UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
Cambridge International Level 3 Pre-U Certificate
Principal Subject

MATHEMATICS **9794/02**

Paper 2 Pure Mathematics 2 **May/June 2012**

2 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF20)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **4** printed pages.

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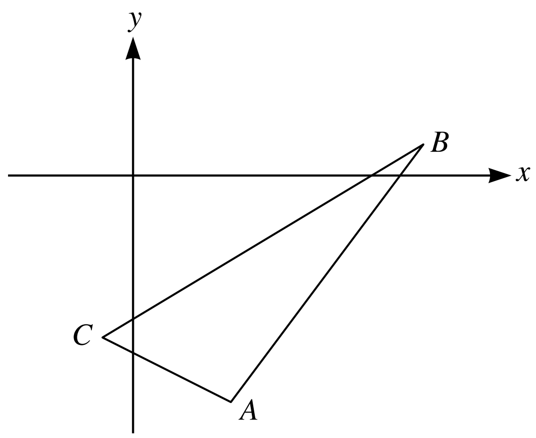
1 **(i)** Solve the equation $x^2 - 8x + 4 = 0$, giving your answer in the form $p \pm q\sqrt{3}$, where p and q are integers. [2]

(ii) Expand and simplify $\left(6 + 2\sqrt{3}\right)\left(2 - \sqrt{3}\right)$. [3]

2



The diagram shows a triangle ABC. The vertices have coordinates A (3, −7), B (9, 1) and C (−1, −5).

(i) **(a)** Find the length of the side AB. [2]

(b) Find the coordinates of the mid-point of AB. [1]

(c) A circle has diameter AB. Find the equation of the circle in the form $(x - a)^2 + (y - b)^2 = r^2$, where a, b and r are constants to be found. [3]

(ii) Find the equation of the line l passing through B parallel to AC. [3]

3 Find the exact value of $\displaystyle\int_0^1 (e^x - x)\,dx$. [4]

4 Use logarithms to solve the equation $2^{2x-1} = 5$. [4]

5 Sketch, on separate diagrams, the graphs of the following functions for $0 \leqslant x \leqslant 2\pi$ giving the coordinates of all points of intersection with the axes.

(i) $y = \sin x$. [1]

(ii) $y = \sin\left(x + \frac{1}{6}\pi\right)$. [2]

6 **(i)** An arithmetic sequence has first term 5 and fifth term 37.

(a) Find an expression for u_n, the nth term of the sequence, in terms of n. [4]

(b) Find an expression for S_n, the sum of the first n terms of this sequence, in terms of n. [2]

(ii) Hence, or otherwise, calculate $\displaystyle\sum_{n=5}^{25}(8n - 3)$. [2]

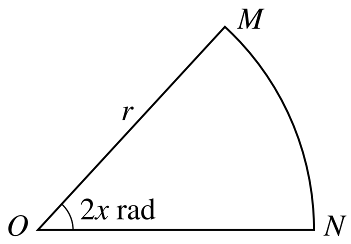
7 Let $y = (2x - 3)e^{-2x}$.

(i) Find $\frac{dy}{dx}$, giving your answer in the form $e^{-2x}(ax + b)$, where a and b are integers. [3]

(ii) Determine the set of values of x for which y is increasing. [2]

8 Solve the differential equation $\frac{dy}{dx} = -y^2x^3$, where $y = 2$ when $x = 1$, expressing your solution in the form $y = f(x)$. [6]

9



The diagram shows a sector of a circle, OMN. The angle MON is $2x$ radians, the radius of the circle is r and O is the centre.

(i) Find expressions, in terms of r and x, for the area, A, and perimeter, P, of the sector. [2]

(ii) Given that $P = 20$, show that $A = \frac{100x}{(1+x)^2}$. [2]

(iii) Find $\frac{dA}{dx}$, and hence find the value of x for which the area of the sector is a maximum. [5]

10 **(i)** **(a)** Find $\int \frac{e^x}{1 + e^x}\, dx$. [2]

(b) Hence evaluate $\int_0^{\ln 3} \frac{e^x}{1 + e^x}\, dx$, giving your answer in the form $\ln k$, where k is an integer. [3]

(ii) **(a)** Using the substitution $u = 1 + e^x$, find $\int \left(\frac{e^x}{1 + e^x}\right)^2 dx$. [5]

(b) Hence find the exact volume of the solid of revolution generated when the curve given by $y = \frac{e^x}{1 + e^x}$, between $x = -\ln 3$ and $x = \ln 3$, is rotated through 2π radians about the x-axis. [2]

[Question 11 is printed on the next page.]

11 The function f is defined by $\mathrm{f} : t \mapsto 2\sin t + \cos 2t$ for $0 \leqslant t < 2\pi$.

(i) Show that $\dfrac{\mathrm{d}\mathrm{f}}{\mathrm{d}t} = 2\cos t(1 - 2\sin t)$. [2]

(ii) Determine the range of f. [5]

A curve C is given parametrically by $x = 2\cos t + \sin 2t$, $y = \mathrm{f}(t)$ for $0 \leqslant t < 2\pi$.

(iii) Show that $x^2 + y^2 = 5 + 4\sin 3t$. [3]

(iv) Deduce that C lies between two circles centred at the origin, and touches both. [2]

(v) Find the gradient of the tangent to C at the point at which $t = 0$. [3]