UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
Cambridge International Level 3 Pre-U Certificate
Principal Subject

MATHEMATICS **9794/01**

Paper 1 Pure Mathematics and Probability **May/June 2011**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF20)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
You are advised to spend no more than 2 hours on Section A and 1 hour on Section B.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 120.

This document consists of **5** printed pages and **3** blank pages.

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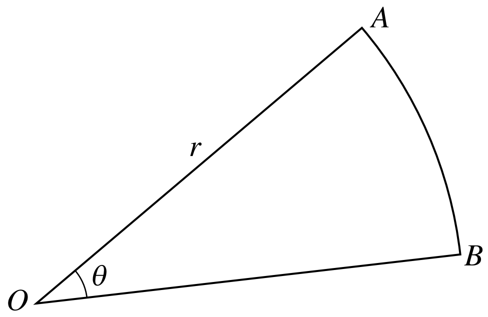
UNIVERSITY *of* CAMBRIDGE
International Examinations

Section A: Pure Mathematics (80 marks)

You are advised to spend no more than 2 hours on this section.

1 Find the equation of the line passing through the points $(-2, 5)$ and $(4, -7)$. Give your answer in the form $y = mx + c$. [3]

2



The diagram shows a sector OAB of a circle with centre O and radius r cm in which angle AOB is θ radians. The sector has a perimeter of 18 cm.

(i) Show that $\theta = \dfrac{18 - 2r}{r}$. [2]

(ii) Find the area of the sector in terms of r, simplifying your answer. [2]

3 Solve the equation $3 + 2x = |7 - 4x|$. [3]

4 **(i)** Show that $4 \ln x - \ln(3x - 2) - \ln x^2 = \ln\left(\dfrac{x^2}{3x - 2}\right)$, where $x > \frac{2}{3}$. [3]

(ii) Hence solve the equation $4 \ln x - \ln(3x - 2) - \ln x^2 = 0$. [3]

5 A circle has equation $x^2 + y^2 = 16$. Find the volume generated when the region in the first quadrant which is bounded by the circle and the lines $x = 1$ and $x = 2$ is rotated through 2π radians about the x-axis. [5]

6 **(i)** Sketch, on a single diagram, the graphs of $y = e^{\frac{1}{5}x}$ and $y = x$ and state the number of roots of the equation $e^{\frac{1}{5}x} = x$. [3]

(ii) Use the Newton-Raphson method with $x_0 = 0$ to determine the value of a root of the equation $e^{\frac{1}{5}x} = x$ correct to 3 decimal places. [4]

7 **(i)** Given that the point $(-1, -2, 4)$ lies on both the lines

$$\mathbf{r} = \begin{pmatrix} 2 \\ -3 \\ a \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 1 \\ 1 \end{pmatrix} \quad \text{and} \quad \mathbf{r} = \begin{pmatrix} 2 \\ 4 \\ b \end{pmatrix} + \mu \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix},$$

find a and b. [3]

(ii) Find the acute angle between the lines. [4]

8 **(i)** Find and simplify the first three terms in the expansion of $(1 - 4a)^{\frac{1}{2}}$ in ascending powers of a, where $|a| < \frac{1}{4}$. [4]

(ii) Hence show that the roots of the quadratic equation $x^2 - x + a = 0$ are approximately $1 - a - a^2$ and $a + a^2$, where a is small. [4]

9 **(i)** Prove that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$ and deduce that

$$\sin\theta + \sin 3\theta = 4\sin\theta\cos^2\theta.$$ [5]

(ii) Hence find the values of θ such that $0^\circ < \theta < 180^\circ$ that satisfy the equation

$$\cot^2\theta = \sin\theta + \sin 3\theta.$$ [4]

10 **(a)** The complex number z is such that $|z| = 2$ and $\arg z = -\frac{2}{3}\pi$. Find the exact value of the real part of z and of the imaginary part of z. [2]

(b) The complex numbers u and v are such that

$$u = 1 + \mathrm{i}a \quad \text{and} \quad v = b - \mathrm{i},$$

where a and b are real and $a < b$. Given that $uv = 7 + 9\mathrm{i}$, find the values of a and b. [7]

11 An arithmetic progression has first term a and common difference d. The first, ninth and fourteenth terms are, respectively, the first three terms of a geometric progression with common ratio r, where $r \neq 1$.

(i) Find d in terms of a and show that $r = \frac{5}{8}$. [7]

(ii) Find the sum to infinity of the geometric progression in terms of a. [2]

12 Find the general solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y}{x(1 + x^2)}$$

giving your answer in the form $y = \mathrm{f}(x)$. [10]

Section B: Probability (40 marks)

You are advised to spend no more than 1 hour on this section.

13 **(a)** A random sample of young people in a certain town comprised 312 boys and 253 girls. Denoting a boy's age by x years and a girl's age by y years, the following data were obtained:

$$\Sigma x = 4618, \qquad \Sigma x^2 = 68\,812, \qquad \Sigma y = 3719, \qquad \Sigma y^2 = 55\,998.$$

(i) Calculate the mean and standard deviation of the ages of the boys in the sample and also of the girls in the sample. [3]

(ii) Use these results to comment on the distribution of the ages of the boys and girls in the sample. [1]

(b) How many arrangements of the letters of the word DEFEATED are there in which the Es are separated from each other? [3]

14 **(a)** The table below relates the values of two variables x and y.

x	1	A	$A+3$	10
y	2	$A-1$	A	5

A is a positive integer and $\Sigma xy = 92$.

(i) Calculate the value of A. [3]

(ii) Explain how you can tell that the product-moment correlation coefficient is 1. [1]

(b) A music society has 300 members. 240 like Puccini, 100 like Wagner and 50 like neither.

(i) Calculate the probability that a member chosen at random likes Puccini but not Wagner. [3]

(ii) Calculate the probability that a member chosen at random likes Puccini given that this member likes Wagner. [2]

15 A firm produces chocolate bars whose weights are normally distributed with mean 120 g and standard deviation 6 g.

(i) Bars which weigh more than 114 g are sold at a profit of 15p per bar. The remaining bars are sold at no profit. Show that the expected profit per 100 bars is £12.62. [5]

(ii) It is subsequently decided that bars which weigh more than x g should be sold at a profit of 20p per bar. Those which weigh x g or less are sold to employees at a profit of 3p per bar. The expected profit per 100 bars is £19.17. Find the value of x. [7]

16 In a factory, computer chips are produced in large batches. A quality control procedure is used for each batch which requires a random sample of 8 chips to be tested. If no faulty chip is found, the batch is accepted. If two or more are faulty, the batch is rejected. If one is faulty, a further sample of 4 is selected and the batch is accepted if none of these is faulty. The probability of any chip being faulty is q.

(i) Show that the probability of accepting a batch is $p^8(1 + 8p^3 - 8p^4)$, where $p = 1 - q$. [6]

(ii) Find the expected number of chips sampled per batch, giving your answer in terms of p. Hence show that when $p = 0.75$, the expected number of chips sampled per batch is approximately 9. [6]

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