CAMBRIDGE INTERNATIONAL EXAMINATIONS

Pre-U Certificate

MARK SCHEME for the May/June 2014 series

9794 MATHEMATICS

9794/02 Paper 2 (Pure Mathematics 2), maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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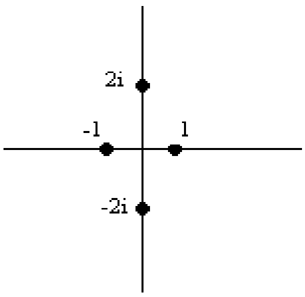
CAMBRIDGE
International Examinations

1 **(i)**	$BC^2 = 10^2 + 7^2 - 2 \times 10 \times 7 \times \cos(100)$ $BC = 13.164\ldots = 13.2$ to 3 sf	M1 A1	 [2]	Must be correct formula attempted
(ii)	Area $= 0.5 \times 10 \times 7 \times \sin(100)$ $= 34.468\ldots = 34.5$ to 3 sf	M1 A1	 [2]	Must be correct formula attempted Allow equiv methods as long as valid use of trig throughout
2 **(i)**	$\Delta = b^2 - 4ac$ $= k^2 - 16$	M1 A1	 [2]	 Simplify to this
(ii)	$k^2 - 16 . 0$ $k . 4$ $k , -4$	M1 A1 A1	 [3]	Must be > seen, or implied by answer Allow incorrect answer from **(i)**, as long as $b^2 - 4ac$ attempted A1A0 for $-4 > k > 4$ or $k > \pm 4$ Allow BOD on 'and' not 'or' $\|k\| > 4$ gets A1A1 Attempting to solve $f'(x) > 0$ can get M1A1A1 as above
3	$(x + h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$ seen anywhere If $f(x) = x^3$, $f(x + h) = (x + h)^3$ $\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^3 - x^3}{h}$ $= 3x^2 + 3xh + h^2$ then $f'(x) = \lim_{h \to 0}(3x^2 + 3xh + h^2) = 3x^2$	B1 M1 M1 A1	 [4]	Or unsimplified equiv Could expand $(x - h)^3$ instead Just recognise that $f(x + h) = (x + h)^3$, or $f(x - h) = (x - h)^3$ Attempt correct process, including division by h Allow $h = 0$ for $h \to 0$ Allow $f'(x) \to 3x^2$ Need to see $f'(x)$ or $\frac{dy}{dx}$ within proof

Question		Answer	Marks		Guidance
4	(i)	$\overrightarrow{AB} = \begin{pmatrix}1\\3\\5\end{pmatrix}$ $\overrightarrow{CB} = \begin{pmatrix}-1\\0\\2\end{pmatrix}$	B1		Any two relevant vectors Allow vectors of inconsistent directions e.g. *AB* and *BC*
		$(\pm)9 = \sqrt{35}\sqrt{5}\cos A\hat{B}C$	M1		Attempt scalar product – allow inconsistent directions
		$\cos A\hat{B}C = \dfrac{9}{\sqrt{35}\sqrt{5}}$	A1		Correct expression involving cos *ABC* – not necessarily with cos *ABC* as the subject
		so $A\hat{B}C = 47.13... = 47.1°$ to 1 dp	A1	[4]	CWO
					Using cosine rule
			B1		Three correct vectors soi
			M1		Attempt correct cosine rule
			A1		Correct expression involving cos *ABC* CWO so A0 if correct surd from incorrect vector
			A1		Obtain 47.1°
	(ii)	$k\overrightarrow{AB} = k\begin{pmatrix}1\\3\\5\end{pmatrix} = \overrightarrow{CD} = \begin{pmatrix}-5\\a-2\\b-3\end{pmatrix}$	M1		Attempt to find at least one of *a* and *b*, by considering at least two components of parallel vectors, including attempt at *k*
		$3 \times -5 = a - 2$ so $a = -13$	A1		
		$5 \times -5 = b - 3$ so $b = -22$	A1	[3]	
5	(i)	68	B1	[1]	
	(ii)	$S_{15} = 7.5 \times (2 \times 5 + 14 \times 7)$ $= 810$	M1 A1	 [2]	Attempting to use correct formula
	(iii)	New series with $a = 11$ and $d = 14$	M1		Either identified explicitly, used in formula or just listing new terms (could be *a* & *l*)
		$S_{15} = 7.5 \times (2 \times 11 + 14 \times 14)$ $= 1635$	M1 A1	 [3]	
		OR	**OR**		
		$\sum_1^{15} 2x_n + 1 = 2\sum_1^{15} x_n + 15$	M1		Allow M1M0 for $\left(2\sum_1^{15} x_n\right) + 1$
		$= 2 \times 810 + 15$ $= 1635$	M1 A1		

6	$\sin\theta = \dfrac{\sqrt{7}}{4}$	M1		Attempt to find numerical value of $\sin\theta$ – from right angled triangle or identities Must be correct triangle/identity
	$\sin 2\theta = 2\sin\theta\cos\theta$	M1		Use $\sin 2\theta = 2\sin\theta\cos\theta$ with numerical values M0 if using numerical value for θ not $\sin\theta$ M0M1 is possible (e.g. assuming 3, 4, 5 Δ)
	$= 2\times\dfrac{\sqrt{7}}{4}\times\dfrac{3}{4} = \dfrac{3\sqrt{7}}{8}$	A1		Obtain correct surd aef (must be single fraction)
	$\cot\theta = \dfrac{\cos\theta}{\sin\theta} = \dfrac{3/4}{\sqrt{7}/4} = \dfrac{3}{\sqrt{7}}$	M1		Attempt to find $\cot\theta$, using numerical values M0 if using numerical value for θ not $\tan\theta$ Could follow first M0
		A1	[5]	Obtain correct surd aef (must be single fraction)
7 (i)	$(z^2+4)(z^2-1)$	B2	[2]	Stating $a = 4$, $b = -1$ (or reverse) gets B2 Allow B1 for $(z^2-4)(z^2+1)$ Allow B1 (BOD) for $(z+4)(z-1)$
(ii)		√B1		At least 2 correct points, following their a & b
		√B1	[2]	All 4 correct points, following their a & b as long as one positive and one negative NIS so B1B0 if locus drawn through points Allow just 2 on axis as long as $2i$ seen in solution, or axis is labelled as Im

8	$\frac{dy}{dx} = 2x - \frac{1}{x}$	M1 A1		Attempt integration – one correct term Fully correct
	solve to obtain $x = (\pm)\frac{1}{\sqrt{2}}$ only one stationary point at $\left(\frac{1}{\sqrt{2}}, \frac{1}{2} - \ln\frac{1}{\sqrt{2}}\right)$ AEF	M1 A1 A1		Equate to 0 and attempt to solve Obtain at least the positive root Obtain correct stationary point having selected the positive root only from $\pm\frac{1}{\sqrt{2}}$ Allow $x = ..., y = ...$ Exact final answer only, else A0
	as x , 0 cannot be valid due to ln	A1	[6]	Explanation of why there is only one root, referring to ln$x$ Only considering +ve solution will get max 4/6
9 (i)	Model 1: Attempt iteration $P_3 = 687$ Model 2 : Attempt iteration $P_3 = 927$	M1 A1 M1 A1	[4]	At least twice Allow decimal values, or 686 At least twice Allow decimal values, or 926
(ii)	Model 1	B1		Identify Model 1, with minimal explanation e.g. decreasing rate of increase
	converges to 693	B1	[2]	Identify that it converges to 693 oe (could justify that $P_t \approx P_{t-1}$)
(iii)	appears to settle down to periodic (values 926, 561, 980 and 429)	B1	[1]	State as periodic oe – need to see 4 values, or refer to period of 4 years

10	**(i)**	$f(1) = 1 - 4 - 10 + 28 - 15$ $= 0$ hence $x = 1$ is a root	M1 A1 [2]	Substitute $x = 1$ into the function, or equivalent process to find remainder Show clearly that it equals zero and conclude, with correct terminology
	(ii)	$f(x) = (x - 5)(x^3 + x^2 - 5x + 3) + 0$	M1 A1 A1 B1 [4]	Attempt complete division or factorisation Quotient correct at least as far as x^2 term Quotient $x^3 + x^2 - 5x + 3$ soi Remainder 0 (allow 'no remainder') soi
	(iii)	$f(x) = (x - 5)(x - 1)(x^2 + 2x - 3)$ $= (x - 5)(x - 1)^2 (x + 3)$	M1 A1 A1	Attempt to write f(x) as product of two linear factors and one quadratic Could go via $(x - 1)(x^3 - 3x^2 - 13x + 15)$ Obtain correct linear and quadratic factors soi Obtain fully correct factorisation
		Sketch showing: a positive quartic Intersecting with the x-axis at –3 and 5 and maximum on the x-axis at 1	M1 A1 [5]	Positive quartic, with 3 turning points Allow $y \le 0$ only x coords indicated, or implied by scale No need to see –15 on y-axis Allow minimum at (0, –15) Need $y > 0$ as well, possibly with one arm truncated

11	**(i)**	$F_1'(x) = x - \frac{3}{4}x^2$	B1	
		$F_2'(x) = \frac{1}{3}\left(2x^2 - 4x + 7\right)^{-\frac{2}{3}}\left(4x - 4\right)$	M1 A1	Attempt chain rule Must include any necessary brackets
		$F_3'(x) = \frac{-4\left(x^2 - 2x\right) - \left(7 - 4x\right)\left(2x - 2\right)}{\left(x^2 - 2x\right)^2}$	M1 A1 [5]	Attempt quotient rule (allow sign muddles in numerator) Could also differentiate partial fractions No need to simplify (isw if done incorrectly)
	(ii)	$F_1'(1.9) = -0.8075$ $F_2'(1.9) = 0.340\ldots$ $F_3'(1.9) = 50.9\ldots$	M1 B1*	Attempt $F'(1.9)$ for all three functions State correct condition for convergence – could be for acceptance or rejection, but must have modulus sign oe
		F_1 and F_2 converge	A1d*	Identify F_1 and F_2 Need $F'(1.9)$ correct for all 3 functions
		Faster convergence is F_2 because the magnitude of the gradient is smallest near the root. WWW	A1 [4]	Must have magnitude soi, not just 'gradient smaller' Accept gradient closer to 0 Need $F'(1.9)$ correct for all 3 functions M1B0A0A1 is possible
	(iii)	$\lvert e_{r+1}\rvert = 0.34^r \lvert e_1\rvert$	M1	Attempt to apply general statement to this question e.g. $e_2 = 0.34 \times e_1$
		$10^{-10}\lvert e_1\rvert > 0.34^r \lvert e_1\rvert$	M1	Attempt to solve $10^{-10}\lvert e_1\rvert = \mathrm{F}`(1.9)^r \lvert e_1\rvert$ Allow index of $r$ or $r - 1$ Could use a more accurate value for $\alpha$ Allow any numerical value for $e_1$, inc 1.9 Allow any $F`(1.9)$ as long as $\lvert F`(1.9)\rvert < 1$
		so $r > \frac{-10}{\log 0.34} > 21.34$, so 22 iterations.	A1 [3]	Obtain 21 / 22 / 23 depending on index and method used If numerical e_1 used, it must have been correct No credit for answer only

12	**(i)**	$\cos t = 0$ or $\sin t = \frac{1}{2}$	M1		Attempt to solve at least one of these
		$\cos t = 0 \Rightarrow t = \frac{1}{2}\pi, \frac{3}{2}\pi$	A1		Obtain both values for t
		y = –2 and –4 respectively	A1		Obtain both values for y SR A1 for one correct t, y pair
		$\sin t = \frac{1}{2} \Rightarrow t = \frac{1}{6}\pi, \frac{5}{6}\pi$	A1		Obtain both values for t
		$y = -\frac{1}{4}$ for both values of t so there is only one point on the y-axis associated with both.	A1	[5]	Obtain $y = \frac{-1}{4}$ for both, and comment that same point – allow just listing $(0, \frac{-1}{4})$ once SR A1 for one correct t, y pair max of 4/5 if working in degrees
	(ii)	$\sin t < 0$ AND $\sin t > \frac{1}{3}$, but this is not possible	B1		
		Identify that $\sin t > 0$ AND $\sin t < \frac{1}{3}$	M1		If equating to 0 and solving then both inequalities must be used/implied later to get M1 Allow ≥ for >
		so $t \in \left(0, \sin^{-1}\frac{1}{3}\right) \cup \left(\pi - \sin^{-1}\frac{1}{3}, \pi\right)$ oe	A1		Obtain at least $0 < t < \sin^{-1}(\frac{1}{3})$ Allow $0 < t < 0.34$ Allow ≥ for >
			A1	[4]	Allow $0 < t < 0.34$, $2.80 < t < 3.14$ working in degrees can get M1A1 only