UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
Cambridge International Level 3 Pre-U Certificate
Principal Subject

MATHEMATICS **9794/02**

Paper 2 Pure Mathematics 2 **May/June 2013**

2 hours

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **4** printed pages.

UNIVERSITY of CAMBRIDGE
International Examinations

[Turn over

1 Vectors **u** and **v** are given by $\mathbf{u} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$.

    (i) Find $\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - \mathbf{v}$. [2]

    (ii) Show that $|\mathbf{u} + \mathbf{v}| = |\mathbf{u} - \mathbf{v}|$. [2]

2   **(i)** An arithmetic sequence has first term 3 and common difference 2. Find the twenty-first term of this sequence. [2]

    (ii) Find the sum to infinity of a geometric progression with first term 162 and second term 54. [3]

    (iii) A sequence is given by the recurrence relation $u_1 = 3$, $u_{n+1} = 2 - u_n$, $n = 1, 2, 3, \ldots$. Find u_2, u_3, u_4, u_5 and describe the behaviour of this sequence. [2]

3   **(i)** Express $x^2 + 2x - 3$ in the form $(x + a)^2 + b$, where a and b are integers to be found. [2]

    (ii) Sketch the graph of $y = x^2 + 2x - 3$ giving the coordinates of the vertex and of any intersections with the coordinate axes. [5]

4   **(i)** Verify that $z = -1$ is a root of the equation $z^3 + 5z^2 + 9z + 5 = 0$. [1]

    (ii) Find the two complex roots of the equation $z^3 + 5z^2 + 9z + 5 = 0$. [4]

    (iii) Show all three roots on an Argand diagram. [1]

5 The curve C has equation $x^2 + xy + y^2 = 19$.

    (i) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{-2x - y}{x + 2y}$. [4]

    (ii) Hence find the equation of the normal to C at the point $(2, 3)$ in the form $ax + by + c = 0$, where a, b and c are integers. [4]

6 The table below gives the population of breeding pairs of red kites in Yorkshire from 2001 to 2008.

Year	2001	2002	2003	2004	2005	2006	2007	2008
Number of breeding pairs	8	10	16	24	33	40	47	69

Source: www.gigrin.co.uk

The following model for the population has been proposed:

$$N = a \times b^t,$$

where N is the number of breeding pairs t years after the year 2000, and a and b are constants.

  (i) Show that the model can be transformed to a linear relationship between $\log_{10} N$ and t.   [2]

  (ii) On graph paper, plot $\log_{10} N$ against t and draw by eye a line of best fit. Use your line to estimate the values of a and b in the equation for N in terms of t.   [6]

  (iii) What values of N does the model give for the years 2008 and 2020?   [2]

  (iv) In which year will the number of breeding pairs first exceed 500 according to the model?   [3]

  (v) Comment on the suitability of the model to predict the population of breeding pairs of red kites in Yorkshire.   [1]

7 It is given that $y = x^2 e^{-x}$.

  (i) Show that $\dfrac{dy}{dx} = xe^{-x}(2 - x)$.   [4]

  (ii) Hence find the exact coordinates of the stationary points on the curve $y = x^2 e^{-x}$.   [3]

8 Evaluate the following, giving your answers in exact form.

  (i) $\displaystyle\sum_{n=1}^{30} \frac{1}{n} - \sum_{n=2}^{29} \frac{1}{n}$.   [2]

  (ii) $\displaystyle\sum_{n=1}^{100} n \times (-1)^n$.   [2]

9   **(i)** Prove that $\operatorname{cosec} 2x - \cot 2x \equiv \tan x$ and hence find an exact value for $\tan\left(\frac{3}{8}\pi\right)$.   [6]

  (ii) Find the exact value of $\displaystyle\int_{\frac{1}{4}\pi}^{\frac{3}{8}\pi} (\operatorname{cosec} 2x - \cot 2x)^2 \, dx$.   [6]

[Question 10 is printed on the next page.]

10 A tank with vertical sides and rectangular cross-section is initially full of water. The water is leaking out of a hole in the base of the tank at a rate which is proportional to the square root of the depth of the water. $V\,\text{m}^3$ is the volume of water in the tank at time t hours.

(i) Show that $\dfrac{\mathrm{d}V}{\mathrm{d}t} = a\sqrt{V}$, where a is a constant. [2]

(ii) Given that the tank is half full after one hour, show that $V = V_0\left(\left(\dfrac{1}{\sqrt{2}} - 1\right)t + 1\right)^2$, where $V_0\,\text{m}^3$ is the initial volume of water in the tank. [7]

(iii) Hence show that the tank will be empty after approximately 3 hours and 25 minutes. [2]