UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
Cambridge International Level 3 Pre-U Certificate
Principal Subject

MATHEMATICS **9794/01**

Paper 1 Pure Mathematics 1 **May/June 2012**

2 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF20)

1404245451

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **3** printed pages and **1** blank page.

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[Turn over

1 The first term of a geometric progression is 16 and the common ratio is 0.8.

(i) Calculate the sum of the first 12 terms. [3]

(ii) Find the sum to infinity. [2]

2 Let $f(x) = x^3 - 3x^2 - 13x + 15$.

(i) Show that $f(1) = 0$ and hence factorise $x^3 - 3x^2 - 13x + 15$ completely. [4]

(ii) Hence solve the equation $x^3 - 3x^2 - 13x + 15 = 0$. [2]

3 The equation of a curve is $y = x^3 + x^2 - x + 3$.

(i) Find $\frac{dy}{dx}$. [2]

(ii) Hence find the coordinates of the stationary points on the curve. [4]

4 (i) Show that the equation $x^3 - 6x + 2 = 0$ has a root between $x = 0$ and $x = 1$. [2]

(ii) Use the iterative formula $x_{n+1} = \frac{2 + x_n^3}{6}$ with $x_0 = 0.5$ to find this root correct to 4 decimal places, showing the result of each iteration. [3]

5 Let $f(x) = x^2$ and $g(x) = 7x - 2$ for all real values of x.

(i) Give a reason why f has no inverse function. [1]

(ii) Write down an expression for $gf(x)$. [2]

(iii) Find $g^{-1}(x)$. [2]

6 The roots of the equation $z^2 - 6z + 10 = 0$ are z_1 and z_2, where $z_1 = 3 + i$.

(i) Write down the value of z_2. [1]

(ii) Show z_1 and z_2 on an Argand diagram. [2]

(iii) Show that $z_1^2 = 8 + 6i$. [2]

7 (i) Show that the first three terms in the expansion of $(1 - 2x)^{\frac{1}{2}}$ are $1 - x - \frac{1}{2}x^2$ and find the next term. [4]

(ii) State the range of values of x for which this expansion is valid. [1]

(iii) Hence show that the first four terms in the expansion of $(2 + x)(1 - 2x)^{\frac{1}{2}}$ are $2 - x + ax^2 + bx^3$ and state the values of a and b. [4]

8 **(i)** Given that $\dfrac{2x+11}{(2x+1)(x+3)} \equiv \dfrac{A}{2x+1} + \dfrac{B}{x+3}$, find the values of the constants A and B. [4]

(ii) Hence show that $\displaystyle\int_0^2 \frac{2x+11}{(2x+1)(x+3)}\,dx = \ln 15$. [5]

9 Three points A, B and C have coordinates (1, 0, 7), (13, 9, 1) and (2, −1, −7) respectively.

(i) Use a scalar product to find angle ACB. [5]

(ii) Hence find the area of triangle ACB. [2]

(iii) Show that a vector equation of the line AB is given by $\mathbf{r} = \mathbf{i} + 7\mathbf{k} + \lambda(4\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})$, where λ is a scalar parameter. [3]

10 **(i)** Prove that

$$\sin^2 2\theta(\cot^2\theta - \tan^2\theta) = 4(\cos^4\theta - \sin^4\theta)$$

and hence show that

$$\sin^2 2\theta(\cot^2\theta - \tan^2\theta) = 4\cos 2\theta.$$ [5]

(ii) Hence solve the equation $\sin^2 2\theta(\cot^2\theta - \tan^2\theta) = 2$ for $0° \leqslant \theta < 360°$. [4]

11 **(i)** Use integration by parts to show that $\displaystyle\int \ln x\,dx = x\ln x - x + c$. [2]

(ii) Find

(a) $\displaystyle\int (\ln x)^2\,dx$, [4]

(b) $\displaystyle\int \frac{\ln(\ln x)}{x}\,dx$. [5]

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