MATHEMATICS

The following question papers for Mathematics are the first papers to be taken by Pre-U students at the end of the two-year course. This also means that they are the first live question papers to be set for Pre-U candidates and in common with all new Pre-U examination questions were subjected to the same rigorous question paper setting procedure, involving subject experts and experienced teachers as well as assessment professionals.

Setting a new standard is always a challenging activity for those involved and CIE was aware that Mathematics and Further Mathematics in particular might be a difficult standard to gauge owing to the wide spread of ability in this subject. Extra work was therefore commissioned to assess the level of difficulty of the examination papers and the outcome reassured CIE that the papers were appropriate for the cohort.

However, it became evident as soon as Paper 1 had been sat that the level of difficulty might have been too high, and this was confirmed after Paper 2 had been taken. The grade boundaries for these papers were set at a level that reflected A level grades A and E at D3 and P3 respectively. Additional statistical analysis led to further work to ensure fairness to all candidates.

CIE is releasing these papers according to its usual practice but would like to point out that future examination papers will include more accessible marks for those in the middle of the distribution.

Paper 9794/01

Pure Mathematics and Probability

General comments

This was the first paper of the Pre-U sat by mathematics candidates and Examiners were impressed by the many excellent scripts seen. There were very few low-scoring scripts. Candidates responded to the challenge of a three-hour paper by showing Examiners a pleasing maturity of response which allowed them to demonstrate what they had learnt. There was some evidence that a sizeable minority of candidates were pressed for time and, this will be taken into consideration in the design of future papers. The standard of presentation was generally very pleasing and almost all candidates showed their working in full. While it was pleasing to see that, with only an odd exception, all candidates scored full marks on the first question, there were several questions which caused a number of candidates some difficulty (**Questions 7**, **8** and **11**) while allowing the most able candidates to produce work of a high quality.

Comments on specific questions

Section A: Pure Mathematics

Question 1

This proved to be a very straightforward question with almost all candidates obtaining a correct value for *x*. Two methods were seen: either a solution based on re-writing 4 as 2^2 was found, or logs were used. Both were, of course, awarded full credit.

Answer: $\frac{-2}{3}$

Question 2

(i) This part of the question saw a correct re-arrangement from almost all candidates.

(ii) Strangely, this part of the question proved to be one of the most difficult aspects of the paper with candidates seemingly unaware of the conditions for convergence of the $x = \mathrm{g}(x)$ method. It is a syllabus item that candidates should be able to recognize the relationship between the magnitude of the derivative of g at the root and the convergence or divergence of the iterative scheme; the question was designed to lead candidates to such a conclusion. It was envisaged that most candidates might wish to take a value of x between 0 and 1 and, by substituting it into $|\mathrm{g}'(x)|$, show that the result was less than 1. Candidates were not required to find the root and were not given credit for attempting this nor for finding the values of g at 0 and 1 and noting a sign change.

Answers: **(i)** $p = \frac{1}{5}$, $q = \frac{3}{5}$ **(ii)** $\left|\frac{3}{5}x^2\right| < 1$

Question 3

This question was generally well answered and it might be helpful to candidates to note that graph sketches should be done with care. A sketch of a polynomial should be a curve and not a succession of linear segments and should not include artistic flourishes at the end. While a plot is certainly not expected, it is important for candidates to indicate the main features of a curve, such as minimum and maximum points, and perhaps an intercept with the y-axis. This is important if the graph has later to be transformed as in this question. In this respect, it was surprising that a minority of candidates were unable to recognize how to plot a cubic from its factorised form. Almost all candidates were able to calculate the inverse function but very few to explain why the inverse existed. Ideally Examiners were hoping to see reference to f being many-one over part of its domain or that f was not one-one. Mentioning that the graph bent back on itself or some such comment was acceptable. Many explanations were either wrong or lacked clarity, e.g. stating that f was a cubic.

Answers: **(ii)** $\mathrm{g}^{-1}(x) = \frac{1}{2}x + \frac{1}{2}$ **(iii)** One real root

Question 4

A large number of elegant and concise solutions were seen to this integration. It was unfortunate that many able candidates who performed well in other parts of the paper struggled with this question. While attempting to make use of the suggested substitution, faulty algebraic manipulation resulted in an impossible integral or, having found $\frac{\mathrm{d}u}{\mathrm{d}x}$, an incomplete substitution was made and integration attempted on a muddled integral still in terms of x and u.

Answer: $2(1 + \sqrt{x} - \ln|1+ \sqrt{x}|) + c$

Question 5

Very many candidates made successful attempts at this question and even those who scored only a few marks on it clearly knew what they were aiming for as they were able to state the correct relationship between the derivatives. The main problem they encountered was in finding $\frac{\mathrm{d}y}{\mathrm{d}t}$ where they were largely defeated by the algebraic manipulation and simplification involved. A few candidates tried to eliminate the parameter and obtain an expression in the form $y = \mathrm{f}(x)$ first before differentiating. These candidates were in general rather less successful in achieving a completely correct solution. The condition for finding a stationary point of a curve seemed universally known.

Answers: **(i)** $\frac{1-t^2}{-2t}$ **(ii)** $\left(\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{2},-\frac{1}{2}\right)$

Question 6

Almost all candidates achieved correct solutions to the first two parts of the question. Candidates who used the sum formula and reached a quartic equation were as successful in showing $r = 2$ to be a root as those who summed individual terms and reached a cubic. Some candidates dealt with their equation by the remainder theorem and a long division process, and most substituted $r = 2$ to show the given result. A large proportion of candidates did not carry the last part through to a successful conclusion as a result of errors in rearranging the formula for the sum of a GP. This process is a familiar one at this level and candidates should be aware that 12×2^n does not simplify to 24^n or a similar error.

Answers: **(ii)** 12 **(iii)** 210

Question 7

Most candidates appreciated the need to factorise the 4 and accurately obtained $\left(1-\frac{3x}{4}\right)^{-\frac{1}{2}}$ but were notably less successful in applying the index to the 4. Expansions were generally accurate though slips occurred in evaluating the third term. Candidates seemed much less familiar with the concept of the validity of their expansions, providing incorrect statements like $|x| < 1$ or $x < \frac{4}{3}$. On the other hand, the last two parts of the question were generally done accurately, albeit on the function that had been obtained. Full marks for the question were rarely obtained as a result.

Answers: **(i)** $\frac{1}{2} + \frac{3}{16}x + \frac{27}{256}x^2$ for $|x| < \frac{4}{3}$ **(ii)** $\frac{1}{2} + \frac{3}{16}x + \frac{155}{256}x^2$ **(iii)** 0.0511

Question 8

The first part of this question proved challenging to many candidates. It cannot be emphasized too often to candidates that a clear and fully-labelled diagram is essential in vector geometry and enables the solution to drop out without too much trouble. Scripts showed that candidates who adopted such a starting point were notably successful. Candidates who did not start thus struggled, particularly in exhibiting the given equation. Credit was also awarded for those who correctly indicated that a vector equation of a line was an equation to define a general point **r** on the line. Candidates who believed that $AX = (1,-1,1) + \lambda(5,1,7)$ or similar were penalised. The second part of the question was more routine and therefore more successful, though candidates who did not obtain the correct expression for the line CD did not gain full marks.

Answers: **(i)** $\mathbf{r} = (3,-3,3) + \mu(5,7,9)$ **(ii)** (5, -0.2, 6.6)

Question 9

Examiners were pleased to see a number of fully correct solutions to this question and all candidates were familiar with implicit differentiation. Differentiation of y^2 was usually correct but errors occurred in differentiating xy. Either the need for a product rule was not seen or there were errors in the sign of the second term. The tendency to preface an attempt at implicit differentiation by starting the working with $\frac{dy}{dx}$ =, although incorrect, seems in practice to have been ignored subsequently. Examiners considered this as an indication to the candidate of what he finally wished to achieve rather than a part of his method. Most candidates made a good attempt at part **(ii)** but many were impeded by an incorrect derivative.

Answers: **(i)** $\frac{dy}{dx} = \frac{2x-y}{x-2y}$ **(ii)** $\left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right), \left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

Question 10

Many candidates struggled on the first part of the question, seemingly unable to deal with the complex number z and the constant n. The most successful candidates either attempted to rationalise the denominator by multiplying numerator and denominator of the expression for z by (2-i) or by writing $z = a + \mathrm{i}b$ and comparing coefficients. In part **(b)** finding z_1 and z_2 was generally much more successful, as was the Argand diagram, though a lack of scale on the diagram did give Examiners pause for thought and it was disappointing to see that some candidates still had problems using the quadratic formula. The last part gave trouble to a sizeable proportion of candidates in interpreting what was asked for. Thus, they seemed familiar with finding the modulus of a complex number but often worked with z_1 and z_2 rather than $z_1 + 1$ and $z_2 + 1$. Similarly, they seemed aware that they should apply the $\tan^{-1}$ function to find the argument of a complex number but took no regard to the quadrant in which the arguments must lie.

Answers: **(a)** $\frac{1}{5}(8 + n) + \mathrm{i}\frac{1}{5}(2n - 4)$ **(b) (i)** $-4 \pm 3\mathrm{i}$ **(ii)** $3\sqrt{2}$, $\frac{3\pi}{4}$, $-\frac{3\pi}{4}$

Question 11

This was the most challenging pure question on the paper and so it proved with only a handful of candidates achieving full marks. For those candidates who may have previously explored the identities for cos3x, sin3x and tan3x, the first part of the question posed few difficulties and it was pleasing to see so many fully-argued solutions. The second part of the question was slightly less successful, with many candidates resorting to calculator justification rather than providing the clear demonstration required using the fact that $\tan\theta = \frac{1}{3}$. It was the last part of the question which proved the biggest challenge; it was intended that a link be established between $\theta = \sin^{-1} x$ and x identified as $\frac{1}{\sqrt{10}}$. Many candidates did not spot this link or, having done so, did not make their reasoning clear. The last root could only be shown convincingly if it was realised that a second root of the equation $3\sin^{-1} x = \tan^{-1}\left(\frac{13}{9}\right)$ occurred after a further π radians and the sin(A + B) formula used to evaluate $x = \sin\left(\sin^{-1}\left(\frac{1}{\sqrt{10}} + \frac{\pi}{3}\right)\right)$. A small number of candidates realised this and achieved full marks. A rather larger number of candidates equated the identity for tan 3θ to $\frac{13}{9}$ and attempted to deal with the resulting quadratic. These candidates received credit for what was a very difficult path to tread even though they were unable to carry their task to completion.

Section B: Probability

Question 12

The first question on the probability section proved more challenging than expected. Most candidates obtained a correct value for the first part and most also got the last part correct, though some used ${}^{49}P_6$ and others found ${}^{50}C_6$. A surprisingly large number were unable to use the P(A U B) relation to obtain the correct fraction in the second part, though most were able to state that $P(A \cap B) = \frac{1}{12}$.

Answer: **(a) (i)** $\frac{5}{6}$ **(ii)** $\frac{2}{3}$ **(b)** 13 983 816

Question 13

There were some surprising misconceptions illustrated by candidates' answers to this question. For example, while the median was usually given correctly, the upper and lower quartiles were often based on using $\frac{1}{4}$ and $\frac{3}{4}$ of n rather than n+1. Candidates also showed they were unaware of the conditions for an outlier, the most common error being to add 1.5 times the IQR onto the median instead of the upper quartile. They were also too ready to discard an outlier without considering whether it might be an item of genuine data. The product-moment correlation coefficient was usually calculated correctly. Many candidates were at pains to show every step in the calculation; in papers where the use of the scientific calculator is encouraged, all that is really required is a statement of the formula and a substitution of the main quantities. Some interesting comments on the correlation coefficient were seen indicating that candidates were aware that there was no implication of causality. Very few indicated that extrapolation beyond the data was unreliable. Candidates should be aware that both comments are expected in questions involving correlation between two sets of data.

Answers: **(a) (i)** Median £41 000, IQR £35 000 **(ii)** Outliers were in excess of £108 500.
(b)(i) 0.73

Question 14

This question was very well answered by the vast majority of candidates, with calculations using the geometric distribution handled very successfully. The only part which caused widespread difficulty was in justifying the use of the geometric distribution. Explanations like "independent", "constant probability", "could only have a success or failure" were widely employed but only a minority of candidates fixed on the crucial point that once a winning ticket was found, the reader stopped buying the newspaper.

Answers: **(a) (i)** 1.89 **(ii)** 0.25 **(iii)** 0.49 **(b) (i)** $F(y) = (0.002)(0.998)^{y-1}$ **(ii)** 0.00193
(iii) 0.00599

Question 15

There was evidence that some candidates found themselves under time pressure to complete this question. This may have accounted for the sizeable proportion of candidates who misread the Normal Distribution tables. Examiners noted the use of 1.36 instead of 1.036 and -1.46 instead of -1.406 for example. Sign errors were frequent, notably 1.406 instead of -1.406. In spite of these errors, candidates seemed to be familiar with this topic and made good attempts at all parts of the question, though unfortunately a wrong μ or σ was seen. Various means of demonstrating inconsistency in part **(iii)** were seen using z values, percentages or lengths. All of these received full credit.

Answers: **(i)** σ =1.23, μ = 13.7 **(ii)** 11.7

MATHEMATICS

Paper 9794/02

Pure Mathematics and Mechanics

General comments

It was pleasing to see a sustained performance by the majority of candidates on the ***Section A*** Pure Mathematics questions. Although many candidates tackled the ***Section B*** Mechanics with knowledge of the relevant principles and competence at applying them, there were also many who found this section difficult.

The Examiners were pleased that many candidates who found early parts of a question difficult went on to complete a later part successfully. The benefits of a three-hour paper, allowing more thinking time and time to go back to a question, were apparent.

Comments on specific questions

Section A: Pure Mathematics

Question 1

Candidates generally found this a straightforward question, and few scored anything other than full marks.

Answer: 110

Question 2

Many candidates made a good start and found the critical values, usually scoring 4 out of 5 marks. The majority of candidates did not consider the case where $x \le 1$.

Answer: $1 < x < \frac{3}{2}$

Question 3

(i) Almost all candidates correctly answered this part.

(ii) (a) The majority of candidates answered this part correctly, although a few only gave the first four terms of one sequence.

(b) It was pleasing to see that a number of candidates were able to provide satisfactory solutions to this slightly unusual question. A few candidates confused 'infinite number' with 'sum of an infinite series'.

Answers: **(i)** $a = 6$, $d = 4.5$ **(ii)(a)** 6, 10.5, 15, 19.5; 1.5, 4.5, 7.5, 10.5

Question 4

(i) There was a very good response to this question, with a variety of approaches, such as factorizing the quartic expression or direct use of the trigonometric version of Pythagoras' Theorem to eliminate either the cosine or the sine function. Some candidates had difficulty with the algebraic manipulations.

(ii) The vast majority of candidates scored full marks here.

Answer: **(ii)** 0°, 120°, 240°, 360°

Question 5

This was a question combining several areas of the syllabus: complex numbers, rational functions, solving a quadratic equation, inequalities and graphical work. The majority of candidates responded well to parts **(i)** and **(ii)**. A few candidates found a very neat proof for part **(ii)**, involving a pre-cursor to the result displayed in part **(i)**, and this was accepted. There was a mixed response to part **(iii)** with many candidates not realising the stem instruction that x and y are positive applied to the whole question.

Question 6

(i) This part was well done by most candidates. A few multiplied by $(x+1)^2$, effectively treating it as $(x+1)^3$, thereby obtaining the wrong algebraic factors for the coefficients A and B.

(ii) The vast majority of candidates correctly stated the quotient and remainder, but quite a number subsequently showed their lack of understanding of these terms.

(iii) The majority of candidates realised the differential equation was separable and were able to use their answers to parts **(i)** and **(ii)**. Those who misunderstood their answer to part **(ii)** lost a mark because their y-integrand simplified the integration process. A few candidates did not evaluate the constant of integration.

Answers: **(i)** $\frac{1}{x+1}-\frac{2}{(x+1)^2}$ **(ii)** $2y-2+\frac{3}{y+1}$

(iii) $y^2-2y+3\ln(y+1)=\ln(x+1)+\frac{2}{x+1}+3\ln 3-2$

Question 7

(i) Much good work was seen in this part, but a surprising number of candidates did not take note of the instruction that y was an increasing function. A few candidates incorrectly differentiated y, and follow-through was allowed in such cases.

(ii) The vast majority of candidates scored full marks for this part.

Answers: **(i)(a)** $1 < x < 2$ **(b)** $x = 2$, maximum **(ii)(c)** 0.6823

Question 8

(i) There were very few full solutions to this part. Nevertheless it was pleasing that most candidates were able to move on and use the displayed equation for the curve C.

(ii) Most candidates stated the correct y-coordinate of the lowest part of the curve. Candidates who sketched a rough diagram often noted that the lowest point on the curve was the midpoint of the interval joining F and the intersection of D with the y-axis. This was sufficient for the mark.

(iii) (a) It was pleasing that many candidates were able to sustain the algebra needed for this part, albeit sometimes rather heavy-handedly. This was particularly obvious with those candidates who did not appear to be familiar with the factorization of a difference of squares.

(b) This part was often well done, weaker candidates being helped by the numerical values of a and b rather that the symbolics of the previous parts.

Question 9

(i) In spite of the rather imposing nature of the displayed result, most candidates took this part in their stride. Surprisingly, some candidates chose not to take advantage of the displayed general result for use in parts **(ii)** and **(iii)**, so wasting time going through unnecessary integration by parts calculations.

(ii) The value $x = 2$ was usually found. Many candidates either found the area between the curves, or got close to it. A common error was not dealing correctly with a product of two minus signs after the limit values were substituted.

(iii) There was a good response to this part. Some candidates who included the factor π early on subsequently lost marks because they did not use it in the final numerical calculation.

Answers: **(ii)(a)** $x = 2$ **(b)** $\frac{7}{9} - \frac{1}{3}\ln 2$ **(iii)** 32.36

Section B: Mechanics

Question 10

(i) This part was almost always tackled well by those candidates who seemed confident in the area of Mechanics. A small minority chose the wrong direction for the acceleration of the particle, but still managed to end up with the given answer by erroneous working.

(ii) The majority of candidates who correctly solved part **(i)**, also correctly solved this part. A few candidates used a conservation of work/energy principle.

Answer: **(ii)** $\frac{V^2\sqrt{2}}{3g}$

Question 11

(i) There was a mixed response to this question. Although there were many good solutions, some candidates had clearly never encountered the notion of finding the resultant of forces given in vector form, and tried to resolve the forces in various directions.

(ii) and **(iii)** Candidates were more successful with these parts.

Answers: **(ii)** 7.5 ms^{-2} **(iii)** 4 seconds

Question 12

This was the highest scoring of the ***Section B*** questions.

(i) The majority of candidates knew how to obtain velocity from acceleration, namely by integration, and so most scored at least 5 marks out of 7. Some candidates assumed a zero value for the first stage constant, without showing why that had to be the case. Those who did not obtain the second stage correctly using $v(10) = 0$, could not also verify that $v(20) = 0$. Nevertheless, many candidates correctly sketched the complete velocity-time graph. A small minority sketched graphs with straight lines, or that were in other ways distorted.

(ii) Nearly all candidates who tackled this part obtained the correct value for the area under the first parabola. There was less success with the area corresponding to the second parabola and what to do with its negative value. A very small number assumed the areas to be the same numerically, which happens to be true but is not obvious.

Answer: **(ii)** 1000m

Question 13

(i) This part was mostly well answered by those candidates who wrote down the two separate equations of motion for the particles. A very small number treated it as a single compound body problem – it can be done this way, but it needs care and justification.

(ii) Candidates needed to realise that the acceleration of *A* changes after impact and also that the total height comes to 1 + 1 + 0.2 metres.

(iii) There were some candidates who assumed a formula involving the quotient of initial height and rebound height. This is inappropriate in this question as the string goes slack after impact so the acceleration of *B* changes.

Answers: **(i)** 0.2*g* **(ii)** 2.2m **(iii)** 0.5