MATHEMATICS

Paper 9794/01
Pure Mathematics 1

Key message

In order to succeed in this paper, candidates need to have an understanding of the complete Pure Mathematics syllabus. They also need to ensure that they can give explanations clearly using mathematical language.

General comments

This paper enabled weaker candidates to demonstrate what they could do and the better candidates to produce some very fine, detailed and accurate solutions. The standard of presentation was generally of an excellent standard. This year, candidates appeared to have greater confidence in dealing with the linear nature of the specification and consequently were more at ease in showing clearly the detailed steps in their solutions. This may have been due to the change in format of the question papers. The majority of the candidates were able to achieve most of the marks in the first few questions and there was no evidence that candidates had insufficient time to complete the paper.

Comments on specific questions

Question 1

This proved a straightforward question with the only errors being due to arithmetic or algebraic slips. Candidates showed themselves to be familiar with the formulae or how to obtain them and showed clearly the values they were substituting into the formulae.

Answers: **(i)** 74.5 **(ii)** 80.

Question 2

This again proved to be a question in which candidates showed a large degree of confidence. All were able to show that f(1) = 0. Examiners have previously made reference to the need to show all steps in the working when a result is given on the paper and candidates put this into practice and most received full marks. Subsequent attempts to find the quadratic factor mostly used division and nearly all candidates did so accurately, although a few lost the final mark for not then factorising the cubic completely as requested in part **(i)**.

Answers: **(i)** $(x-1)(x+3)(x-5)$ **(ii)** −3, 1, 5.

Question 3

This question was again done extremely well. The derivative was correct in all cases, and this was then equated to 0 and solved using a variety of methods. The only errors seen were the odd slip when finding the *y*-coordinates, or omitting to attempt them.

Answers: **(i)** $3x^2+2x-1$ **(ii)** (−1, 4) $\left(\frac{1}{3}, \frac{76}{27}\right)$

Question 4

The majority of candidates understood clearly what was being asked in this question and attempted the relevant values. Explanations varied however. At their best, reference was made to the continuity of the function while at the other extreme, short and sometimes very unclear phrases were unable to receive credit. Candidates should always be encouraged to write in sentences using correct mathematical terminology. A graph drawn with the values clearly shown was accepted as sufficient explanation but attempting to find a root by the Newton-Raphson process was not.

The iteration itself was usually done correctly, and the majority of candidates showed the results of each iteration in sufficient detail. Some candidates however, failed to gain the final mark as they did not distinguish between an iterate and a statement of the value of the root, and a few others rounded incorrectly.

Answer: **(ii)** 0.3399

Question 5

Many candidates could give a clear and concise explanation of why there was no inverse. The majority stated that it was many-one, or that it was not one-one, and others used specific examples. Others focused on the fact that the inverse would have to be one-many and thus not satisfy the definition of a function. A few candidates conveyed this idea well, but many lacked clarity. For example, brief statements that x had more than one value did not make it explicit whether the function or its inverse was being considered. When reference was made to the impossibility of square roots being obtained for negative numbers or that x could not be made the subject, candidates may have had the correct idea but were unable to express this in a sufficiently coherent form to receive any credit. It is essential that candidates can express themselves clearly, using both words and mathematical symbols. In the remainder of the question, finding gf(x) or $g^{-1}(x)$ posed no problems for the vast majority.

Answers: **(i)** many-one **(ii)** $7x^2 - 2$ **(iii)** $\frac{1}{7}(x+2)$

Question 6

Most candidates could correctly write down the value of z_2, although a few felt the need to find it by solving the given quadratic. On the whole, Argand diagrams were drawn correctly, including labelled axes and some indication of scale. However, there was evidence of insecure knowledge or carelessness in that a few did not label either the point or mark a scale. Scales were seen in which points were marked i, 2i etc. but were called x and y and a surprising minority transposed the Real and Imaginary axes. Axes which were clearly Cartesian, however, were not accepted.

Finding z_1^2 was usually done correctly, although a number of candidates failed to gain the second mark as they did not show that i^2 was –1 and went straight from their expansion to the given answer.

Answer: **(i)** 3 – i

Question 7

Candidates were helped by being given the first three terms of the expansion and the working shown to arrive at these was usually detailed enough to be convincing. Most candidates could then make a reasonable attempt at the fourth term. However, there were instances where omission of brackets led to sign or coefficient errors. In some cases, however, the correct coefficient and sign was given as the final answer and it was often unclear whether this was a genuine recovery or a correct result following incorrect working. Candidates must ensure that they are meticulous when including essential brackets in algebraic work.

As seems often to be the case, many candidates did not appear to be familiar with the condition for such expansions to be valid. Many suggested $x < 0.5$ or $x \le 0.5$. A few quoted $|x| < 1$, and others stated $|2x| < 1$ but then failed to progress further. However, a disappointing number started with an incorrect initial statement, often $1 - 2x < 0$.

Most candidates attempted the product required in the last part but many did not extend this to include enough terms to obtain the value of *b*. They therefore obtained the $2 - x$ correctly and found the value of *a*, but failed to gain any further credit.

Answers: **(i)** $-0.5x^3$ **(ii)** $|x| < 0.5$ **(iii)** $a = -2$, $b = -1.5$

Question 8

The first part of this question was done very well, with most candidates gaining full marks. In the second part of the question, candidates successfully showed that the fractions integrated to natural logarithms and usually got the second term correct. The first term was more problematic however. Candidates should be encouraged to achieve an easy facility in dealing correctly with coefficients in these situations. This would have prevented them using $4\ln(2x + 1)$ in subsequent work. Limits were almost invariably used correctly, and most then attempted to use log laws to achieve the given result. Examiners did note, however, that the steps involved were not always shown as explicitly as might have been desirable and there was also evidence on a number of scripts that the given answer allowed some candidates to go back and correctly amend their work. This was allowed if the work was corrected consistently throughout their solution.

Answer: **(i)** $A = 4$, $B = -1$

Question 9

The last questions on the paper are designed to be quite challenging and in the main, this was found to be true. Many candidates find it difficult to apply their knowledge of vectors to specific situations and thus it was clear that many candidates had learnt the formula for the scalar product as $\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$ and unfortunately used these as their vectors in answering the question. These candidates were able to be rewarded for finding the length of one of their vectors. Candidates should also be clear that the scalar product requires the arrow heads of both vectors either to point into the angle or away from it. Those who were not consistent in this practice found the obtuse angle instead of the angle required. They were not penalized in finding the area of the triangle however, as the obtuse angle also gave the same area.

Most candidates were fully confident in finding the area by using the expected method and very few tried to form altitudes to use the base and height formula.

As in a previous question, the final part of the question showed that candidates should try to develop facility in explaining their mathematics. Almost all were familiar with the vector equation of a straight line and most formed the appropriate direction vector but were very unsure how to explain its reduction to the form given in the question. Statements such as $\lambda = 3$ did not speak of confidence in this respect and many lost marks by omitting the general position vector **r** in the equation or leaving their calculation in column form.

Answer: **(i)** 62.1° **(ii)** 105 units^2

Question 10

A large number of candidates saw that the ratios of $\cot\theta$ and $\tan\theta$ should be turned into ratios involving $\sin\theta$ and $\cos\theta$ and were able to use the correct double angle formula to show the first part of the proof convincingly. The second part proved unexpectedly difficult however to most candidates. Factorising $\cos^4\theta - \sin^4\theta$ as $(\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)$ was carried out infrequently and, even if this were seen, the $\cos^2\theta - \sin^2\theta$ was often then subject to more modification to achieve another form of the double angle identity before the required result was exhibited. Rather more usual was to see $\cos^4\theta - \sin^4\theta$ written as $\cos^2\theta\cos^2\theta - \sin^2\theta\sin^2\theta$ and eventually achieving the required result by using products of $(1 - \sin^2\theta)$ and applying various double angle formulae. A few candidates started the initial proof by using identities involving $\sec^2\theta$ and $\mathrm{cosec}^2\theta$, a procedure which regrettably bypassed the intermediate result requested. Solving the given equation in part **(ii)** was usually done well, although a number of candidates could not find all four solutions. Some did not consider solutions to $\cos 2\theta$ beyond 360° and others who went via a $\cos^2\theta$ route did not consider the negative square root.

Answer: **(ii)** 30°, 150°, 210°, 330°

Question 11

Most candidates could handle the first part of the question very well, but various approaches were seen in the remaining parts, some of which were concise and accurate and others rather longer. There is no doubt that candidates regarded this question as challenging and indeed, it was intended to be so. In part **(ii)(a)** the most common approaches were using integration by parts on ln x and ln x, integration by parts on 1 and $(\ln x)^2$ and using a substitution $u^2 = (\ln x)^2$ before a double integration by parts. It was a pity that some very good solutions of all these types were often spoilt by careless use of brackets shown by solutions including a – 2x instead of a + 2x in the final answer. In part **(b)** the most successful and concise method was to use a substitution method putting $u = \ln x$ but equally impressive attempts were seen using integration by parts on ln(ln(x)) and $\frac{1}{x}$. The most creative method was probably $\ln x = e^u$, giving $x = \exp(\exp u)$).

Answers: **(ii)(a)** $x(\ln x)^2 - 2x\ln x + 2x + c$ **(b)** $(\ln x)(\ln(\ln x)) - \ln x + c$

MATHEMATICS

Paper 9794/02
Pure Mathematics 2

Key message

In order to succeed in this paper, candidates need to have a good knowledge of the content of the syllabus and an awareness of the importance of accurate algebraic manipulation and an ability to carry this out.

General comments

This paper allowed candidates of all abilities to demonstrate their mathematical knowledge. All questions appear to have been accessible, although parts of **Question 11** were only attempted by the strongest candidates. Many problems were caused by the candidates' inability to manipulate their algebra accurately.

The standard of presentation was good in general, although some candidates continue to present their work in a way that is difficult for Examiners to interpret.

Comments on specific questions

Question 1

These two requests were answered well by the majority of candidates. Sign errors were the most common cause of lost marks in both parts.

Answers: **(i)** $4 \pm 2\sqrt{3}$ **(ii)** $6 - 2\sqrt{3}$

Question 2

(i)(a)(b)(c) These basic techniques of coordinate geometry were understood by most candidates. Some gave their answer to part **(b)** as a vector rather than coordinates, which was condoned on this occasion.

(ii) This was well answered by most candidates. Some found the gradient of the perpendicular line to *AC* which could still obtain 2 of the 3 marks. A fair number of candidates chose to use vector notation which was, of course, acceptable in this case since no given form had been requested.

Answers: **(i)(a)** 10 **(b)** (6, –3) **(c)** $(x-6)^2 + (y+3)^2 = 25$ **(ii)** $y = -\frac{1}{2}x + \frac{11}{2}$

Question 3

This definite integral was evaluated correctly by the majority of candidates. Many of the weaker candidates gave an inexact answer rather than the exact value requested.

Answer: $\mathrm{e} - \frac{3}{2}$

Question 4

Many different approaches were seen to the solution of this equation, involving logarithms to base 10, 2 and e, and the correct answer was usually obtained. An exact answer was not required here, and working to a suitable degree of accuracy might have allowed some candidates to avoid some of their manipulation errors.

Answer: $x = \dfrac{1}{\log 4} = 1.66$

Question 5

When asked to sketch these graphs, many candidates chose to present their answers on graph paper. This is appropriate for a "plot", but not a "sketch". All that is required is a hand drawn graph in the standard answer booklet with correct graph shape and approximately consistent scales.

(i) Solutions to this part were excellent. Almost all candidates knew the shape of the graph and gave the intercepts with the x-axis correctly. Axis labels in degrees were condoned in this part only.

(ii) This part was much less well attempted. Most candidates appreciated that the required graph is a translation of that in part **(i)** in the negative x-direction, but they usually either omitted one of the intercepts, commonly the y-intercept, or failed to draw the curve over the whole domain.

Question 6

(i)(a)(b) Most candidates knew how to find the common difference in this case and could then substitute their values correctly into the given formulae.

(ii) In contrast, few candidates attempted this part satisfactorily. Some simply found either the 25$^{\text{th}}$ or 5$^{\text{th}}$ terms, or the difference between them. Even those who found the difference between two sums often found $S_{25} - S_5$. A few candidates tried the time consuming approach of finding all terms and summing which worked, although it must have taken a little while to write out.

Answers: **(i)(a)** $u_n = 8n - 3$ **(b)** $S_n = n(1 + 4n)$ **(ii)** 2457

Question 7

(i) Most candidates understood that the product rule was necessary here and could apply it accurately. However, many then went on to make errors in their manipulation into the required form.

(ii) The concept of an increasing function was not well understood and even candidates who understood that the differential needed to be non-negative seemed unsure of how to proceed. Many considered the value of x that made the gradient zero and decided which side to choose by substituting values rather than by considering an inequality. Strict inequalities were condoned.

Answers: **(i)** $(8 - 4x)\mathrm{e}^{-2x}$ **(ii)** $\dfrac{\mathrm{d}y}{\mathrm{d}x} \geq 0$ when $x \leq 2$

Question 8

This was generally well answered for 5 out of the 6 marks. The technique of separation of variables was well understood and generally well executed, but most candidates could not rearrange into explicit form without making errors, the most common being to find the reciprocal of two fractions separately instead of collectively.

Answer: $y = \dfrac{4}{x^4 + 1}$

Question 9

(i) Almost all candidates got the correct expressions for the perimeter and the area.

(ii) The method for this was well understood by many and the given answer achieved by most.

(iii) This part caused many candidates some difficulty. Most understood that the quotient rule was needed and could apply it, but it was very common to see $x = -1$ as one of the solutions because they did not simplify their expression sufficiently. Very few candidates thought to show that the stationary point they had found was a maximum. Those that did either used the second differential test and made errors in the long manipulations, or used the first differential test, but failed to give numerical values for the gradient on either side of their point.

Answers: **(i)** $P = 2r + 2rx$ **,** $A = r^2 x$ **(iii)** $\frac{dA}{dx} = \frac{100(1-x)}{(1+x)^3}$, $x = 1$

Question 10

(i) (a)(b) This simple integrand of the form $\frac{f'(x)}{f(x)}$ was spotted by most candidates and was followed up by correct use of limits and log rules by many.

(ii) (a) Candidates found this part of the question more difficult. Candidates who attempted the question could usually perform the substitution, but found it difficult to deal with the fraction that resulted, either cancelling incorrectly or using integration by parts and making errors.

(b) Regardless of their answer in part **(a)** most candidates knew how to find the volume of revolution, although it was relatively common to see a multiple of 2π rather than π.

Answers: **(i)(a)** $\ln(1+e^x) + c$ **(b)** ln 2 **(ii)(a)** $\ln(1+e^x) + \frac{1}{1+e^x} + c$ **(b)** $\pi(\ln 3 - \frac{1}{2})$

Question 11

With the exception of part **(i)**, most weaker candidates omitted this question.

(i) Most candidates made some attempt at this and, of those attempting it, all achieved the given answer although often with insufficient working. Many candidates chose to expand $\sin 2t$ with the double angle formula and use the product rule rather than the much simpler application of the chain rule.

(ii) Complete answers to this part were very rare, the most common omission being the values of f at the endpoints. Only the strongest candidates found all four of the turning points *and* the values of f at those points. Many went on to use the second differential test to confirm which were maxima and minima which is unnecessary as f is the sum of two bounded functions so is clearly bounded.

(iii) Some very long answers were seen here due to candidates expanding everything in terms of powers of $\sin x$ and $\cos x$. It was common, particularly among the weaker candidates attempting this part, for there to be no cross terms in the expansion of a squared bracket. Those that managed to multiply out correctly and spot the use of the addition formula produced some very neat and concise solutions.

(iv) Only the very best candidates managed to get any marks here. The key was to spot that the equation of the curve was similar to that of a circle, and it would therefore intersect any circle with a radius between the maximum and minimum of the right hand side. Candidates that made this connection usually found the relevant maximum and minimum and concluded well.

(v) This question seemed familiar to candidates and many correct answers were seen.

Answers: **(ii)** $-3 \le f(x) \le 1.5$ **(v)** 1

MATHEMATICS

Paper 9794/03

Applications of Mathematics

Key Message

In ***Section A***: ***Probability***, candidates should be encouraged to make full use of the resources available to them. They should know how to use the statistical functions on their calculators efficiently and to best advantage and they should also appreciate when and how to make the most of the statistical tables that are provided. In ***Section B***: ***Mechanics***, the most successful candidates are those who begin their solutions with a simple diagram and then write clear statements showing how they have applied the basic principles and models relevant to the situation.

General comments

This was the first sitting of the new structure of assessment that included a separate "Applications of Mathematics" paper. The paper as a whole appeared to have been well received by candidates and there were many high scoring scripts. There was no evidence to suggest that candidates were short of time. The Probability Section proved to be largely accessible to most candidates while the later questions of the Mechanics Section were found to be more demanding. Candidates are reminded that they should remember to heed the instruction about the accuracy of final answers, while at the same time bearing in mind the consequences of premature approximation, and the instruction about the value of *g*.

Comments on specific questions

Section A: Probability

Question 1

This question was answered correctly by almost all candidates. In order to do so efficiently candidates should be encouraged to enter the raw data into the calculator (set to Statistics mode) and then extract the summary data and the statistics from it. It is appropriate to show a certain amount of working in order to demonstrate an understanding of the method but there should be no need for candidates to do the calculations manually if they have used the calculator properly.

Answers: Mean 154.3   Standard deviation 10.0

Question 2

Most candidates recognised the geometric distribution in part **(i)** and most candidates worked out the probabilities correctly in parts **(ii)** and **(iii)**. Only very few made slips in setting things up.

Answers: **(i)** Geometric   **(ii)** 0.128   **(iii)** 0.36

Question 3

This question was answered correctly by the majority of candidates. It is expected that, when referring to the standard normal tables, candidates will make proper use of the difference columns, interpolating if appropriate. Neglecting to do so is likely to be regarded as an accuracy error. To assist with this the standardised "z-value" should be quoted (and subsequently used) to an accuracy of at least 3 decimal places.

Answer: 0.952(2)

Question 4

(i) Most candidates recognised that this question was about combinations and so had little difficulty in obtaining the correct answer.

(ii) This part was almost always answered correctly.

(iii) Although this part was usually answered correctly, candidates can do much more to help themselves. It was helpful if they identified the binomial distribution (including parameters) clearly. Next they needed to realise that $P(X < 5)$ is the same as $P(X \leq 4)$. Then all that remained was to look this up in the relevant tables. Many chose to work out each point probability from $X = 0$ to $X = 4$ using their calculator – a strategy that is risky and wastes time. There was also evidence of a few candidates finding $P(X = 5)$.

Answers: **(i)** 3060 **(ii)** 5 **(iii)** 0.997(4)

Question 5

Pleasingly, most candidates took the suggestion to use a tree diagram and consequently correct answers to parts **(i)**, **(ii)** and **(iii)** were usually obtained easily. Part **(iv)** turned out to be rather more problematic. By and large this seemed to be due to uncertainty about conditional probability and/or how to apply what they did know to this situation.

Answers: **(i)** 0.187 **(ii)** 0.363 **(iii)** 0.813 **(iv)** 0.406

Question 6

(i) Many candidates misunderstood the definition of the random variable, X, the number of coins gained, as given in the question; they tabulated the distribution of the numbers of coins returned by the machine instead. This led to problems in subsequent parts of the question.

(ii) It was clear that many candidates knew how to find the expectation and variance of a discrete random variable but not so many managed to find the correct values for this context. If the wrong table was constructed in part **(i)** then in this part there was likely to be much falsification as candidates attempted to reconcile their $E(X)$ (= 0.75) with the answer given in the question. Furthermore, candidates seemed unprepared to review their work and attempt to correct it and so the problems persisted in respect of the calculation of the variance.

(iii) In this part candidates should have realised that it is not appropriate to round an expected number to a whole number (of coins).

(iv) This part was found to be quite challenging for the vast majority of candidates and, usually, little, if any, progress was made. Careful thought was needed to realise that James must either win at least one game (i.e. 1 – P(not win any)) or draw all ten games in order that he does not finish out of pocket.

Answers: **(i)** Table showing (-1, 0.7), (0, 0.25), (9, 0.05) **(ii)** 4.69 **(iii)** 7.5 **(iv)** 0.401

Section B: Mechanics

Question 7

Most candidates managed to do this question well, especially parts **(i)** and **(ii)**. From time to time there was some confusion over whether to use sine or cosine to find each of the components. For part **(iii)** there were many correct answers obtained via a variety of sensible means. Just occasionally the direction of the resultant force was either omitted or not given due consideration.

Answers: **(i)** 7.5N **(ii)** 13.0N **(iii)** 21.8N, 36.6° below horizontal

Question 8

There were many correct solutions to this question. Nonetheless there were quite a few candidates who used “speed = distance/time” in order to then try to find the acceleration of the crate. There were also candidates who seemed unable to write down a correct equation of motion for it.

Answer: 225N

Question 9

There were many candidates who had a good grasp of how friction is modelled and how to resolve forces and so had little difficulty in scoring full marks for this question. More widely, part **(i)** seemed to present little difficulty. In part **(ii)** and, more particularly, part **(iii)**, resolving parallel to the slope to write down a complete equilibrium equation was found to be a little more tricky. The need to reverse the direction of the friction force in part **(iii)** was a particular issue. Candidates who take the trouble to provide themselves with a simple clear force diagram are likely to be the most successful.

Answers: **(i)** 16.4N **(ii)** 73.7N **(iii)** 41.0N

Question 10

In this question it was important to make careful use of both the Principle of Conservation of Momentum and Newton’s Law of Restitution. Candidates who could set these up properly at the outset experienced little difficulty in scoring high marks. The best solutions made good use of a simple, clear diagram which can make a big difference. It was evident from many scripts that the use/application of the Law of Restitution was not as well understood as it ought to have been.

(i) Candidates with a poor understanding of the Law of Restitution usually fell into the trap of using the circular argument of assuming that particle *A* came to rest in order to conclude that it did.

(ii) There was less “help” by way of a given answer in this part, and so those who struggled with the basic model found it difficult to make much meaningful progress.

(iii) A bit more progress was made here, but a minority of candidates either tried to analyse a third collision or tried to use “suvat” to find a non-existent acceleration.

Answers: **(i)** 7 ms^{-1} **(ii)** –0.5 ms^{-1} **(iii)** 7 m

Question 11

(i) Many candidates struggled with this question, despite having been given one of the answers. Instead of resolving along the slope they tried to treat the first part of the motion of this particle as a projectile by resolving vertically and horizontally, all to no avail.

(ii) There was potential for more progress here since many candidates seemed able to write down the basic projectile model. Difficulties came about when they tried to apply the model to the particular situation. Relatively few realised that they could use the equation for vertical displacement to find the total time of flight to land 5 metres below the take-off point. After that it was a matter of finding the corresponding horizontal displacement to which should be added the horizontal distance while travelling up the slope. Many unsuccessful attempts involved splitting the flight up in a variety of (often incomplete) ways thus creating additional difficulties.

Answers: **(i)** $4 - 2\sqrt{3}$ (≈ 0.536)s **(ii)** 41.5 m