Cambridge Pre-U

**Cambridge International Examinations**
Cambridge Pre-U Certificate

**FURTHER MATHEMATICS (PRINCIPAL)** **9795/02**

Paper 2 Further Application of Mathematics **For Examination from 2016**

SPECIMEN MARK SCHEME

**3 hours**

# MAXIMUM MARK: 120

The syllabus is approved for use in England, Wales and Northern Ireland as a Cambridge International Level 3 Pre-U Certificate.

This document consists of **7** printed pages and **1** blank page.

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**Mark Scheme Notes**

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

The following abbreviations may be used in a mark scheme:

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
aef Any equivalent form
art Answers rounding to
cwo Correct working only (emphasising that there must be no incorrect working in the solution)
ft Follow through from previous error is allowed
o.e. Or equivalent

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| 1 | **(i)** | $\mathrm{M}_X(t) = \int_0^\infty \mathrm{e}^{tx} k\mathrm{e}^{-kx}\mathrm{d}x$ (Limits required) | M1 |
| | | $= k\int_0^\infty \mathrm{e}^{(t-k)x}\mathrm{d}x = k\int_0^\infty \mathrm{e}^{-(k-t)x}\mathrm{d}x$ (Limits not required) | M1 |
| | | $= \dfrac{-k}{k-t}\left[e^{-(k-t)x}\right]_0^\infty = \dfrac{k}{k-t}$ AG | A1 |
| | **(ii)** | $\mathrm{M}_X{}'(t) = \dfrac{k}{(k-t)^2} \Rightarrow \mathrm{E}(X) = \mathrm{M}_X{}'(0) = \dfrac{1}{k}$ | M1 A1 |
| | | $\mathrm{M}_X{}''(t) = \dfrac{2k}{(k-t)^3} \Rightarrow \mathrm{E}(X^2) = \mathrm{M}_X{}''(0) = \dfrac{2}{k^2}$ | M1 A1 |
| | | (A1 ft if double sign error when differentiating twice, but CAO) | A1 |
| | | **Alternatively:** | |
| | | $\mathrm{M}_X(t) = \left(1-\dfrac{t}{k}\right)^{-1} = 1 + \dfrac{t}{k} + \dfrac{t^2}{k^2} + ... = 1 + \dfrac{1}{k}t + \dfrac{2}{k^2}t^2 + ...$ | M1 A1 |
| | | $\mathrm{E}(X) = \dfrac{1}{k}$ | A1 |
| | | $\mathrm{E}(X^2) = \dfrac{2}{k^2} \Rightarrow \mathrm{Var}(X) = \dfrac{2}{k^2} - \left(\dfrac{1}{k}\right)^2 = \dfrac{1}{k^2}$ | M1 A1 |
| 2 | **(i)** | $\mathrm{E}(a\overline{X} + b\overline{Y}) = \mu$ | M1 |
| | | $\mathrm{E}(a\overline{X} + b\overline{Y}) = a\mathrm{E}(\overline{X}) + b\mathrm{E}(\overline{Y})$ | M1 |
| | | $\Rightarrow a\mu + 3b\mu = \mu \Rightarrow a + 3b = 1$ | A1 |
| | **(ii)** | $\mathrm{Var}(a\overline{X} + b\overline{Y}) = a^2\mathrm{Var}(\overline{X}) + b^2\mathrm{Var}(\overline{Y}) = a^2\dfrac{\sigma^2}{n} + 4b^2\dfrac{\sigma^2}{n}$ | M1 |
| | | $= \dfrac{\sigma^2}{n}(a^2 + 4b^2) = \dfrac{\sigma^2}{n}(1 - 6b + 9b^2 + 4b^2) = \dfrac{\sigma^2}{n}(1 - 6b + 13b^2)$ AG | M1 A1 |
| | **(iii)** | $\dfrac{\mathrm{d}}{\mathrm{d}b}\mathrm{Var}(a\overline{X} + b\overline{Y}) = -6 + 26b = 0 \Rightarrow b = \dfrac{3}{13}$ | M1 A1 |
| | | $\Rightarrow \mathrm{Var}_{\min}(a\overline{X} + b\overline{Y}) = \dfrac{\sigma^2}{n}\left(1 - 6\times\dfrac{3}{13} + 13\times\dfrac{9}{169}\right) = \dfrac{4\sigma^2}{13n}$ | A1 |

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| **3** | **(i)** | $\bar{x} = 1.675$<br>99% confidence limits are $1.675 \pm 2.576 \times \frac{0.1}{\sqrt{6}}$ (ft on wrong mean)<br>99% confidence interval is (1.57, 1.78) art | B1<br>M1<br>A1ft<br>A1 |
| | **(ii)** | $s_n = 0.09215$ or $s_{n-1} = 0.1009$<br>$v = 5 \Rightarrow t_5\,(0.99) = 4.032$<br>99% confidence limits are $1.675 \pm 4.032 \times \frac{0.1009}{\sqrt{6}}$ or $1.675 \pm 4.032 \times \frac{0.09215}{\sqrt{5}}$<br>99% confidence interval is (1.51, 1.84) art | B1<br>B1<br>M1 A1<br>A1 |
| | **(iii)** | Sensible comment referring to the fact that 1.8 is outside the 1st interval but inside 2nd. (ft on their confidence intervals) | B1ft |
| **4** | **(i)** | $\mathrm{P}(X = r) = \mathrm{e}^{-\lambda} \frac{\lambda^r}{r!}$ (may be implied by next line)<br>$\mathrm{G}_X(t) = \sum_0^\infty p_r t^r = \sum_0^\infty \mathrm{e}^{-\lambda} \frac{(\lambda t)^r}{r!} = \mathrm{e}^{-\lambda}\mathrm{e}^{\lambda t} = \mathrm{e}^{\lambda(t-1)}$ AG | B1<br>M1 A1 |
| | **(ii)** | $\mathrm{G}_{X+Y}(t) = \mathrm{e}^{\lambda(t-1)}.\mathrm{e}^{\mu(t-1)} = \mathrm{e}^{(\lambda+\mu)(t-1)} \quad \Rightarrow (X+Y) \sim \mathrm{Po}(\lambda+\mu)$ | M1 A1 |
| | **(iii)** | P( [0, 4] or [1, 3] or [2, 2] ) = 0.2231 × 0.1336 + 0.3347 × 0.2138 + 0.2510 × 0.2565 (B1 for any two correct)<br>$\mathrm{P}(X \leqslant 2 \mid X + Y = 4) = \frac{0.1657}{0.1954}$<br>$= 0.848$ art | B2,1,0<br>M1<br>A1ft<br>A1 |
| **5** | **(i)** | $np = 100 \times \frac{1}{5} = 20$ and $npq = 20 \times \frac{4}{5} = 16$<br>Standardisation in either **(a)** or **(b)** | B1<br>M1 |
| | **(a)** | $z = \frac{14.5 - 20}{4} = -1.375 \Rightarrow \mathrm{P}(\geqslant 15) = 0.915$ (within range [0.915,0.916])<br>(ft on variance of 20) | A1ft A1 |
| | **(b)** | $z = \frac{12.5 - 20}{4} = -1.875 \Rightarrow \mathrm{P}(<12) = 1 - 0.9696 = 0.0304$ (within range [0.0303,0.0304]) | A1 A1 |
| | **(ii)** | mean = variance = 36 $\Rightarrow$ S.D. = 6<br>$z = 1.645$<br>$\frac{\left(N + \frac{1}{2}\right) - 36}{6} > 1.645$ (Allow working with equality, but must be $\left(N + \frac{1}{2}\right)$)<br>$\Rightarrow N > 45.37 \quad \therefore$ least $N = 46$ | B1<br>B1<br>M1 A1<br>A1 |

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| 6 | **(i)** | **Above** $x$-axis between (0, 0) to (3, 0)<br>Correct concavity. (Do not condone parabolas) | B1<br>B1 |
| | **(ii)** | $\mu = \frac{4}{27}\int_0^3 \left(3x^3 - x^4\right)\mathrm{d}x$ (Limits required)<br>$= \frac{4}{27}\left[3\frac{x^4}{4} - \frac{x^5}{5}\right]_0^3 = 1.8$<br>$\mathrm{f}'(x) = \frac{4}{27}(6x - 3x^2) = 0$<br>$\Rightarrow x = 0, 2 \quad \therefore$ Mode = 2 | M1<br>A1 A1<br>M1 A1<br>A1 |
| | **(iii)** | Mean less than mode in **(ii)** matches negative skew in sketch. | B1 |
| | **(iv)** | $\mathrm{P}(\lvert X - \mu\rvert < \sigma) = \frac{4}{27}\int_{1.2}^{2.4}\left(3x^2 - x^3\right)\mathrm{d}x$ (Limits required)<br>$= \frac{4}{27}\left[x^3 - \frac{x^4}{4}\right]_{1.2}^{2.4} = 0.64$ | M1<br>A1 A1 |
| 7 | **(i)** | Tractive force = Resistance at maximum speed $\Rightarrow \frac{75}{10} = 10k \Rightarrow k = \frac{3}{4}$ AG | B1 |
| | **(ii)** | $F = ma \Rightarrow \frac{75}{v} - \frac{3}{4}v = 90\frac{\mathrm{d}v}{\mathrm{d}t} \Rightarrow \frac{25}{v} - \frac{1}{4}v = 30\frac{\mathrm{d}v}{\mathrm{d}t}$ AG (3 terms required for M1) | M1 A1 |
| | **(iii)** | $\int_0^t \mathrm{d}t = \int_3^7 \frac{120v}{100 - v^2}\,\mathrm{d}v$<br>$t = -60\int_3^7 \frac{-2v}{100 - v^2}\,\mathrm{d}v = \left[-60\ln\lvert 100 - v^2\rvert\right]_3^7$ (Limits not required)<br>$= -60\ln 51 + 60\ln 91 = 60\ln\left(\frac{91}{51}\right)$ ( = 34.7) seconds. | M1<br>M1 A1<br>M1 A1 |
| 8 | **(i)** | Resolve vertically $N_B = 2 \times 10$<br>Take moments about e.g. intersection of normals: $N_B \times 0.2\cos 60 = F \times 0.4\sin 60$<br>moments equation correct (if in equilibrium)<br>$F = 5.77$, $N_B = 20$<br>$F > \mu N_B$<br>Correctly deduce not in equilibrium therefore rod does slip | M1<br>M1<br>A1<br>A1<br>M1<br>A1 |
| | **(ii)** | Consider forces horizontally $N_A$ non-zero<br>Correctly deduce impossibility hence leftwards force required for equilibrium | M1<br>A1 |

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| 9 | **(i)** | $T\cos\theta = mg$ (Must be seen to score in part **(i)**)<br>$\theta = 90° \Rightarrow mg = 0 \quad \therefore AP$ can never be horizontal.<br>$\therefore 0 < \cos\theta < 1 \quad \Rightarrow T\left(=\dfrac{mg}{\cos\theta}\right) > mg$ | B1<br>B1<br>B1 |
| | **(ii)** | $T\sin\theta = ml\sin\theta\varpi^2 \Rightarrow \mathrm{T} = ml\varpi^2$ | M1 A1 |
| | **(iii)** | $T\dfrac{h}{l} = mg \Rightarrow T = \dfrac{mgl}{h}$<br>$\Rightarrow ml\varpi^2 = \dfrac{mgl}{h}$<br>$\Rightarrow \omega^2 h = g$ .<br>$h = 0.5 \Rightarrow \varpi = \sqrt{20} = 4.47$ Angular speed is 4.47 rad/s. | B1<br>B1<br>B1<br>B1 |
| 10 | **(i)** | Let $u$ denote speed of sphere $Q$ before impact, $v_1$ and $v_2$ the speeds of spheres $Q$ and $P$, respectively, after impact and $\alpha$ the angle between $Q$'s initial direction of motion and the line of centres.<br>After impact, if moving perpendicularly, $Q$ moves perpendicular to line of centres and $P$ moves along line of centres. (Stated or implied)<br><br>Conservation of linear motion: $mu\cos\alpha = 0 + 3mv_2$ or $mu_x = 3mv$<br>Newtons experimental law: $eu\cos\alpha = v_2$ or $eu_x = v$.<br>$\therefore e = \dfrac{1}{3}$ | B1<br><br>M1 A1<br>A1<br>A1 |
| | **(ii)** | $v_1 = u\sin\alpha$ and $v_2 = \dfrac{1}{3}u\cos\alpha$.<br>Loss in kinetic energy is<br>$\dfrac{1}{2}mu^2 - \dfrac{1}{2}mu^2\sin^2\alpha - \dfrac{1}{2}\cdot 3m\dfrac{u^2\cos^2\alpha}{9}$<br>$= \dfrac{1}{12}mu^2$ (Or remaining kinetic energy is 5/6 of initial kinetic energy etc.)<br>But $\cos^2\alpha + \sin^2\alpha = 1$ (used)<br>$\Rightarrow \ldots \Rightarrow \sin^2\alpha = \dfrac{3}{4} \quad \Rightarrow \sin\alpha = \dfrac{\sqrt{3}}{2} \Rightarrow \alpha = 60°$ | B1 (both)<br><br>M1 A1<br>A1<br>M1<br>M1 A1 |

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| **11** | **(i)** | $T = \dfrac{6mg\left(\sqrt{9a^2+x^2}-a\right)}{a}$ <br> Let $\theta$ be the angle between each string and line of motion of particle. <br> $m\ddot{x} = -2T\cos\theta = -\dfrac{12mg}{a}\left(\sqrt{9a^2+x^2}-a\right)\times\dfrac{x}{\sqrt{9a^2+x^2}}$ <br> $\Rightarrow \ddot{x} = -\dfrac{12gx}{a}\left(1-\dfrac{a}{\sqrt{9a^2+x^2}}\right)$ | M1 A1 <br> M1 <br> A1 A1 <br> A1 |
| | **(ii)** | $\therefore \ddot{x} \approx (-12g+4g)\dfrac{x}{a} = -\dfrac{8g}{a}x$ <br> Which is simple harmonic motion of period $2\pi\sqrt{\dfrac{a}{8g}}$ or $\pi\sqrt{\dfrac{a}{2g}}$. | M1 A1 <br> A1 |
| | **(iii)** | $v_{\max} = \varpi a \Rightarrow \dfrac{ga}{200} = \dfrac{8g}{a}A^2 \Rightarrow A^2 = \dfrac{a^2}{1600} \Rightarrow A = \dfrac{a}{40}$ <br> where $A$ is the amplitude. | M1 A1 |
| **12** | **(i)** | $x = 20\cos\alpha t \quad y = 20\sin\alpha t - 5t^2$ <br> $y = 20\sin\alpha\cdot\dfrac{x}{20\cos\alpha} - 5\left(\dfrac{x}{20\cos\alpha}\right)^2 = x\tan\alpha - \dfrac{x^2}{80}(1+\tan^2\alpha)$ AG | B1 B1 <br> M1 A1 |
| | **(ii)** | $x^2\tan^2\alpha - 80x\tan\alpha + x^2 + 80y = 0$ (Can be implied by what follows.) <br> Real roots $\Rightarrow 6400x^2 - 4x^2(x^2+80y) \geqslant 0$ <br> $\Rightarrow 1600 - x^2 - 80y \geqslant 0 \Rightarrow y \leqslant 20 - \dfrac{x^2}{80}$, $(x \neq 0)$. | B1 <br> M1 A1 <br> A1 |
| | **(iii)** | $x = R\cos\beta$ and $y = R\sin\beta \Rightarrow y = x\tan\beta$. <br> In $y = 20 - \dfrac{x^2}{80} \Rightarrow R\sin\beta = 20 - \dfrac{R^2(1-\sin^2\beta)}{80}$ <br> $\therefore$ <br> $R^2(1-\sin^2\beta) + 80R\sin\beta - 1600 = 0 \Rightarrow (R[1-\sin\beta]+40)(R[1+\sin\beta]-40) = 0$ <br> $\Rightarrow R = \dfrac{40}{1+\sin\beta}$ (up) or $\dfrac{-40}{1-\sin\beta}$ (down) | <br> M1 A1 <br> M1 <br> A1 |
| | **(iii)** | **Alternative solution:** <br> $x^2 + 80\tan\beta x - 1600 = 0$ <br> $\Rightarrow x = -40\tan\beta \pm 40\sec\beta$ <br> $\Rightarrow R_{\text{up}} = \dfrac{-40\sin\beta+40}{\cos^2\beta} = \dfrac{40(1-\sin\beta)}{1-\sin^2\beta} = \dfrac{40}{1+\sin\beta}$ <br> $\Rightarrow R_{\text{down}} = \left\lvert\dfrac{-40\sin\beta-40}{\cos^2\beta}\right\rvert = \dfrac{40(1+\sin\beta)}{1-\sin^2\beta} = \dfrac{40}{1+\sin\beta}$ | <br> B1 <br> B1 <br> B1 <br> B1 |

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