U Diploma. More details about the Diploma requirements and the core components can be found in a separate Diploma syllabus. The results of the individual Principal Subjects are reported on a separate certificate to the Diploma result.

Classification Code for UK Centres

In the UK, every syllabus is assigned to a national classification code that indicates the subject area to which it belongs. UK Centres should be aware that candidates who enter for more than one qualification with the same classification code will have only one grade (the highest) counted for the purpose of the School and College Performance Tables.

The classification code for this syllabus is **2210**.

Language

This syllabus and the associated assessment materials are available currently in English only.

Procedures and Regulations

This syllabus complies with the *CIE Code of Practice* and *The Statutory Regulation of External Qualifications 2004*.

Further information about the administration of Cambridge Pre-U qualifications can be found in the CIE *Handbook for Cambridge Pre-U Centres* available from CIE Publications or by contacting international@cie.org.uk

Spiritual, Moral, Ethical, Social, Legislative, Economic and Cultural Issues

Candidates are required to examine arguments critically and so to distinguish between truth and falsehood. They are also expected to interpret the results of modelling exercises and there are times when this could raise some of the above issues. Such issues are not assessed in examination questions.

Sustainable Development, Environmental Education, Health and Safety Considerations, European Dimension and International Agreements

Candidates are expected to interpret the results of modelling exercises and there are times when this could raise some of the above issues. Such issues are not assessed in examination questions.

Avoidance of Bias

CIE has taken great care in the preparation of this syllabus and assessment materials to avoid bias of any kind.

Key Skills

This syllabus provides opportunities for the development of evidence for the Key Skills of: *Communication*, *Application of Number*, *Information Technology*, *Working with Others*, *Improving Own Learning and Performance* and *Problem Solving* at Levels 2 and/or 3. However, the extent to which this evidence fulfils the Key Skills criteria at these levels will be totally dependent on the style of teaching and learning adopted for each section.

The Key Skills awarding bodies and the regulatory authorities have produced a suite of example portfolios that will help to give candidates and practitioners a clear understanding of the requirements for the Key Skills portfolio. These are available on the QCA Key Skills website (www.qca.org.uk/keyskills). Full details of the requirements for certification can be obtained from the awarding bodies that are approved to offer Key Skills. For further information about Key Skills assessment, including the current standards, please see the document *The Key Skills Qualifications Standards and Guidance* published by the Qualifications and Curriculum Authority 2004 (ISBN 1 85838 548 2).

The following table indicates where opportunities may exist for at least some coverage of the various Key Skills criteria at Levels 2 and/or 3 for each section.

Paper	Communication	Application of Number	IT	Working with Others	Improving own Learning and Performance	Problem Solving
Paper 1	✓	✓	✓	✓	✓	✓
Paper 2		✓			✓	✓

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