Cambridge Pre-U

Cambridge International Examinations
Cambridge Pre-U Certificate Principal Subject

MATHEMATICS **9794/01**

Paper 1 Pure Mathematics 1 **May/June 2014**

2 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF20)

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READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **4** printed pages.

1 **(i)** Express $x^2 - 8x + 10$ in the form $(x - a)^2 + b$ where a and b are integers to be found. [3]

(ii) Hence write down the minimum value of $x^2 - 8x + 10$ and the corresponding value of x. [2]

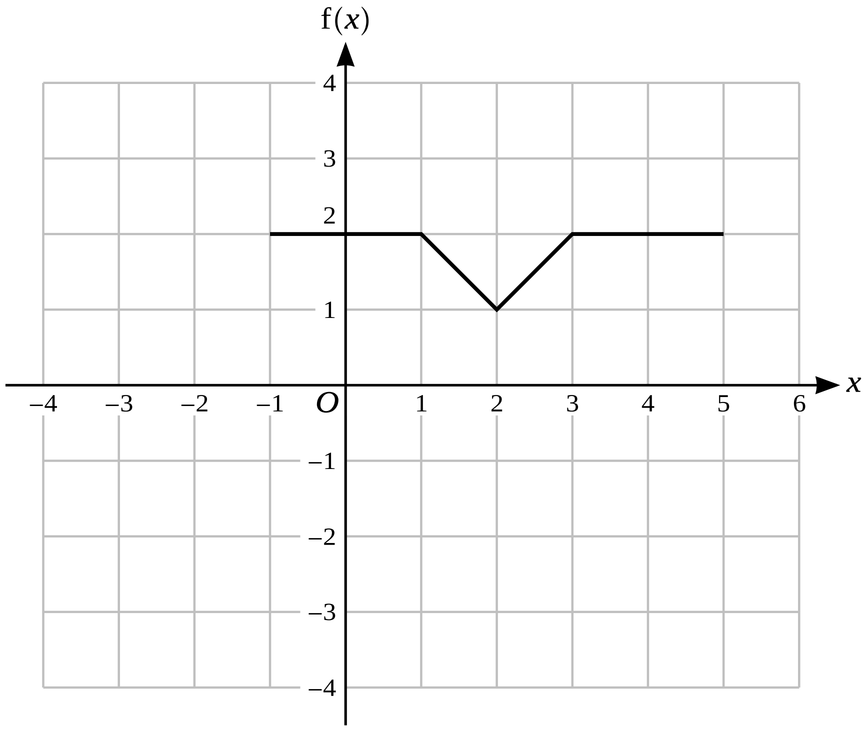
2 Sketch the curve with equation $y = \tan x$ for $-\frac{1}{2}\pi < x < \frac{1}{2}\pi$.

On the same diagram, sketch the curve with equation $y = \tan^{-1} x$ for all x.

State the geometrical relationship between the curves. [3]

3 Solve the inequality $|2x - 1| < 3$. [3]

4 The graph of $f(x)$ is shown below.



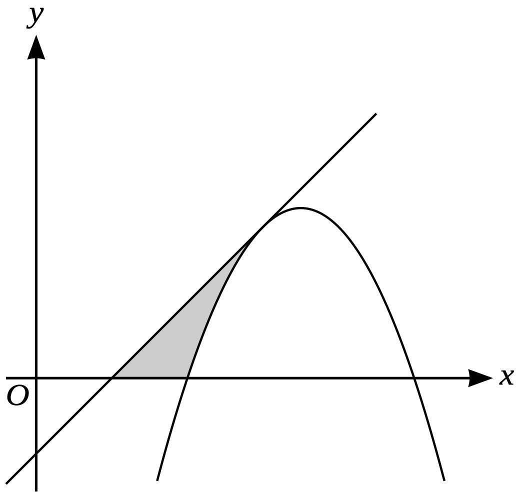
Draw the graphs of

(i) $f(x + 2) + 1$, [2]

(ii) $-\frac{1}{2}f(x)$. [2]

5 A root of the equation $z^2 + pz + q = 0$ is $3 + i$, where p and q are real. Write down the other root of the equation and hence calculate the values of p and q. [4]

6 The diagram shows the curve with equation $y = 7x - 10 - x^2$ and the tangent to the curve at the point where $x = 3$.



(i) Show that the curve crosses the x-axis at $x = 2$. [1]

(ii) Find $\frac{dy}{dx}$ and hence find the equation of the tangent to the curve at $x = 3$.

Show that the tangent crosses the x-axis at $x = 1$. [5]

(iii) Evaluate $\int_2^3 (7x - 10 - x^2)\,dx$ and hence find the exact area of the shaded region bounded by the curve, the tangent and the x-axis. [7]

7 Taking $x = 2$ as a first approximation, use the Newton-Raphson process to find a root of the equation $\frac{1}{x^2} - 0.119 - 0.018x = 0$. Give your answer correct to 3 significant figures. [4]

8 The parametric equations of a curve are given by

$$x = e^t - 2t, \quad y = e^t - 5t.$$

(i) Find $\frac{dy}{dx}$ in terms of t. [2]

(ii) Show that $t = -\ln 2$ at the point on the curve where the gradient is 3. [4]

9 It is given that x, 6 and $x + 5$ are consecutive terms of a geometric progression.

(i) Show that $x^2 + 5x - 36 = 0$ and find the possible values of x. [3]

(ii) Hence find the possible values of the common ratio. [2]

Furthermore, x, 6 and $x + 5$ are the second, third and fourth terms of a geometric progression for which the sum to infinity exists.

(iii) Find the first term and the sum to infinity. [4]

10 **(a)** Show that $\displaystyle\int_0^2 \frac{x}{x^2+5}\,\mathrm{d}x = \ln\left(\frac{3}{\sqrt{5}}\right)$. [4]

(b) Find $\displaystyle\int x\sqrt{x-2}\,\mathrm{d}x$. [4]

11 A differential equation is given by $2\dfrac{\mathrm{d}y}{\mathrm{d}x} = y(1-y)$.

(i) Express $\dfrac{2}{y(1-y)}$ in partial fractions. [3]

(ii) Hence show by integration that $\dfrac{y^2}{(1-y)^2} = A\mathrm{e}^x$. [5]

(iii) Given that $x = 0$ when $y = 2$, find the value of A and express y in terms of x. [3]

12 **(i)** Use the identity $\tan 2x \equiv \dfrac{2\tan x}{1-\tan^2 x}$ to show that $\tan 4x \equiv \dfrac{4(1-\tan^2 x)\tan x}{1-6\tan^2 x+\tan^4 x}$. [6]

(ii) Hence, given that $x = \frac{1}{16}\pi$ is a root of the equation $\tan^4 x + p\tan^3 x - 6\tan^2 x - p\tan x + 1 = 0$ where p is a positive constant, find the value of p. [4]