Cambridge Pre-U Specimen Papers
and Mark Schemes

Cambridge Pre-U

Cambridge International Level 3
Pre-U Certificate in
MATHEMATICS

For use from 2008 onwards

UNIVERSITY of CAMBRIDGE
International Examinations

Cambridge Pre-U

Specimen Materials

Mathematics (9794)

Cambridge International Level 3
Pre-U Certificate in Mathematics (Principal)

For use from 2008 onwards

QAN 500/3789/4

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Syllabus Updates

This booklet of specimen materials is for use from 2008. It is intended for use with the version of the syllabus that will be examined in 2010, 2011 and 2012. The purpose of these materials is to provide Centres with a reasonable idea of the general shape and character of the planned question papers in advance of the first operational examination.

If there are any changes to the syllabus CIE will write to centres to inform them. The syllabus and these specimen materials will also be published annually on the CIE website (www.cie.org.uk/cambridgepreu). The version of the syllabus on the website should always be considered as the definitive version.

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
Cambridge International Level 3 Pre-U Certificate
Principal Subject

MATHEMATICS **9794/01**

Paper 1 Pure Mathematics and Probability **For Examination from 2010**

SPECIMEN PAPER

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF16)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 120.

This document consists of **5** printed pages and **1** blank page.

Section A: Pure Mathematics (79 marks)

1 Find $\int x^2(x+1)\,dx$. [3]

2 Find all values of x for which $0^\circ < x < 360^\circ$ that satisfy the equation

$$\sin\left(\tfrac{1}{2}x\right) = \tfrac{1}{4}.$$ [3]

3 Expand fully $(a+2b)^6$, simplifying the coefficients. [3]

Hence, or otherwise, find the term independent of x in the expansion of

$$\left(x^3 - 2x^{-\frac{3}{2}}\right)^6.$$ [2]

4 Show that the curve $y = 4x^2 + \dfrac{1}{x}$ has only one stationary point, and determine whether it is a maximum or a minimum. [6]

5 Express 9^x in terms of y, where $y = 3^x$. [1]

Verify that the equation

$$6(27^x) - 5(9^x) - 2(3^x) + 1 = 0$$

has $x = 0$ as one of its solutions and find all the other solutions. [6]

6 The function f is defined by

$$\mathrm{f} : x \mapsto x^3 - 1, \quad x \in \mathbb{R}.$$

(i) Find an expression for $\mathrm{f}^{-1}(x)$. [2]

(ii) Sketch on a single diagram the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, making clear the relationship between these graphs. [2]

(iii) Write down $\mathrm{f}'(x)$ and hence determine the gradient of $y = \mathrm{f}^{-1}(x)$ at the point where it crosses the y-axis. [3]

7 The equation of a circle is $x^2 + y^2 - 8y = 9$. Find the coordinates of the centre of the circle, and the radius of the circle. [3]

A straight line has equation $x = 3y + k$, where k is a constant. Show that the y-coordinates of the points of intersection of the line and the circle are given by

$$10y^2 + (6k-8)y + (k^2 - 9) = 0.$$ [2]

Hence determine the exact values of k for which the line is a tangent to the circle. [4]

8 Prove, without using any decimal approximations, that

$$\sqrt{\left(\tfrac{2}{3}\right)} > \tfrac{3}{4} > \sqrt{\left(\tfrac{1}{2}\right)} > \tfrac{2}{3}. \qquad [3]$$

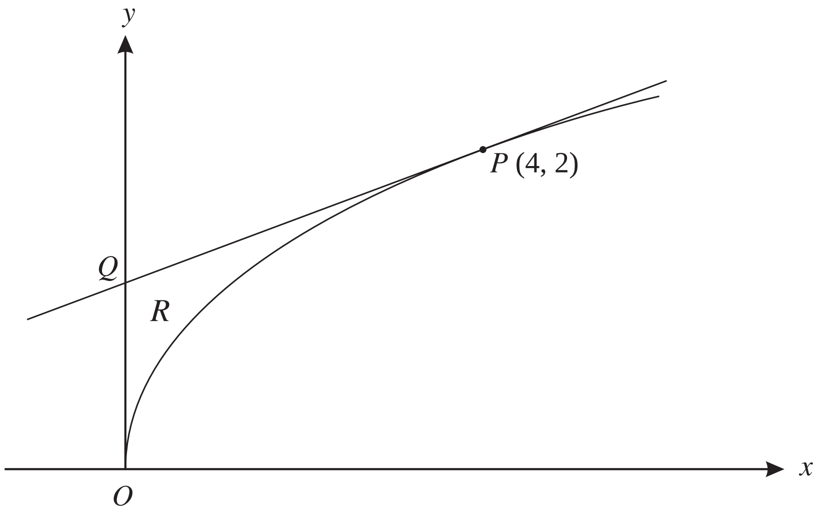
Prove that

$$\sqrt{\left(\frac{n}{n+1}\right)} > \frac{n+1}{n+2}$$

for all positive integer values of n. [4]

Is the inequality valid for all positive rational values of n? [2]

9



The diagram shows the graph of $y = x^{\frac{1}{2}}$. The point P on the graph has coordinates $(4, 2)$. The tangent at P meets the y-axis at Q. Find the area of the region R bounded by the curve, the y-axis and the tangent PQ. [9]

10 The line l has equation $y = \frac{1}{2}x$ and the point P has coordinates $(5, 0)$. Find the equation of the line through P perpendicular to l. Hence find the coordinates of the point N which is the foot of the perpendicular from P to l. [4]

By sketching a diagram showing l, P, N and the origin O, find the coordinates of the point Q which is the reflection of P in l. [3]

By considering the gradients of l and OQ, show that

$$\tan^{-1}\left(\tfrac{4}{3}\right) = 2\tan^{-1}\left(\tfrac{1}{2}\right). \qquad [3]$$

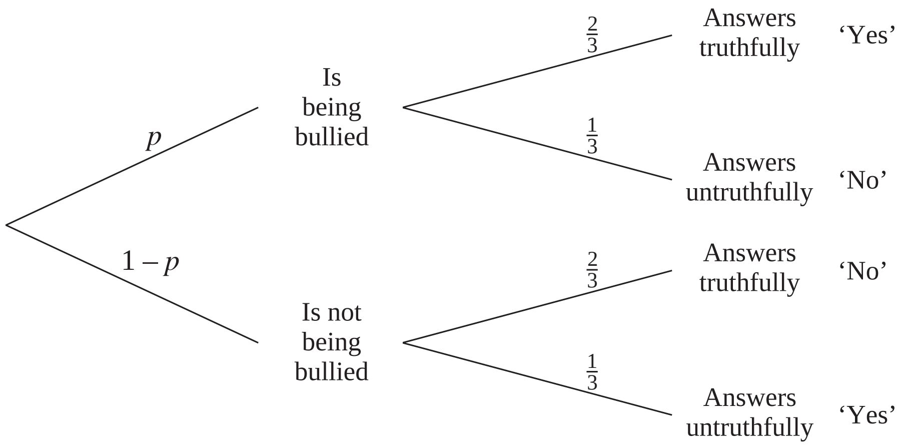
11 **(i)** Show that

$$\tan^{-1}\left(\tfrac{1}{2}\right) + \tan^{-1}\left(\tfrac{1}{3}\right) = \tfrac{1}{4}\pi. \qquad [3]$$

(ii) *ABCD* is a square field in which a goat is tethered to the corner *A* by means of a rope. The rope is just long enough for the goat to be able to reach the mid-points of *BC* and *CD*. Find the proportion of the area of the field that the goat cannot reach. Express your answer in the form $a + b\tan^{-1}\left(\tfrac{1}{3}\right)$, where a and b are rational numbers. [8]

Section B: Probability (41 marks)

12 A researcher is investigating the proportion p of children who are being bullied at school. To overcome any reluctance children might have to answering questions about being bullied, the following procedure is used. The researcher asks 'Are you being bullied at school?'. Before answering, the child being interviewed throws an unbiased die (unseen by the researcher); if the score on the die is 1, 2, 3 or 4 the child answers the question truthfully and if the score is 5 or 6 the child answers untruthfully. This procedure is illustrated in the tree diagram below.



(i) Show that the probability that a child answers 'Yes' to the researcher's question is $\tfrac{1}{3}(1+p)$. [3]

(ii) The researcher finds that 35% of children, on average, answer 'Yes'. Find the conditional probability that a child who answers 'No' is answering truthfully. [4]

13 After extensive testing, it was found that the lifetimes of Osric light bulbs had a mean of 2408 hours and a standard deviation of 101 hours. Assuming that the lifetime of a bulb is modelled by a normal distribution, find

(i) the probability that an Osric light bulb will have a lifetime of more than 2600 hours, [3]

(ii) the percentage of bulbs having a lifetime of between 2200 hours and 2500 hours, [4]

(iii) the lifetime, correct to the nearest hour, exceeded by 5% of bulbs. [3]

14 Four married couples are to be seated in a row of eight seats numbered 1 to 8.

(i) If no two men and no two women are to sit together, find the number of different ways in which the eight people can be arranged. [3]

(ii) If the four men sit down in seats 1, 3, 5 and 7, show that there are 3 ways in which the wives can sit so that none of them sits next to her husband. [2]

(iii) Hence or otherwise find the total number of arrangements in which the men and women occupy alternate seats and no wife sits next to her husband. [4]

(iv) If husbands and wives are to sit together, find the number of ways in which the eight people can be arranged. [3]

15 It is known that the proportion of men who are right-handed is 0.8 and that the proportion of women who are right-handed is 0.8.

(i) Three women are chosen at random and three men are chosen at random.

(a) Find the probability that exactly two women and exactly two men are right-handed. [3]

(b) Find the probability that equal numbers of men and women are right-handed. [4]

(ii) A man is chosen at random, then a woman is chosen at random, then another man, and so on alternately. The process continues until a right-handed person is chosen.

(a) Find the probability that the right-handed person is the second woman chosen. [2]

(b) By summing an appropriate series, or otherwise, show that the probability that the right-handed person is a woman is $\frac{1}{6}$.

[3]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
Cambridge International Level 3 Pre-U Certificate
Principal Subject

MATHEMATICS **9794/01**

Paper 1 Pure Mathematics and Probability **For Examination from 2010**

SPECIMEN MARK SCHEME

3 hours

MAXIMUM MARK: 120

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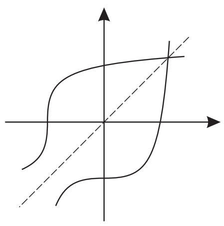
UNIVERSITY of CAMBRIDGE International Examinations

1	Expand brackets and attempt to integrate each of the resulting two terms	M1	
	Obtain $\frac{1}{4}x^4 + \frac{1}{3}x^3$	A1	
	Include arbitrary constant in answer	B1	**3**
2	State, or show evidence of use of, $\sin^{-1}\left(\frac{1}{4}\right)$	M1	
	Obtain correct answer $x = 29°$	A1	
	Obtain second correct answer $x = 331°$ and no others in range	A1	**3**
3	State 7 terms, each of the form $ka^r b^{6-r}$, starting with a^6 and finishing with Kb^6	M1	
	Obtain at least 5 correct coefficients (1, 12, 60, 160, 240, 192, 64)	A1	
	Obtain correct expansion $a^6 + 12a^5b + 60a^4b^2 + 160a^3b^3 + 240a^2b^4 + 192ab^5 + 64b^6$	A1	**3**
	Substitute $a = x^3$ and $b = -x^{-\frac{3}{2}}$ into the a^2b^4 term	M1	
	Obtain 240	A1√	**2**
4	Differentiate both terms and equate to zero	M1	
	Obtain $8x$ and $-x^{-2}$ (or equivalent) correctly	A1, A1	
	Obtain $x = \frac{1}{2}$, stating that it is the only solution	A1	
	Consider sign of $y''(x)$ at $x = \frac{1}{2}$, or use any other valid method	M1	
	State that the point is a minimum, following completely correct working	A1	**6**
5	State or obtain $9^x = y^2$	B1	**1**
	Show correct verification $6(27^0) - 5(9^0) - 2(3^0) + 1 = 6 - 5 - 2 + 1 = 0$	B1	
	State correct y-equation $6y^3 - 5y^2 - 2y + 1 = 0$	B1	
	State or imply factor $(y - 1)$ and attempt factorisation of cubic	M1	
	Obtain correct three factors $(y-1)(2y+1)(3y-1)$	A1	
	Attempt solution for x from second and third factors	M1	
	State no solution from second factor and $x = -1$ from third	A1	**6**
6	**(i)** Let $x = y^3 - 1$ and attempt to change the subject, or equivalent	M1	
	Obtain $\mathrm{f}^{-1}(x) = (x+1)^{\frac{1}{3}}$	A1	**2**
	(ii)  Show correct sketch of $y = \mathrm{f}(x)$	B1	
	Show correct sketch of $y = \mathrm{f}^{-1}(x)$, together with $y = x$ or explanation of the relationship between the graphs	B1	**2**
	(iii) State $\mathrm{f}'(x) = 3x^2$	B1	
	Evaluate reciprocal gradient of $\mathrm{f}(x)$ at (1, 0)	M1	
	Obtain correct answer $\frac{1}{3}$	A1	**3**
7	*EITHER*: Write circle equation in the form $x^2 + (y-4)^2 = 25$	M1	
	State that centre is (0, 4)	A1	
	State that radius is 5	A1	
	OR: Quote correct formula $\sqrt{(g^2 + f^2 - c)}$ and use $f^2 = 16$	M1	
	State that radius is 5	A1	
	State that centre is (0, 4)	A1	**3**
	Substitute for x and attempt expansion of $(3y+k)^2$	M1	
	Obtain given answer $10y^2 + (6k-8)y + (k^2 - 9) = 0$ correctly	A1	**2**
	State the condition $(6k-8)^2 - 40(k^2 - 9) = 0$, or equivalent	B1*	
	Expand and form 3-term quadratic in k	M1(dep*)	
	Attempt exact solution of the resulting quadratic equation	M1	
	Obtain answer $k = -12 \pm 5\sqrt{10}$, or any equivalent exact form	A1	**4**

8 State sequence of squares $\frac{2}{3}, \frac{9}{16}, \frac{1}{2}, \frac{4}{9}$	M1	
Evaluate differences $\frac{5}{48}, \frac{1}{16}, \frac{1}{18}$	A1	
State that as these are all positive the original numbers are in decreasing order	A1	**3**
Consider the difference of squares $\frac{n}{n+1} - \frac{(n+1)^2}{(n+2)^2}$	M1	
Obtain correct simplified expression $\frac{n^2+n-1}{(n+1)(n+2)^2}$	A1	
Use completing the square for the numerator, or equivalent	M1	
Justify given result for positive integer n, e.g. via $\frac{(n+\frac{1}{2})^2 - \frac{5}{4}}{(n+1)(n+2)^2}$	A1	**4**
[Other approaches are possible, e.g. n a positive integer $\Rightarrow n^2 + n > 1$ $\Rightarrow n^3 + 4n^2 + 4n > n^3 + 3n^2 + 3n + 1 \Rightarrow n(n+2)^2 > (n+1)^3 \Rightarrow \frac{n}{n+1} > \frac{(n+1)^2}{(n+2)^2}$, and hence the result]		
State that numerator is negative for $n = \frac{1}{2}$ (e.g.)	M1	
Conclude that result is not true for all positive rational n	A1	**2**
9 Attempt to differentiate $x^{\frac{1}{2}}$	M1	
Obtain correct value $\frac{1}{4}$ for the derivative at $x = 4$	A1	
Carry out complete method for finding the y-coordinate of Q	M1	
Obtain $y_Q = 1$	A1	
EITHER: Attempt (indefinite) integration of $x^{\frac{1}{2}}$	M1	
Use limits 4 and 0 correctly in the integral	M1	
Obtain $\frac{16}{3}$ or equivalent	A1	
Calculate area of R by subtracting $\frac{16}{3}$ from area of the trapezium	M1	
Obtain answer $\frac{2}{3}$ correctly	A1	
OR: Attempt (indefinite) integration of y^2	M1	
Use limits 2 and 0 correctly in the integral	M1	
Obtain $\frac{8}{3}$ or equivalent	A1	
Calculate area of R by subtracting the area of the triangle from $\frac{8}{3}$	M1	
Obtain answer $\frac{2}{3}$ correctly	A1	**9**
10 State or imply that the gradient of the perpendicular is -2	B1	
State the equation as $y = -2(x-5)$ or equivalent	B1	
Solve the two relevant equations simultaneously	M1	
Obtain answer (4, 2)	A1	**4**
Sketch diagram with N on l in first quadrant, P on x-axis, angle $ONP \approx 90^\circ$	B1	
Use mid-point property to find Q	M1	
State that Q is (3, 4)	A1	**3**
Identify the equal reflection angles (on the sketch, or otherwise)	B1	
Identify the gradient of l with $\tan^{-1}\left(\frac{1}{2}\right)$ and/or the gradient of OQ with $\tan^{-1}\left(\frac{4}{3}\right)$	B1	
Demonstrate the given result completely correctly	B1	**3**

11	**(i)**	Use compound angle formula to find $\tan\{\tan^{-1}(\frac{1}{2})+\tan^{-1}(\frac{1}{3})\}$	M1	
		Obtain $\dfrac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\times\frac{1}{3}}$	A1	
		Obtain given answer $\frac{1}{4}\pi$ correctly	A1	**3**
	(ii)	State length of rope is $x\sqrt{5}$ (where side of square is $2x$)	B1	
		Obtain angle of the relevant sector as $\frac{1}{2}\pi - 2\tan^{-1}(\frac{1}{2})$	B1	
		Use correct formula for area, S, of sector	M1	
		Obtain $S=\frac{1}{2}(x\sqrt{5})^2\{\frac{1}{2}\pi-2\tan^{-1}(\frac{1}{2})\}$	A1	
		State or imply unreachable area is $4x^2-x^2-x^2-S$	M1	
		Obtain unreachable proportion as $\dfrac{2x^2-S}{4x^2}$, or equivalent	A1	
		Use (i) to obtain numerical value involving $\tan^{-1}(\frac{1}{3})$	M1	
		Obtain answer $\frac{1}{2}-\frac{5}{4}\tan^{-1}(\frac{1}{3})$	A1	**8**
12	**(i)**	Multiply two relevant probabilities for a ‘Yes’ branch	M1*	
		Add the two relevant two-factor cases, i.e. $\frac{2}{3}p+\frac{1}{3}(1-p)$	M1(dep*)	
		Obtain given answer $\frac{1}{3}(1+p)$ correctly	A1	**3**
	(ii)	Solve $\frac{1}{3}(1+p)=0.35$ and find $p=0.05$	B1	
		Divide attempted P(‘No’ and ‘Truthful’) by P(‘No’)	M1	
		State or imply answer is $\dfrac{\frac{2}{3}(1-0.05)}{1-0.35}$	A1$\surd$	
		Obtain answer 0.974 or $\frac{38}{39}$	A1	**4**
13	**(i)**	State or imply $z=\dfrac{2600-2408}{101}$	B1	
		Carry out method for obtaining the correct tail probability	M1	
		Obtain answer 0.029	A1	**3**
	(ii)	State or imply z-values 0.9109 and –2.0594	B1	
		Carry out correct method for obtaining $P(-a<Z<+b)$	M1	
		Obtain probability 0.799	A1	
		State answer in required percentage form, i.e. 79.9% or 80%	A1$\surd$	**4**
	(iii)	Use correct z-value 1.645	B1	
		Attempt solution of equation of the form $\dfrac{T-\mu}{\sigma}=z$	M1	
		Obtain answer 2574 (hours)	A1	**3**
14	**(i)**	Number of ways is $2\times(4!)^2=1152$	B3	**3**
		[If correct value 1152 is not obtained, allow B2 for $2\times(4!)^2$ or B1 for $(4!)^2$.]		
	(ii)	Consider possible schemes, i.e. $M_1(W_3\|W_4)M_2(W_1\|W_4)M_3(W_1\|W_2)M_4(W_1\|W_2\|W_3)$	M1	
		Conclude clearly that only $W_4W_1W_2W_3$ or $W_3W_4W_1W_2$ or $W_3W_4W_2W_1$ can work	A1	**2**
	(iii)	State that the number of ways of arranging the men in seats 1, 3, 5, 7 is 4!	B1	
		State that then arranging the wives gives $3\times4!$ possibilities	M1	
		Consider the alternative case where the men sit in seats 2, 4, 6, 8	M1	
		Obtain correct answer 144	A1	**4**
	(iv)	State that there are 4! ways of arranging the four MW units	M1	
		State that each MW pair can sit in 2 ways, giving 2^4 possibilities	M1	
		Obtain answer 384	A1	**3**

15	**(i) (a)**	State the binomial probability $3(0.8)^2(0.2)$ for either men or women	M1	
		Square this probability	M1	
		Obtain correct answer 0.147	A1	**3**
	(b)	State or imply that required probability is P(0, 0) + P(1, 1) + P(2, 2) + P(3, 3)	M1	
		State correct expression $\left(0.2^3\right)^2 + \left(3(0.2)^2(0.8)\right)^2 + \left(3(0.2)(0.8)^2\right)^2 + \left(0.8^3\right)^2$	A2	
		Obtain answer 0.419	A1	**4**
		[If the P(0, 0) term is omitted, allow M1A1A0 if otherwise correct.]		
	(ii) (a)	State or imply required event is M_L, W_L, M_L, W_R	M1	
		Obtain answer $0.2^3 \times 0.8 = 0.0064$	A1	**2**
	(b)	State required probability as $(0.2)(0.8) + (0.2)^3(0.8) + (0.2)^5(0.8) + \ldots$	M1	
		Use GP sum formula with appropriate a and r	M1	
		Obtain given answer $\frac{1}{6}$ correctly	A1	**3**

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
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MATHEMATICS **9794/02**

Paper 2 Pure Mathematics and Mechanics **For Examination from 2010**

SPECIMEN PAPER

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF16)

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You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value for the acceleration due to gravity is needed, use $10\,\mathrm{m\,s^{-2}}$.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 120.

Section A: Pure Mathematics (80 marks)

1 Show that the sum of the first n multiples of 8 is one less than the square of an odd number, for all positive integers n. [4]

2 Solve the inequality $|x+1| < |2x+1|$. [4]

3 **(i)** Find the exact solution of the equation $9^{x+1} = 10^x$, giving your answer in terms of $\log_{10} 9$. [2]

(ii) Determine the number of digits in the expansion of 386^{386}. [3]

4 Find the value of $\displaystyle\int_0^2 \frac{x-1}{x^2-2x-3}\,\mathrm{d}x$. [6]

5 **(i)** Show that the equation $x = 1 - x\cos x$ has at least two roots in the interval $0 \leqslant x \leqslant 3$. [2]

(ii) Use the iteration $x_{n+1} = 1 - x_n \cos x_n$, with $x_0 = 0$, to find an approximation to one of the roots to 1 decimal place. [3]

(iii) Comment on the use of the same iteration to attempt to find a different root starting with $x_0 = 3$. [2]

6 Prove the identity

$$\tan A + \cot A \equiv \frac{2}{\sin 2A}.$$ [3]

Hence, or otherwise, solve the equation

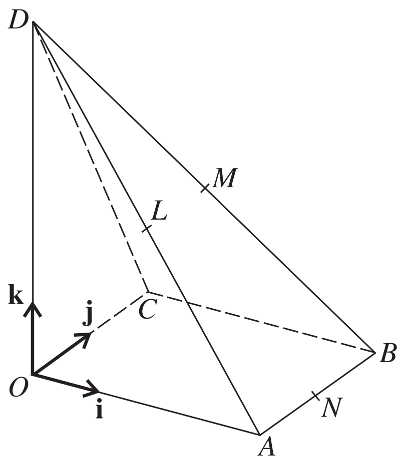
$$\tan(\theta + 45^\circ) + \cot(\theta + 45^\circ) = 4,$$

giving all the solutions in the interval $0^\circ < \theta < 360^\circ$. [5]

7 **(i)** Determine the roots p and q of the equation $x^2 - 6x + 13 = 0$. If p is the root whose argument is an acute angle, find an equation whose roots are $\mathrm{i}p$ and $-\mathrm{i}q$. [5]

(ii) Sketch on the same Argand diagram the loci of points representing $|z - \mathrm{i}| = 2$ and $\arg z = \frac{1}{4}\pi$. Determine the intersection of these loci. [5]

8



The diagram shows a pyramid $DOABC$. Taking unit vectors $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ as shown, the position vectors of A, B, C, D are given by

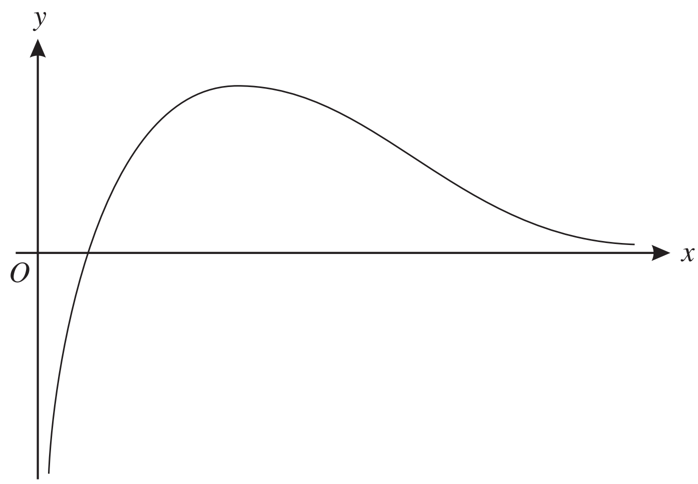
$$\overrightarrow{OA} = 4\mathbf{i}, \quad \overrightarrow{OB} = 4\mathbf{i} + 2\mathbf{j}, \quad \overrightarrow{OC} = 2\mathbf{j}, \quad \overrightarrow{OD} = 6\mathbf{k}.$$

The mid-points of AD, BD and AB are L, M and N respectively.

(i) Find the vector $\overrightarrow{MN}$ and the angle between the directions of $\overrightarrow{MN}$ and $\overrightarrow{OB}$. [4]

(ii) The point P lying on OD has position vector $p\mathbf{k}$. Determine the value of p for which the line through P and B intersects the line through C and L. [7]

9



The diagram shows a sketch of the curve $y = \dfrac{\ln x}{x}$. Show that the maximum value of y occurs when $x = \mathrm{e}$. [3]

State the set of values of the constant k for which the equation

$$\frac{\ln x}{x} = k$$

has two distinct real roots for x. [2]

These roots are denoted by a and b, where $a < b$.

(i) Explain why $1 < a < \mathrm{e}$, and state an inequality satisfied by b. [3]

(ii) Show that $a^b = b^a$. [2]

(iii) Given that a and b are positive integers, deduce from parts **(i)** and **(ii)** the values of a and b. [2]

10 Waste material from a mining operation is dumped on a 'slag-heap', which is a large mound, roughly conical in shape, which continually increases in size as more waste material is added to the top. In a mathematical model, the rate at which the height h of the slag-heap increases is inversely proportional to h^2. Express this statement as a differential equation relating h with the time t. [2]

Show by integration that the general solution of the differential equation relating h and t may be expressed in the form

$$h^3 = At + B,$$

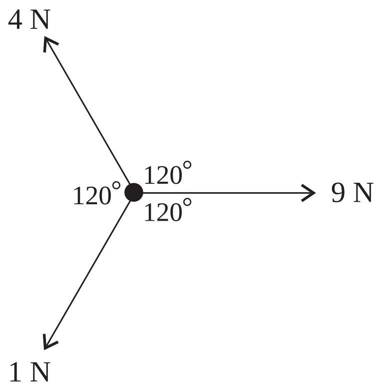
where A and B are constants. [3]

A new slag-heap was started at time $t = 0$, and after 2 years its height was 18 m. Find the time by which its height would grow to 30 m. [4]

The assumptions underlying this mathematical model are that the volume V of the slag-heap increases at a constant rate, and that the slag-heap remains the same shape as it grows, so that V is proportional to h^3. Show how these assumptions lead to the model described in the first paragraph. [4]

Section B: Mechanics (40 marks)

11

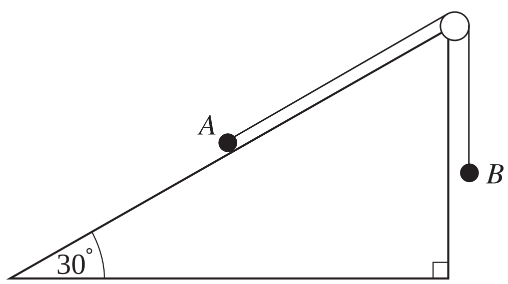


Coplanar forces of magnitudes 1 N, 4 N, 9 N act on a particle, as shown in the diagram; the angle between the directions of each pair of the forces is 120°. The resultant of the three forces has components X N parallel to the 9 N force and Y N perpendicular to the 9 N force.

(i) Find X and Y. [3]

(ii) Hence show that the magnitude of the resultant is 7 N, and calculate the angle that the resultant makes with the 9 N force. [3]

12



Particles A and B, each of mass m kg, are connected by a light inextensible string. Particle A rests on an inclined plane, particle B hangs freely, and the string passes over a smooth pulley at the top of the plane (see diagram).

(i) Given that the plane is smooth, show that the acceleration of each particle has magnitude $\frac{1}{4}g$ and express the tension in the string in terms of m and g. [5]

(ii) Given instead that the plane is rough and that the system is in limiting equilibrium, find the coefficient of friction between A and the plane. [4]

13 A ball is dropped from rest from the top of a tower H metres above a horizontal plane. At the first rebound, the ball rises to a height of 24 m above the plane, and at the second rebound it rises to a height of 6 m above the plane.

(i) Calculate the coefficient of restitution between the ball and the plane, and deduce that $H = 96$. [8]

(ii) Show that the total distance travelled by the ball before it comes to rest is 160 m. [3]

14 A particle is projected from the point O, on horizontal ground, with initial speed V at an angle θ above the horizontal. At time t after projection the coordinates of the particle are (x, y) referred to horizontal and vertical axes at O. Obtain the cartesian equation of the trajectory. [4]

The particle is projected directly towards a vertical wall at distance a from O, and strikes the wall at a height h above the ground. The speed of projection is $\sqrt{(2ga)}$. Letting $p = \tan\theta$, show that

$$ap^2 - 4ap + a + 4h = 0. \qquad [3]$$

Find the two values of θ for which the particle strikes the wall at ground level. [3]

Find the maximum value of h as p varies, and the corresponding value of θ. [4]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
Cambridge International Level 3 Pre-U Certificate
Principal Subject

MATHEMATICS **9794/02**

Paper 2 Pure Mathematics and Mechanics **For Examination from 2010**

SPECIMEN MARK SCHEME

3 hours

MAXIMUM MARK: 120

This document consists of **5** printed pages and **1** blank page.

1	State required sum is $8 + 16 + 24 + 32 + \ldots + 8n$ Use appropriate AP sum formula Obtain $4n(n+1)$ Justify given result, via $4n(n+1) \equiv (2n+1)^2 - 1$	B1 M1 A1 A1	 **4**
2	*EITHER*: Square both sides, leading to two 3-term quadratics Simplify to $3x^2 + 2x > 0$, or equivalent 2-term inequality or equation Carry out complete method leading to two critical values State answer $x > 0$ or $x < -\frac{2}{3}$ *OR*: Sketch graphs of both $y = \lvert x+1 \rvert$ and $y = \lvert 2x+1 \rvert$ on a single diagram Show two correct graphs Solve $y = x + 1$ and $y = -2x - 1$ to obtain the non-zero critical value State answer $x > 0$ or $x < -\frac{2}{3}$ *OR*: State both possibilities $x + 1 = \pm(2x+1)$, as either equations or inequalities Obtain two correct equations (or with inequality signs) Solve both to obtain two critical values State answer $x > 0$ or $x < -\frac{2}{3}$	M1* A1 M1(dep*) A1 M1* A1 M1(dep*) A1 M1* A1 M1(dep*) A1	 **4**
3	**(i)** Take logs of both sides to base 10 Obtain $x = \dfrac{\log_{10} 9}{1 - \log_{10} 9}$ **(ii)** Attempt to relate the number of digits in N to $\log_{10} N$ Evaluate $\log_{10}(386^{386}) = 998.4$ Deduce correctly that the required number of digits is 999	M1 A1 M1 A1 A1	 **2** **3**
4	State or imply correct form $\dfrac{A}{x+1} + \dfrac{B}{x-3}$ for partial fractions Use 'cover-up' or equivalent to evaluate A and B Obtain $A = B = \frac{1}{2}$ Integrate partial fractions to obtain $\left[\frac{1}{2}\lvert x+1 \rvert + \frac{1}{2}\ln\lvert x-3 \rvert\right]_0^2$ Use limits correctly (no logs of negative quantities) Obtain answer 0 correctly	B1 M1 A1 A1 M1 A1	 **6**
5	**(i)** Evaluate $x + x\cos x - 1$, or equivalent, at e.g. $x = 0,\ 1.5,\ 3$ Demonstrate two sign changes and deduce given result correctly **(ii)** Use given iteration to obtain at least x_2 $(= 0.459\,69\ldots)$ Continue to $x_6 = 0.5286\ldots$ and $x_7 = 0.5435\ldots$, using at least 3sf Deduce correctly that the root is 0.5 correct to 1 decimal place **(iii)** State that x_1 is now outside the interval $0 \leqslant x \leqslant 3$ Explain that there appears to be no convergence in this case	M1 A1 M1 A1 A1 B1 B1	 **2** **3** **2**

6	Make relevant use of $\cos^2 A + \sin^2 A = 1$ or $1 + \tan^2 A = \sec^2 A$	M1	
	Make relevant use of a double-angle formula	M1	
	Complete the proof of the identity correctly	A1	**3**
	EITHER: Use identity and obtain equation in $\cos 2\theta$ or $\sin(2\theta + 90°)$, in any form	M1	
	Obtain a simplified equation, e.g. $2\cos 2\theta = 1$ or $4\cos^2\theta = 3$ or $4\sin^2\theta = 1$	A1	
	Show evidence of use of $\cos^{-1}\left(\frac{1}{2}\right)$, or equivalent	M1	
	Obtain one correct answer in range, e.g. 30°	A1	
	Obtain the other three correct values (only), e.g. 150°, 210°, 330°	A1	
	OR: Use $\tan(A + B)$ to obtain an equation in $\tan\theta$	M1	
	Obtain simplified equation, e.g. $6\tan^2\theta = 2$	A1	
	Show evidence of use of $\tan^{-1}(1/\sqrt{3})$	M1	
	Obtain one correct answer in range, e.g. 30°	A1	
	Obtain the other three correct values (only), e.g. 150°, 210°, 330°	A1	
	OR: Form a quadratic equation in $\tan(\theta + 45°)$	M1	
	Obtain $\tan^2(\theta + 45°) - 4\tan(\theta + 45°) + 1 = 0$	A1	
	Solve the quadratic, and show evidence of $\tan^{-1}$ usage	M1	
	Obtain one correct answer in range, e.g. 30°	A1	
	Obtain the other three correct values (only), e.g. 150°, 210°, 330°	A1	**5**
7	**(i)** Obtain p and q as $3 \pm 2\mathrm{i}$	B1	
	Identify p as $3 + 2\mathrm{i}$ and evaluate the required new roots	M1	
	Obtain $-2 + 3\mathrm{i}$ and $-2 - 3\mathrm{i}$	A1	
	Substitute and expand $(x - \mathrm{i}p)(x + \mathrm{i}q) = 0$	M1	
	Obtain $x^2 + 4x + 13 = 0$	A1	**5**
	(ii) Sketch circle with centre (0, 1) and radius 2	B1	
	Sketch the half-line $y = x$ in the first quadrant	B1	
	Set up and solve an appropriate equation for the intersection, e.g. $x^2 + (x - 1)^2 = 4$	M1	
	Obtain roots $x = \frac{1}{2}(1 \pm \sqrt{7})$	A1	
	State that the point of intersection is $\left(\frac{1}{2}(1 + \sqrt{7}), \frac{1}{2}(1 + \sqrt{7})\right)$	A1	**5**
8	**(i)** State $\overrightarrow{OM} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ or $\overrightarrow{ON} = 4\mathbf{i} + \mathbf{j}$ or $\overrightarrow{DA} = 4\mathbf{i} - 6\mathbf{k}$ or $\overrightarrow{DB} = 4\mathbf{i} + 2\mathbf{j} - 6\mathbf{k}$	B1	
	Obtain $\overrightarrow{MN} = 2\mathbf{i} - 3\mathbf{k}$	B1	
	Show completely correct method for evaluating $\cos^{-1}\left(\dfrac{(2 \times 4) + (0 \times 2) + (-3 \times 0)}{\sqrt{(2^2 + (-3)^2)} \times \sqrt{(4^2 + 2^2)}}\right)$	M1	
	Obtain answer 60.3°	A1√	**4**
	(ii) State $\overrightarrow{PB} = 4\mathbf{i} + 2\mathbf{j} - p\mathbf{k}$	M1	
	Obtain the equation of PB as $\mathbf{r} = (4\mathbf{i} + 2\mathbf{j}) + \lambda(4\mathbf{i} + 2\mathbf{j} - p\mathbf{k})$, or equivalent	A1	
	State or imply $\overrightarrow{OL} = 2\mathbf{i} + 3\mathbf{k}$ and hence $\overrightarrow{CL} = 2\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$	A1	
	Obtain the equation of CL as $\mathbf{r} = 2\mathbf{j} + \mu(2\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$	A1	
	Equate the expressions for $\mathbf{r}$	M1	
	Obtain $4 + 4\lambda = 2\mu$, $2 + 2\lambda = 2 - 2\mu$, $-p\lambda = 3\mu$	A1	
	Obtain (e.g. via $\lambda = -\frac{2}{3}$, $\mu = \frac{2}{3}$) correct answer $p = 3$	A1	**7**

9	Obtain $\frac{dy}{dx} = \frac{x.(1/x) - \ln x}{x^2}$	B1	
	Equate numerator of the derivative to zero	M1	
	Obtain given answer $x = e$ correctly	A1	**3**
	Obtain $y = \frac{1}{e}$ at the maximum	B1	
	State $0 < k < \frac{1}{e}$	B1√	**2**
	(i) State that smaller root is to the left of the maximum, so $a < e$	B1	
	State that graph cuts the axis at $x = 1$, so $a > 1$	B1	
	State $b > e$	B1	**3**
	(ii) State $\frac{\ln a}{a} = \frac{\ln b}{b}$ and use log laws	M1	
	Obtain given result $a^b = b^a$ correctly	A1	**2**
	(iii) State that $a = 2$	B1	
	Deduce that $b = 4$	B1	**2**
10	State a differential equation of the form $\frac{dh}{dt} = f(h)$	M1	
	State correct equation $\frac{dh}{dt} = \frac{k}{h^2}$, or equivalent	A1	**2**
	Make recognisable attempt to separate the variables	M1*	
	Attempt to integrate each side w.r.t. the appropriate variable	M1(dep*)	
	Obtain the given form of answer following correct integration	A1	**3**
	Use $18^3 = 2A + B$ and/or $h = 0$ when $t = 0$	M1	
	Obtain $B = 0$ and $A = 2916$	A1	
	Evaluate t when $h = 30$	M1	
	Obtain answer 9.26 (years)	A1	**4**
	EITHER: State $\frac{dV}{dt} = k_1$ and $V = k_2h^3$	B1	
	Differentiate the relation between V and h	M1	
	Use the chain rule	M1	
	Obtain $\frac{dh}{dt} = \frac{k}{h^2}$ correctly	A1	
	OR: State that assumptions mean that h^3 increases at a constant rate	M1	
	Express this symbolically as $\frac{d}{dt}\left(h^3\right) = k$ or $h^3 = At + B$ or equivalent	A1	
	Use appropriate implicit differentiation	M1	
	Obtain $\frac{dh}{dt} = \frac{k}{h^2}$ correctly	A1	**4**
11	**(i)** Show evidence of at least one correct resolution, i.e. use of '$F\cos\theta$'	M1	
	Obtain $X = 6.5$	A1	
	Obtain $Y = \frac{3}{2}\sqrt{3}$	A1	**3**
	(ii) Obtain given answer 7 following correct use of $\sqrt{(X^2 + Y^2)}$	B1	
	State correct trigonometric equation involving two of (numerical) X, Y, 7	M1	
	Obtain correct angle 21.8°	A1	**3**

12	**(i)** State any one of $mg - T = ma$, $T - mg\cos 60 = ma$, $mg - mg\cos 60 = (m+m)a$	B1	
	State another of the above equations	B1	
	Carry out complete solution method for both a and T	M1	
	Obtain given answer $a = \frac{1}{4}g$ correctly	B1	
	Obtain $T = \frac{3}{4}mg$	A1	**5**
	(ii) State any one correct resolving equation for A, e.g. $R = mg\cos 30$	B1	
	State a second correct resolving equation, e.g. $T = mg\sin 30 + F$	B1	
	Use $T = mg$, $F = \mu R$ and solve equations to find μ	M1	
	Obtain answer $\mu = \dfrac{1}{\sqrt{3}}$ or equivalent	A1	**4**
13	**(i)** Use $v^2 = u^2 + 2as$ to find speed V before first impact	M1	
	Obtain correct relation $V^2 = 2gH$	A1	
	Use initial rebound speed eV and relate to height of first bounce	M1	
	Obtain correct relation $(eV)^2 = 2g \times 24$	A1	
	State analogous relation $(e^2V)^2 = 2g \times 6$ for second bounce	A1	
	Solve simultaneous equations for e	M1	
	Obtain $e = 0.5$	A1	
	Deduce given answer $H = 96$ correctly	A1	**8**
	(ii) State total distance is $H + 2(e^2H + e^4H + e^6H + \dots)$	B1	
	Use sum to infinity of GP with ratio e^2	M1	
	Obtain given answer 160 m correctly	A1	**3**
14	State $x = Vt\cos\theta$ and $y = Vt\sin\theta - \frac{1}{2}gt^2$	M1, A1	
	Eliminate t from these two equations	M1	
	Obtain equation $y = x\tan\theta - \dfrac{gx^2}{2V^2\cos^2\theta}$	A1	**4**
	Substitute $x = a$, $y = h$, $V^2 = 2ga$ in trajectory equation	M1	
	Use $\dfrac{1}{\cos^2\theta} = 1 + p^2$	M1	
	Obtain given answer $ap^2 - 4ap + a + 4h = 0$ correctly	A1	**3**
	Substitute $h = 0$ and solve quadratic for p	M1	
	Obtain $p = 2 \pm \sqrt{3}$, or equivalent	A1	
	Obtain both angles 15° and 75°	A1	**3**
	EITHER: Attempt differentiation with respect to p	M1	
	Equate $\dfrac{dh}{dp}$ to zero and solve for p	M1	
	Obtain $p = 2$ and hence $\theta = 63.4°$	A1	
	Obtain $h_{max} = \frac{3}{4}a$	A1	
	OR: Attempt to complete the square	M1	
	Equate squared term to zero and solve for p	M1	
	Obtain $p = 2$ and hence $\theta = 63.4°$	A1	
	Obtain $h_{max} = \frac{3}{4}a$	A1	**4**

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MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

1. Marks are of the following three types.

   M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula.

   A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied).

   B Mark for a correct result or statement independent of Method marks.

   The marks indicated in the scheme may not be subdivided. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

2. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation 'dep*' is used to indicate that a particular M or B mark is dependent on an earlier, asterisked, mark in the scheme. Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

3. The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A and B marks are not given for 'correct' answers or results obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable.

4. Where alternative methods of solution, not covered in the mark scheme, are used, full marks will be given for a correct result obtained by any valid method, with equivalent partial credit for equivalent stages. (This does not however apply if candidates are directed in the question to use a particular method.)

5. The following abbreviations may be used in a mark scheme.

   AEF Any Equivalent Form (of answer or result is equally acceptable).

   AG Answer Given on the question paper (so extra care is needed in checking that the detailed working leading to the result is valid).

   BOD Benefit Of Doubt (allowed for work whose validity may not be absolutely plain).

   CAO Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed).

   ISW Ignore Subsequent Working.

   MR Misread.

   PA Premature Approximation (resulting in basically correct work that is numerically insufficiently accurate).

   SOS See Other Solution (the candidate makes a better attempt at the same question).

   SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance).

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