Cambridge Pre-U Syllabus

Cambridge International Level 3
Pre-U Certificate in
FURTHER MATHEMATICS

For examination in 2013, 2014 and 2015

UNIVERSITY *of* **CAMBRIDGE**
International Examinations
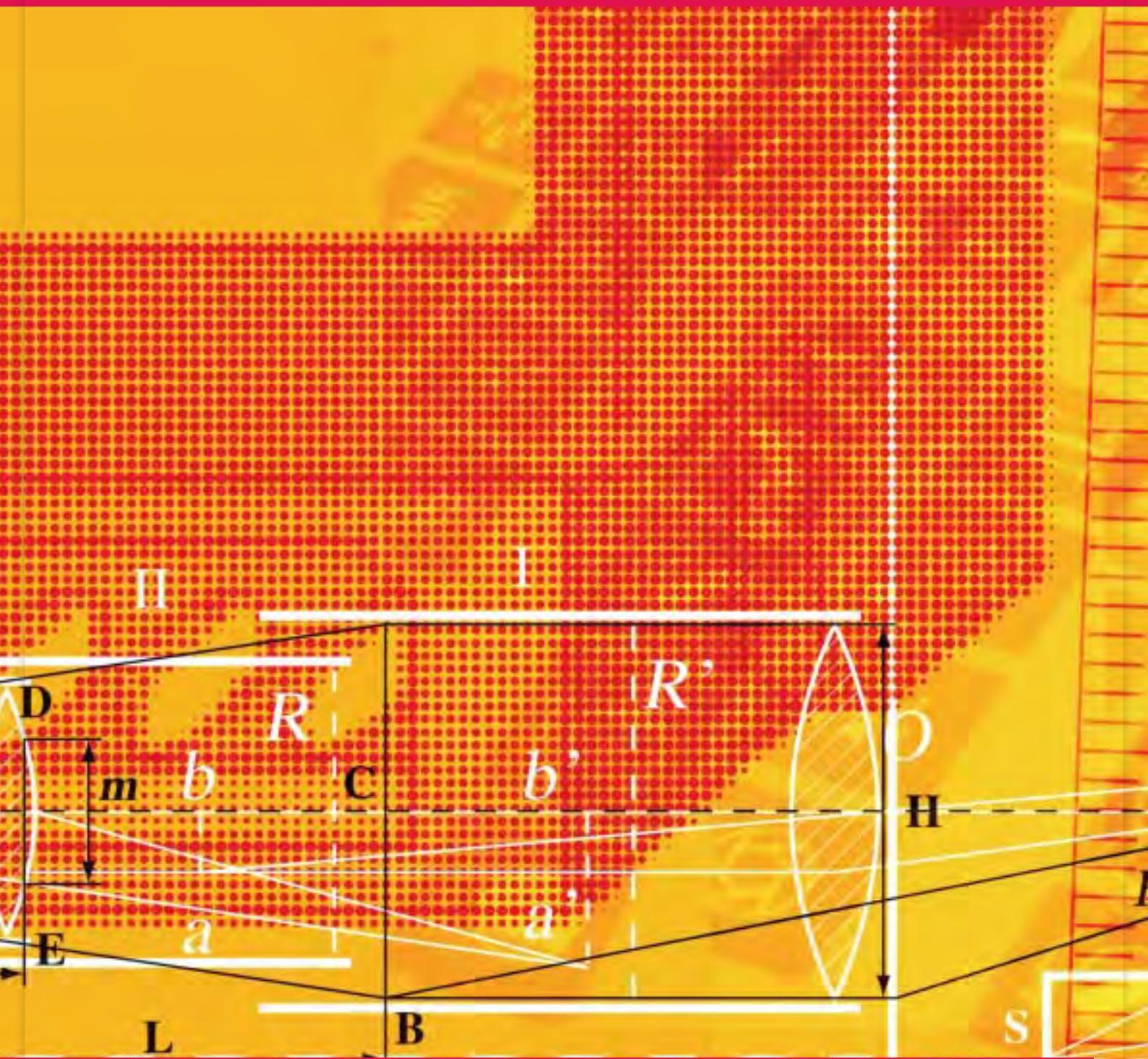
Further Mathematics (9795)

**Cambridge International Level 3
Pre-U Certificate in Further Mathematics (Principal)**

For examination in 2013, 2014 and 2015

QN 500/3829/1

Support

CIE provides comprehensive support for all its qualifications, including the Cambridge Pre-U. There are resources for teachers and candidates written by experts. CIE also endorses a range of materials from other publishers to give a choice of approach. More information on what is available for this particular syllabus can be found at **www.cie.org.uk**.

Syllabus updates

This syllabus is for teaching from 2011 and is valid for examination in 2013, 2014 and 2015.

If there are any changes to this syllabus, CIE will write to Centres to inform them. This syllabus will also be published annually on the CIE website (**www.cie.org.uk/cambridgepreu**). The version of the syllabus on the website should always be considered as the definitive version.

Further copies of this, or any other Cambridge Pre-U syllabus, can be obtained by either downloading from our website **www.cie.org.uk/cambridgepreu**

or contacting:
Customer Services, University of Cambridge International Examinations,
1 Hills Road, Cambridge CB1 2EU
Telephone: +44 (0)1223 553554
Fax: +44 (0)1223 553558
E-mail: **international@cie.org.uk**

Cambridge International Level 3 Pre-U Certificate

Further Mathematics

9795

Contents

Introduction

Cambridge Pre-U syllabuses aim to equip candidates with the skills required to make a success of their subsequent studies at university, involving not only a solid grounding in each specialist subject at an appropriate level, but also the ability to undertake independent and self-directed learning and to think laterally, critically and creatively. The Cambridge Pre-U curriculum is underpinned by a core set of educational principles:

- A programme of study which supports the development of well-informed, open and independent-minded individuals capable of applying their skills to meet the demands of the world as they will find it and over which they may have influence.
- A curriculum which retains the integrity of subject specialisms and which can be efficiently, effectively and reliably assessed, graded and reported to meet the needs of universities.
- A curriculum which is designed to recognise a wide range of individual talents, interests and abilities and which provides the depth and rigour required for a university degree course.
- A curriculum which encourages the acquisition of specific skills and abilities, in particular the skills of problem solving, creativity, critical thinking, team working and effective communication.
- The encouragement of 'deep understanding' in learning – where that deep understanding is likely to involve higher order cognitive activities.
- The development of a perspective which equips young people to understand a range of different cultures and ideas and to respond successfully to the opportunity for international mobility.

All Cambridge Pre-U Principal Subject syllabuses are linear. A candidate taking a Principal Subject must take all the components together at the end of the course in the same examination session.

Studying mathematics involves the acquisition of skills that can be applied in a wide range of contexts. The syllabus is designed to encourage teaching and learning which enable candidates to develop a positive attitude towards the subject by developing an understanding of mathematics and mathematical processes in a way that promotes confidence and enjoyment. Throughout this course candidates are expected to develop two parallel strands of mathematics: pure mathematics and applications of mathematics.

The study of mathematics encourages the development of logical thought and problem solving skills. Whilst it is a satisfying subject in its own right, mathematics is also a prerequisite for further study in an increasing range of subjects. For this reason, candidates following this course will be expected to apply their mathematical knowledge in the contexts of both mechanics and probability and will also be presented with less familiar scenarios. In conjunction with the Pre-U Mathematics syllabus, this syllabus provides a sound foundation in mathematics for higher education courses or other career pathways where mathematics is used extensively.

Aims

1. To enable candidates to develop a range of mathematical skills and techniques, appreciating their applications in a wide range of contexts, and to apply these techniques to problem solving in familiar and less familiar contexts.
2. To enable candidates to develop an understanding of how different branches of mathematics are connected.
3. To enable candidates to recognise how a situation may be represented mathematically and understand how mathematical models can be refined.
4. To encourage candidates to use mathematics as an effective means of communication, through the use of correct mathematical language and notation and through the construction of sustained logical arguments, including an appreciation of the limitations of calculator use in relation to obtaining exact solutions.

Scheme of assessment

Candidates taking the Cambridge Pre-U Principal Subject qualification in Further Mathematics take both papers together in the same session.

Component	Component name	Duration	Weighting (%)	Type of assessment
1	Further Pure Mathematics	3 hours	50	Written paper, externally set and marked
2	Further Applications of Mathematics	3 hours	50	Written paper, externally set and marked

Assessment objectives

Candidates will be expected to:

AO1	manipulate mathematical expressions accurately, round answers to an appropriate degree of accuracy and understand the limitations of solutions obtained using calculators
AO2	construct rigorous mathematical arguments and proofs through the use of precise statements and logical deduction, including extended arguments for problems presented in unstructured form
AO3	recall, select and apply their knowledge of mathematical facts, concepts and techniques in a variety of contexts
AO4	understand how mathematics can be used to model situations in the real world and solve problems in relation to both standard models and less familiar contexts, interpreting their results

Relationship between scheme of assessment and assessment objectives

The question papers will be constructed so that, for the assessment as a whole, the relative weightings of the assessment objectives fall within the ranges shown in the following table. The marks give an example of how the assessment objectives can be split across the two components.

Assessment objective	Component 1 (marks)	Component 2 (marks)	Weighting %
AO1	34	41	30–35
AO2	24	13	13–18
AO3	62	24	35–40
AO4	0	42	16–21
Total	120	120	240 / 100

Description of components

For both components, knowledge of the content of the Cambridge Pre-U Mathematics syllabus is assumed.

Component 1 – Further Pure Mathematics

Rational functions
Roots of polynomials
Complex numbers
De Moivre's theorem
Polar coordinates
Summation of series
Mathematical induction
Calculus
Hyperbolic functions
Differential equations
Vector geometry
Matrices
Groups

The written paper will consist of a mixture of short, medium and longer questions with a total of 120 marks. In addition to the topics listed, candidates will be expected to apply their knowledge of pure mathematics to questions set in less familiar contexts. Candidates will be expected to answer all questions.

Component 2 – Further Applications Of Mathematics

Mechanics
- Energy, work and power
- Motion in a circle
- Relative motion
- Elastic strings and springs
- Simple harmonic motion
- Further particle dynamics
- Linear motion under a variable force

Probability
- Poisson distribution
- Normal distribution as approximation
- Continuous random variables
- Linear combinations of random variables
- Estimation
- Probability generating functions
- Moment generating functions

The written paper will consist of a mixture of short, medium and longer questions with a total of 120 marks, with approximately equal marks for mechanics and probability. In addition to the topics listed, candidates will be expected to apply their knowledge of pure mathematics to questions set in less familiar contexts. Candidates will be expected to answer all questions.

Use of calculators

The use of scientific calculators will be permitted in all papers. Graphic calculators will not be permitted. Candidates will be expected to be aware of the limitations inherent in the use of calculators.

Mathematical tables and formulae

Candidates will be provided with a booklet of mathematical formulae and tables for use in the examination.

Curriculum content

Component 1: Further Pure Mathematics

Candidates should be able to:

Rational functions
- Sketch graphs of simple rational functions, including the determination of oblique asymptotes, in cases where the degree of the numerator and the denominator are at most 2 (detailed plotting of curves will not be expected, but sketches will generally be expected to show significant features, such as turning points, asymptotes and intersections with axes; the use of a discriminant in determining the set of values taken by the function is included).

Roots of polynomial equations
- Use the relations between the symmetric functions of the roots of polynomial equations and the coefficients.
- Use a substitution to obtain an equation whose roots are related in a simple way to those of the original equation.

Complex numbers
- Illustrate simple equations and inequalities involving complex numbers by means of loci in an Argand diagram, for example $|z - a| < b$, $|z - a| = |z - b|$ and $\arg(z - a) = b$, and combinations of such forms.
- Find square roots of complex numbers in cartesian form, by forming and solving appropriate quadratic equations.
- Recognise the modulus-argument form of a complex number, and convert a complex number in this form into cartesian form, and *vice versa*.
- Multiply and divide complex numbers in modulus-argument form, and calculate integral powers of complex numbers.
- Represent a complex number as $r\mathrm{e}^{i\theta}$.
- Use the relation $\mathrm{e}^{i\theta} = \cos\theta + i\sin\theta$.

De Moivre's theorem
- Understand de Moivre's theorem, for positive and negative integer exponent, in terms of the geometrical effect of multiplication and division of complex numbers.
- Use de Moivre's theorem to express trigonometric ratios of multiple angles in terms of powers of trigonometric ratios of the fundamental angle.
- Use expressions for $\sin\theta$ and $\cos\theta$ in terms of $\mathrm{e}^{i\theta}$, e.g. for expressing powers of $\sin\theta$ and $\cos\theta$ in terms of multiple angles or for summing series.
- Find and use the nth roots of complex numbers.

Polar coordinates

- Understand the relations between cartesian and polar coordinates (using the convention $r \geq 0$), and convert equations of curves from cartesian to polar form and *vice versa*.

- Sketch simple polar curves, for $0 \leq \theta < 2\pi$ or $-\pi < \theta \leq \pi$ or a subset of these intervals (detailed plotting of curves will not be expected, but sketches will generally be expected to show significant features, such as symmetry, the form of the curve at the pole and least/greatest values of r).

- Use the formula $\frac{1}{2}\int_{\alpha}^{\beta} r^2 \mathrm{d}\theta$ for the area of a sector in simple cases.

Summation of series

- Use the standard results for $\sum r$, $\sum r^2$ and $\sum r^3$ to find related sums.

- Use the method of differences to obtain the sum of a finite series, e.g. by expressing the general term in partial fractions.

- Recognise, by direct consideration of a sum to n terms, when a series is convergent, and find the sum to infinity in such cases.

Mathematical induction

- Use the method of mathematical induction to establish a given result (questions set may involve sums of series, divisibility or inequalities, for example).

- Recognise situations where conjecture based on a limited trial followed by inductive proof is a useful strategy, and carry this out in simple cases, e.g. to find the nth derivative of xe^x.

Calculus

- Derive and use reduction formulae for the evaluation of definite integrals in simple cases.

- Differentiate inverse trigonometric functions.

- Recognise integrals of functions of the form $(a^2 - x^2)^{-\frac{1}{2}}$ and $(a^2 + x^2)^{-1}$ and be able to integrate associated functions by using trigonometric substitutions.

- Derive and use Maclaurin and Taylor series.

- Calculate the arc length of a curve using cartesian, parametric or polar forms.

- Calculate the area of a surface of revolution about either the *x*- or *y*-axis using cartesian, parametric or polar forms.

Hyperbolic functions

- Understand the definitions of the hyperbolic functions sinh *x*, cosh *x*, tanh *x*, sech *x*, cosech *x* and coth *x* in terms of the exponential function.

- Sketch the graphs of hyperbolic functions.

- Use identities involving hyperbolic functions.

- Differentiate and integrate hyperbolic functions.

- Understand and use the definitions of the inverse hyperbolic functions and derive and use their logarithmic forms.

- Recognise integrals of functions of the form $(x^2 + a^2)^{-\frac{1}{2}}$ and $(x^2 - a^2)^{-\frac{1}{2}}$ and integrate associated functions by using hyperbolic substitutions.

Differential equations

- Find an integrating factor for a first-order linear differential equation, and use an integrating factor to find the general solution.

- Use a given substitution to reduce a first-order equation to linear form or to a form in which the variables are separable.

- Recall the meaning of the terms 'complementary function' and 'particular integral' in the context of linear differential equations, and use the fact that the general solution is the sum of the complementary function and a particular integral.

- Find the complementary function for a first- or second-order linear differential equation with constant coefficients.

- Recall the form of, and find, a particular integral for a first- or second-order linear differential equation in the cases where $ax^2 + bx + c$ or ae^{bx} or $a \cos px + b \sin px$ is a suitable form, and in other cases find the appropriate coefficient(s) given a suitable form of particular integral.

- Use initial conditions to find a particular solution to a differential equation, and interpret the solution in the context of a problem modelled by the differential equation.

- Use the Taylor series method to obtain series solutions to differential equations.

Vector geometry

- Calculate the vector product of two vectors.

- Understand and use the equation of a plane expressed in any of the forms $(\mathbf{r} - \mathbf{a}).\mathbf{n} = 0$ or $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$ or $ax + by + cz = d$.

- Find the intersection of two planes.

- Calculate the shortest distance of a point from a line or from a plane.

- Calculate the scalar triple product and use it to find the volume of a parallelepiped or a tetrahedron.

- Calculate the shortest distance between two skew lines.

Matrices

- Carry out operations of matrix addition, subtraction and multiplication, and recognise the terms null (or zero) matrix and identity (or unit) matrix.

- Recall the meaning of the terms 'singular' and 'non-singular' as applied to square matrices, and for 2×2 and 3×3 matrices, evaluate determinants and find inverses of non-singular matrices.

- Understand and use the result, for non-singular matrices, that $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$.

- Understand the use of 2×2 matrices to represent certain geometrical transformations in the *x-y* plane, and in particular:

  - Recognise that the matrix product **AB** represents the transformation that results from the transformation represented by **B** followed by the transformation represented by **A**.

  - Recall how the area scale-factor of a transformation is related to the determinant of the corresponding matrix.

  - Find the matrix that represents a given transformation or sequence of transformations (understanding of the terms 'rotation', 'reflection', 'enlargement', 'stretch' and 'shear' will be required).

- Formulate a problem involving the solution of 2 linear simultaneous equations in 2 unknowns, or 3 equations in 3 unknowns, as a problem involving the solution of a matrix equation, and *vice versa*.
- Understand the cases that may arise concerning the consistency or inconsistency of 2 or 3 linear simultaneous equations, relate them to the singularity or otherwise of the corresponding square matrix, and solve consistent systems.

Groups

- Recall that a group consists of a set of elements together with a binary operation which is closed and associative, for which an identity exists in the set, and for which every element has an inverse in the set.
- Use the basic group properties to show that a given structure is, or is not, a group (questions may be set on, for example, groups of matrices, transformations, permutations, integers modulo *n*).
- Use algebraic methods to establish properties in abstract groups in easy cases, e.g. to show that any group in which every element is self-inverse is commutative (abelian).
- Recall the meaning of the term 'order' as applied both to groups and to elements of a group, and determine the order of elements in a given group.
- Understand the idea of a subgroup of a given group, find subgroups in simple cases, and show that given subsets are, or are not, (proper) subgroups.
- Recall and apply Lagrange's theorem concerning the order of a subgroup of a finite group (the proof of the theorem is not required).
- Recall the meaning of the term 'cyclic' as applied to groups, and show familiarity with the structure of finite groups up to order 7 (questions on groups of higher order are not excluded, but no particular prior knowledge of such groups is expected).
- Understand the idea of isomorphism between groups and determine whether given finite groups are, or are not, isomorphic.

Component 2: Further Applications of Mathematics

Candidates should be able to:

MECHANICS

Energy, work and power

- Understand the concept of the work done by a force, and calculate the work done by a constant force when its point of application undergoes a displacement not necessarily parallel to the force.
- Understand the concepts of gravitational potential energy and kinetic energy, and use appropriate formulae.
- Understand and use the relationship between the change in energy of a system and the work done by the external forces, and use in appropriate cases the principle of conservation of energy.
- Use the definition of power as the rate at which a force does work, and use the relationship between power, force and velocity for a force acting in the direction of motion.

Motion in a circle

- Understand the concept of angular speed for a particle moving in a circle, and use the relation $v = r\omega$.
- Understand that the acceleration of a particle moving in a circle with constant speed is directed towards the centre of the circle, and use the formulae $a = r\omega^2$ and $a = \dfrac{v^2}{r}$.
- Solve problems which can be modelled by the motion of a particle moving in a horizontal circle with constant speed.
- Use formulae for the radial and transverse components of acceleration for a particle moving in a circle with variable speed.
- Solve problems which can be modelled as a particle, or a pair of connected particles, moving without loss of energy in a vertical circle (including the determination of points where circular motion breaks down, e.g. through loss of contact with a surface or a string becoming slack).

Relative motion

- Understand the concept of the displacement or velocity of one point relative to another, and the representation of these quantities by vectors (including the use of $_P\mathbf{V}_Q + {}_Q\mathbf{V}_R = {}_P\mathbf{V}_R$, where $_P\mathbf{V}_Q$ denotes the velocity of P relative to Q, etc).
- Solve problems involving relative displacements and velocities, including interception or the determination of the course for closest approach.

Elastic strings and springs

- Use Hooke's law as a model relating the force in an elastic string or spring to the extension or compression, and understand the term 'modulus of elasticity'.
- Use the formula for the elastic potential energy stored in a string or spring.
- Solve problems involving forces due to elastic strings or springs, including those where considerations of work and energy are needed.

Simple harmonic motion

- Recall a definition of SHM, understand the concepts of period and amplitude, and use standard SHM formulae in solving problems.

- Set up the differential equation of motion in problems leading to SHM, quote appropriate forms of solution, and identify the period and amplitude of the motion.

- Use Newton's second law, together with the approximation $\sin\theta \approx \theta$, to show that small oscillations of a simple pendulum may be modelled as SHM, solve associated problems and understand the limitations of this model.

Further particle dynamics

- Understand the vector nature of impulse and momentum for motion in two dimensions.

- Solve problems that may be modelled as the oblique impact of two smooth spheres or as the oblique impact of a smooth sphere with a fixed surface.

- Solve harder problems involving projectile motion, for example projectiles launched and landing on an inclined plane and points accessible to the projectile.

Linear motion under a variable force

- Use $\dfrac{\mathrm{d}x}{\mathrm{d}t}$ for velocity, and $\dfrac{\mathrm{d}v}{\mathrm{d}t}$ or $v\,\dfrac{\mathrm{d}v}{\mathrm{d}x}$ for acceleration, as appropriate.

- Solve problems which can be modelled as the linear motion of a particle under the action of a variable force, by setting up and solving an appropriate differential equation.

PROBABILITY

Poisson distribution

- Calculate probabilities for the distribution Po(λ), both directly from the formula and also by using tables of cumulative Poisson probabilities.

- Use the result that if $X \sim$ Po(λ) then the mean and variance of X are each equal to λ.

- Understand informally the relevance of the Poisson distribution to the distribution of random events, and use the Poisson distribution as a model.

- Use the Poisson distribution as an approximation to the binomial distribution where appropriate.

Normal distribution as approximation

- Recall conditions under which the normal distribution can be used as an approximation to the binomial distribution, and use this approximation, with a continuity correction, in solving problems.

- Use the normal distribution, with continuity correction, as an approximation to the Poisson distribution where appropriate.

Continuous random variables

- Understand the concept of a continuous random variable, and recall and use properties of a probability density function (defined over a single interval or piecewise).

- Use a probability density function to solve problems involving probabilities, and to calculate the mean and variance of a continuous random variable.

- Use, in simple cases, the general result $E(g(X)) = \int f(x)g(x)\,dx$, where $f(x)$ is the probability density function of the continuous random variable X and $g(X)$ is a function of X.

- Understand and use the relationship between the probability density function and the cumulative distribution function, and use either to evaluate the median, quartiles and other percentiles.

- Use cumulative distribution functions of related variables in simple cases, e.g. given the c.d.f. of a variable X, to find the c.d.f. and hence the p.d.f. of Y, where $Y = X^3$.

Linear combinations of random variables

- Use, in the course of solving problems, the results that:
  - $E(aX + b) = aE(X) + b$ and $Var(aX + b) = a^2\,Var(X)$.
  - $E(aX + bY) = aE(X) + bE(Y)$.
  - $Var(aX + bY) = a^2 Var(X) + b^2 Var(Y)$ for independent X and Y.
  - If X has a normal distribution then so does $aX + b$.
  - If X and Y are independent random variables with normal distributions then $aX + bY$ has a normal distribution.
  - If X and Y are independent random variables with Poisson distributions then $X + Y$ has a Poisson distribution.

- Recognise that the sample mean $\overline{X}$ has a normal distribution if X is normally distributed, and has an approximately normal distribution in other cases for sufficiently large sample size (Central Limit Theorem), and use the results $E(\overline{X}) = \mu$, $Var(\overline{X}) = \dfrac{\sigma^2}{n}$.

Estimation

- Understand the distinction between a sample and a population, and appreciate the benefits of randomness in choosing samples.

- Understand that an estimator of a parameter of a distribution is a random variable, and understand the meaning of 'biased' and 'unbiased' estimators (both discrete and continuous distributions are included).

- Determine whether a given estimator is biased or unbiased, and construct estimators in simple cases.

- Calculate unbiased estimates of the population mean and variance from a random sample, using either raw or summarised data.

- Determine a confidence interval for a population mean, using a normal distribution in the context of:
  - A random sample drawn from a normal population of known variance.
  - A large random sample, using the Central Limit Theorem, and an unbiased estimate of the variance of the population derived from the sample.

- Determine, from a large random sample, an approximate confidence interval for a population proportion.

- Use a *t*-distribution with the appropriate number of degrees of freedom, in the context of a small random sample drawn from a normal population of unknown variance to determine a confidence interval for the population mean.

- Calculate a pooled estimate of a population variance based on the data from two random samples.

- Calculate a confidence interval for a difference of population means, using a normal or a t-distribution, as appropriate.

- Comment about assumptions that have been made, for example randomness or normality of a distribution, and comment critically on statements in the light of the confidence interval that has been calculated.

Probability generating functions

- Understand the concept of a probability generating function and construct and use the probability generating function for given distributions (including the discrete uniform, binomial, geometric and Poisson).

- Use formulae for the mean and variance of a discrete random variable in terms of its probability generating function, and use these formulae to calculate the mean and variance of probability distributions.

- Use the result that the probability generating function of the sum of independent random variables is the product of the probability generating functions of those random variables.

Moment generating functions

- Understand the concept of a moment generating function for both discrete and continuous random variables, construct and use the moment generating function for given distributions.

- Use the moment generating function of a given distribution to find the mean and variance.

- Use the result that the moment generating function of the sum of independent random variables is the product of the moment generating functions of those random variables.

Mathematical formulae and statistical tables

PURE MATHEMATICS

Mensuration

Surface area of sphere = $4\pi r^2$

Area of curved surface of cone = $\pi r \times$ slant height

Trigonometry

$a^2 = b^2 + c^2 - 2bc \cos A$

Arithmetic series

$u_n = a + (n-1)d$

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$

Geometric series

$u_n = ar^{n-1}$

$S_n = \dfrac{a(1-r^n)}{1-r}$

$S_\infty = \dfrac{a}{1-r}$ for $|r| < 1$

Summations

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$$

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

Binomial Series

$$\binom{n}{r} + \binom{n}{r+1} = \binom{n+1}{r+1}$$

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n, \ (n \in \mathbb{N}), \text{ where } \binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1.2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1.2\ldots r}x^r + \ldots \ (|x| < 1, n \in \mathbb{R})$$

Logarithms and exponentials

$e^{x \ln a} = a^x$

Complex Numbers

$\{r(\cos\theta + i\sin\theta)\}^n = r^n(\cos n\theta + i\sin n\theta)$

$e^{i\theta} = \cos\theta + i\sin\theta$

The roots of $z^n = 1$ are given by $z = e^{\frac{2\pi k i}{n}}$, for $k = 0, 1, 2, \ldots, n-1$

Maclaurin's Series

$$f(x) = f(0) + x\,f'(0) + \frac{x^2}{2!}\,f''(0) + \dots + \frac{x^r}{r!}\,f^{(r)}(0) + \dots$$

$$e^x = \exp(x) = 1 + x + \frac{x^2}{2!} + \dots + \frac{x^r}{r!} + \dots \quad \text{for all } x$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + (-1)^{r+1}\frac{x^r}{r} + \dots \quad (-1 < x \le 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \dots \quad \text{for all } x$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots + (-1)^r \frac{x^{2r}}{(2r)!} + \dots \quad \text{for all } x$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots + (-1)^r \frac{x^{2r+1}}{2r+1} + \dots \quad (-1 \le x \le 1)$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots + \frac{x^{2r+1}}{(2r+1)!} + \dots \quad \text{for all } x$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots + \frac{x^{2r}}{(2r)!} + \dots \quad \text{for all } x$$

$$\tanh^{-1} x = x + \frac{x^3}{3} + \frac{x^5}{5} + \dots + \frac{x^{2r+1}}{2r+1} + \dots \quad (-1 < x < 1)$$

Hyperbolic Functions

$$\cosh^2 x - \sinh^2 x = 1$$
$$\sinh 2x = 2 \sinh x \cosh x$$
$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

$$\cosh^{-1} x = \ln\{x + \sqrt{x^2 - 1}\} \quad (x \ge 1)$$

$$\sinh^{-1} x = \ln\{x + \sqrt{x^2 + 1}\}$$

$$\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \quad (|x| < 1)$$

Coordinate Geometry

The perpendicular distance from (h, k) to $ax + by + c = 0$ is $\dfrac{|ah + bk + c|}{\sqrt{a^2 + b^2}}$

The acute angle between lines with gradients m_1 and m_2 is $\tan^{-1}\left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$

Trigonometric Identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad (A \pm B \ne (k + \tfrac{1}{2})\pi)$$

For $t = \tan\frac{1}{2}A$: $\sin A = \dfrac{2t}{1 + t^2}$, $\cos A = \dfrac{1 - t^2}{1 + t^2}$

$$\sin A + \sin B = 2 \sin\frac{A+B}{2} \cos\frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos\frac{A+B}{2} \sin\frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos\frac{A+B}{2} \cos\frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin\frac{A+B}{2} \sin\frac{A-B}{2}$$

Vectors

The resolved part of **a** in the direction of **b** is $\dfrac{\mathbf{a}.\mathbf{b}}{|\mathbf{b}|}$

The point dividing AB in the ratio $\lambda : \mu$ is $\dfrac{\mu\mathbf{a} + \lambda\mathbf{b}}{\lambda + \mu}$

Vector product: $\mathbf{a} \times \mathbf{b} = |\mathbf{a}||\mathbf{b}| \sin\theta\,\hat{\mathbf{n}} = \begin{vmatrix} \mathbf{i} & a_1 & b_1 \\ \mathbf{j} & a_2 & b_2 \\ \mathbf{k} & a_3 & b_3 \end{vmatrix} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$

If A is the point with position vector $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and the direction vector **b** is given by
$\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$, then the straight line through A with direction vector **b** has cartesian equation

$$\frac{x - a_1}{b_1} = \frac{y - a_2}{b_2} = \frac{z - a_3}{b_3} \ (= \lambda)$$

The plane through A with normal vector $\mathbf{n} = n_1\mathbf{i} + n_2\mathbf{j} + n_3\mathbf{k}$ has cartesian equation
$n_1x + n_2y + n_3z + d = 0$ where $d = -\mathbf{a}.\mathbf{n}$

The plane through non-collinear points A, B and C has vector equation
$\mathbf{r} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a}) + \mu(\mathbf{c} - \mathbf{a}) = (1 - \lambda - \mu)\mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$

The plane through the point with position vector **a** and parallel to **b** and **c** has equation $\mathbf{r} = \mathbf{a} + s\mathbf{b} + t\mathbf{c}$

The perpendicular distance of (α, β, γ) from $n_1x + n_2y + n_3z + d = 0$ is $\dfrac{|n_1\alpha + n_2\beta + n_3\gamma + d|}{\sqrt{n_1^2 + n_2^2 + n_3^2}}$

Matrix Transformations

Anticlockwise rotation through θ about O: $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

Reflection in the line $y = (\tan\theta)x$: $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k\sec^2 kx$
$\sin^{-1} x$	$\dfrac{1}{\sqrt{1 - x^2}}$
$\cos^{-1} x$	$-\dfrac{1}{\sqrt{1 - x^2}}$
$\tan^{-1} x$	$\dfrac{1}{1 + x^2}$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\operatorname{sech}^2 x$
$\sinh^{-1} x$	$\dfrac{1}{\sqrt{1 + x^2}}$
$\cosh^{-1} x$	$\dfrac{1}{\sqrt{x^2 - 1}}$
$\tanh^{-1} x$	$\dfrac{1}{1 - x^2}$

Integration *(+ constant; a > 0 where relevant)*

f(x)	$\int$ f(x) dx
$\sec^2 kx$	$\dfrac{1}{k}\tan kx$
$\tan x$	$\ln\lvert\sec x\rvert$
$\cot x$	$\ln\lvert\sin x\rvert$
$\operatorname{cosec} x$	$-\ln\lvert\operatorname{cosec} x + \cot x\rvert = \ln\lvert\tan(\tfrac{1}{2}x)\rvert$
$\sec x$	$\ln\lvert\sec x + \tan x\rvert = \ln\lvert\tan(\tfrac{1}{2}x + \tfrac{1}{4}\pi)\rvert$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\ln\cosh x$
$\dfrac{1}{\sqrt{a^2 - x^2}}$	$\sin^{-1}\left(\dfrac{x}{a}\right) \quad (\lvert x\rvert < a)$
$\dfrac{1}{a^2 + x^2}$	$\dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right)$
$\dfrac{1}{\sqrt{x^2 - a^2}}$	$\cosh^{-1}\left(\dfrac{x}{a}\right) = \ln\left\{x + \sqrt{x^2 - a^2}\right\} \quad (x > a)$
$\dfrac{1}{\sqrt{a^2 + x^2}}$	$\sinh^{-1}\left(\dfrac{x}{a}\right) = \ln\left\{x + \sqrt{x^2 + a^2}\right\}$
$\dfrac{1}{a^2 - x^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{a + x}{a - x}\right\rvert = \dfrac{1}{a}\tanh^{-1}\left(\dfrac{x}{a}\right) \quad (\lvert x\rvert < a)$
$\dfrac{1}{x^2 - a^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{x - a}{x + a}\right\rvert$

$$\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$$

Area of a sector

$$A = \tfrac{1}{2}\int r^2\,\mathrm{d}\theta \quad \text{(polar coordinates)}$$

$$A = \tfrac{1}{2}\int\left(x\frac{\mathrm{d}y}{\mathrm{d}t} - y\frac{\mathrm{d}x}{\mathrm{d}t}\right)\mathrm{d}t \quad \text{(parametric form)}$$

Arc Length

$$s = \int\sqrt{1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\,\mathrm{d}x \qquad \text{(cartesian coordinates)}$$

$$s = \int\sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\,\mathrm{d}t \qquad \text{(parametric form)}$$

$$s = \int\sqrt{r^2 + \left(\frac{\mathrm{d}r}{\mathrm{d}\theta}\right)^2}\,\mathrm{d}\theta \qquad \text{(polar form)}$$

Surface area of revolution

$$S_x = 2\pi \int y \, ds$$

$$S_y = 2\pi \int x \, ds$$

Numerical Solution of Equations

The Newton-Raphson iteration for solving f$(x) = 0$: $x_{n+1} = x_n - \dfrac{\text{f}(x_n)}{\text{f}'(x_n)}$

MECHANICS

Motion in a circle

Transverse velocity: $v = r\dot{\theta}$

Transverse acceleration: $\dot{v} = r\ddot{\theta}$

Radial acceleration: $-r\dot{\theta}^2 = -\dfrac{v^2}{r}$

PROBABILITY

Probability

$\text{P}(A \cup B) = \text{P}(A) + \text{P}(B) - \text{P}(A \cap B)$

$\text{P}(A \cap B) = \text{P}(A)\,\text{P}(B \mid A)$

$$\text{P}(A \mid B) = \frac{\text{P}(B \mid A)\,\text{P}(A)}{\text{P}(B \mid A)\,\text{P}(A) + \text{P}(B \mid A')\,\text{P}(A')}$$

Bayes' Theorem: $\text{P}(A_j \mid B) = \dfrac{\text{P}(A_j)\,\text{P}(B \mid A_j)}{\Sigma\text{P}(A_i)\,\text{P}(B \mid A_i)}$

Discrete distributions

For a discrete random variable X taking values x_i with probabilities p_i

Expectation (mean): $\text{E}(X) = \mu = \Sigma x_i p_i$

Variance: $\text{Var}(X) = \sigma^2 = \Sigma(x_i - \mu)^2 p_i = \Sigma x_i^2 p_i - \mu^2$

For a function g(X) : $\text{E}(\text{g}(X)) = \Sigma\, \text{g}(x_i)\, p_i$

The probability generating function of X is $\text{G}_X(t) = \text{E}(t^X)$, and $\text{E}(X) = \text{G}'_X(1)$,

$\text{Var}(X) = \text{G}''_X(1) + \text{G}'_X(1) - \{\text{G}'_X(1)\}^2$

For $Z = X + Y$, where X and Y are independent: $\text{G}_Z(t) = \text{G}_X(t)\,\text{G}_Y(t)$

Standard discrete distributions

Distribution of X	P$(X = x)$	Mean	Variance	P.G.F.
Binomial B(n, p)	$\dbinom{n}{x} p^x (1-p)^{n-x}$	np	$np(1-p)$	$(1 - p + pt)^n$
Poisson Po(λ)	$\text{e}^{-\lambda} \dfrac{\lambda^x}{x!}$	λ	λ	$\text{e}^{\lambda(t-1)}$
Geometric Geo(p) on 1, 2, ...	$p(1-p)^{x-1}$	$\dfrac{1}{p}$	$\dfrac{1-p}{p^2}$	$\dfrac{pt}{1 - (1-p)t}$

Continuous distributions

For a continuous random variable X having probability density function f

Expectation (mean): $E(X) = \mu = \int x\, f(x)\, dx$

Variance: $\text{Var}(X) = \sigma^2 = \int (x - \mu)^2\, f(x)\, dx = \int x^2\, f(x)\, dx - \mu^2$

For a function $g(X)$: $E(g(X)) = \int g(x)\, f(x)\, dx$

Cumulative distribution function: $F(x) = P(X \le x) = \int_{-\infty}^{x} f(t)\, dt$

The moment generating function of X is $M_X(t) = E(e^{tX})$ and $E(X) = M_X'(0)$, $E(X^n) = M_X^{(n)}(0)$,
$\text{Var}(X) = M_X''(0) - \{M_X'(0)\}^2$
For $Z = X + Y$, where X and Y are independent: $M_Z(t) = M_X(t)\, M_Y(t)$

Standard continuous distributions

Distribution of X	P.D.F.	Mean	Variance	M.G.F
Uniform (Rectangular) on $[a, b]$	$\dfrac{1}{b-a}$	$\dfrac{1}{2}(a+b)$	$\dfrac{1}{12}(b-a)^2$	$\dfrac{e^{bt} - e^{at}}{(b-a)t}$
Exponential	$\lambda\, e^{-\lambda x}$	$\dfrac{1}{\lambda}$	$\dfrac{1}{\lambda^2}$	$\dfrac{\lambda}{\lambda - t}$
Normal $N(\mu, \sigma^2)$	$\dfrac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$	μ	σ^2	$e^{\mu t + \frac{1}{2}\sigma^2 t^2}$

Expectation algebra

For independent random variables X and Y

$E(XY) = E(X)\, E(Y)$, $\text{Var}(aX \pm bY) = a^2\, \text{Var}(X) + b^2\, \text{Var}(Y)$

Sampling Distributions

For a random sample $X_1, X_2, \ldots, X_n$ of n independent observations from a distribution having mean μ and variance σ^2

$\overline{X}$ is an unbiased estimator of μ, with $\text{Var}(\overline{X}) = \dfrac{\sigma^2}{n}$

S^2 is an unbiased estimator of σ^2, where $S^2 = \dfrac{\Sigma(X_i - \overline{X})^2}{n - 1}$

For a random sample of n observations from $N(\mu, \sigma^2)$

$\dfrac{\overline{X} - \mu}{\sigma / \sqrt{n}} \sim N(0,1)$

$\dfrac{\overline{X} - \mu}{S / \sqrt{n}} \sim t_{n-1}$ (also valid in matched-pairs situations)

If X is the observed number of successes in n independent Bernoulli trials in each of which the probability of success is p, and $Y = \dfrac{X}{n}$, then $E(Y) = p$ and $\text{Var}(Y) = \dfrac{p(1-p)}{n}$

For a random sample of n_x observations from $N(\mu_x, \sigma_x^2)$ and, independently, a random sample of n_y observations from $N(\mu_y, \sigma_y^2)$

$$\frac{(\overline{X} - \overline{Y}) - (\mu_x - \mu_y)}{\sqrt{\dfrac{\sigma_x^2}{n_x} + \dfrac{\sigma_y^2}{n_y}}} \sim N(0,1)$$

If $\sigma_x^2 = \sigma_y^2 = \sigma^2$ (unknown) then $\dfrac{(\overline{X} - \overline{Y}) - (\mu_x - \mu_y)}{\sqrt{S_p^2\left(\dfrac{1}{n_x} + \dfrac{1}{n_y}\right)}} \sim t_{n_x + n_y - 2}$, where $S_p^2 = \dfrac{(n_x - 1)S_x^2 + (n_y - 1)S_y^2}{n_x + n_y - 2}$

Correlation and Regression

For a set of n pairs of values (x_i, y_i)

$$S_{xx} = \Sigma(x_i - \overline{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}$$

$$S_{yy} = \Sigma(y_i - \overline{y})^2 = \Sigma y_i^2 - \frac{(\Sigma y_i)^2}{n}$$

$$S_{xy} = \Sigma(x_i - \overline{x})(y_i - \overline{y}) = \Sigma x_i y_i - \frac{(\Sigma x_i)(\Sigma y_i)}{n}$$

The product-moment correlation coefficient is

$$r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{\Sigma(x_i - \overline{x})(y_i - \overline{y})}{\sqrt{\left\{\Sigma(x_i - \overline{x})^2\right\}\left\{\Sigma(y_i - \overline{y})^2\right\}}} = \frac{\Sigma x_i y_i - \dfrac{(\Sigma x_i)(\Sigma y_i)}{n}}{\sqrt{\left(\Sigma x_i^2 - \dfrac{(\Sigma x_i)^2}{n}\right)\left(\Sigma y_i^2 - \dfrac{(\Sigma y_i)^2}{n}\right)}}$$

The regression coefficient of y on x is $b = \dfrac{S_{xy}}{S_{xx}} = \dfrac{\Sigma(x_i - \overline{x})(y_i - \overline{y})}{\Sigma(x_i - \overline{x})^2}$

Least squares regression line of y on x is $y = a + bx$ where $a = \overline{y} - b\overline{x}$

CUMULATIVE BINOMIAL PROBABILITIES

n = 5

p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
x = 0	0.7738	0.5905	0.4437	0.4019	0.3277	0.2373	0.1681	0.1317	0.1160	0.0778	0.0503	0.0313	0.0185	0.0102	0.0053	0.0041	0.0024	0.0010	0.0003	0.0001	0.0001	0.0000	0.0000
1	0.9774	0.9185	0.8352	0.8038	0.7373	0.6328	0.5282	0.4609	0.4284	0.3370	0.2562	0.1875	0.1312	0.0870	0.0540	0.0453	0.0308	0.0156	0.0067	0.0033	0.0022	0.0005	0.0000
2	0.9988	0.9914	0.9734	0.9645	0.9421	0.8965	0.8369	0.7901	0.7648	0.6826	0.5931	0.5000	0.4069	0.3174	0.2352	0.2099	0.1631	0.1035	0.0579	0.0355	0.0266	0.0086	0.0012
3	1.0000	0.9995	0.9978	0.9967	0.9933	0.9844	0.9692	0.9547	0.9460	0.9130	0.8688	0.8125	0.7438	0.6630	0.5716	0.5391	0.4718	0.3672	0.2627	0.1962	0.1648	0.0815	0.0226
4	1.0000	1.0000	0.9999	0.9999	0.9997	0.9990	0.9976	0.9959	0.9947	0.9898	0.9815	0.9688	0.9497	0.9222	0.8840	0.8683	0.8319	0.7627	0.6723	0.5981	0.5563	0.4095	0.2262
5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

n = 6

p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
x = 0	0.7351	0.5314	0.3771	0.3349	0.2621	0.1780	0.1176	0.0878	0.0754	0.0467	0.0277	0.0156	0.0083	0.0041	0.0018	0.0014	0.0007	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000
1	0.9672	0.8857	0.7765	0.7368	0.6554	0.5339	0.4202	0.3512	0.3191	0.2333	0.1636	0.1094	0.0692	0.0410	0.0223	0.0178	0.0109	0.0046	0.0016	0.0007	0.0004	0.0001	0.0000
2	0.9978	0.9842	0.9527	0.9377	0.9011	0.8306	0.7443	0.6804	0.6471	0.5443	0.4415	0.3438	0.2553	0.1792	0.1174	0.1001	0.0705	0.0376	0.0170	0.0087	0.0059	0.0013	0.0001
3	0.9999	0.9987	0.9941	0.9913	0.9830	0.9624	0.9295	0.8999	0.8826	0.8208	0.7447	0.6563	0.5585	0.4557	0.3529	0.3196	0.2557	0.1694	0.0989	0.0623	0.0473	0.0159	0.0022
4	1.0000	0.9999	0.9996	0.9993	0.9984	0.9954	0.9891	0.9822	0.9777	0.9590	0.9308	0.8906	0.8364	0.7667	0.6809	0.6488	0.5798	0.4661	0.3446	0.2632	0.2235	0.1143	0.0328
5	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9993	0.9986	0.9982	0.9959	0.9917	0.9844	0.9723	0.9533	0.9246	0.9122	0.8824	0.8220	0.7379	0.6651	0.6229	0.4686	0.2649
6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

n = 7

p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
x = 0	0.6983	0.4783	0.3206	0.2791	0.2097	0.1335	0.0824	0.0585	0.0490	0.0280	0.0152	0.0078	0.0037	0.0016	0.0006	0.0005	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.9556	0.8503	0.7166	0.6698	0.5767	0.4449	0.3294	0.2634	0.2338	0.1586	0.1024	0.0625	0.0357	0.0188	0.0090	0.0069	0.0038	0.0013	0.0004	0.0001	0.0001	0.0000	0.0000
2	0.9962	0.9743	0.9262	0.9042	0.8520	0.7564	0.6471	0.5706	0.5323	0.4199	0.3164	0.2266	0.1529	0.0963	0.0556	0.0453	0.0288	0.0129	0.0047	0.0020	0.0012	0.0002	0.0000
3	0.9998	0.9973	0.9879	0.9824	0.9667	0.9294	0.8740	0.8267	0.8002	0.7102	0.6083	0.5000	0.3917	0.2898	0.1998	0.1733	0.1260	0.0706	0.0333	0.0176	0.0121	0.0027	0.0002
4	1.0000	0.9998	0.9988	0.9980	0.9953	0.9871	0.9712	0.9547	0.9444	0.9037	0.8471	0.7734	0.6836	0.5801	0.4677	0.4294	0.3529	0.2436	0.1480	0.0958	0.0738	0.0257	0.0038
5	1.0000	1.0000	0.9999	0.9999	0.9996	0.9987	0.9962	0.9931	0.9910	0.9812	0.9643	0.9375	0.8976	0.8414	0.7662	0.7366	0.6706	0.5551	0.4233	0.3302	0.2834	0.1497	0.0444
6	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9995	0.9994	0.9984	0.9963	0.9922	0.9848	0.9720	0.9510	0.9415	0.9176	0.8665	0.7903	0.7209	0.6794	0.5217	0.3017
7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

n = 8

p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
x = 0	0.6634	0.4305	0.2725	0.2326	0.1678	0.1001	0.0576	0.0390	0.0319	0.0168	0.0084	0.0039	0.0017	0.0007	0.0002	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.9428	0.8131	0.6572	0.6047	0.5033	0.3671	0.2553	0.1951	0.1691	0.1064	0.0632	0.0352	0.0181	0.0085	0.0036	0.0026	0.0013	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000
2	0.9942	0.9619	0.8948	0.8652	0.7969	0.6785	0.5518	0.4682	0.4278	0.3154	0.2201	0.1445	0.0885	0.0498	0.0253	0.0197	0.0113	0.0042	0.0012	0.0004	0.0002	0.0000	0.0000
3	0.9996	0.9950	0.9786	0.9693	0.9437	0.8862	0.8059	0.7414	0.7064	0.5941	0.4770	0.3633	0.2604	0.1737	0.1061	0.0879	0.0580	0.0273	0.0104	0.0046	0.0029	0.0004	0.0000
4	1.0000	0.9996	0.9971	0.9954	0.9896	0.9727	0.9420	0.9121	0.8939	0.8263	0.7396	0.6367	0.5230	0.4059	0.2936	0.2586	0.1941	0.1138	0.0563	0.0307	0.0214	0.0050	0.0004
5	1.0000	1.0000	0.9998	0.9996	0.9988	0.9958	0.9887	0.9803	0.9747	0.9502	0.9115	0.8555	0.7799	0.6846	0.5722	0.5318	0.4482	0.3215	0.2031	0.1348	0.1052	0.0381	0.0058
6	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9987	0.9974	0.9964	0.9915	0.9819	0.9648	0.9368	0.8936	0.8309	0.8049	0.7447	0.6329	0.4967	0.3953	0.3428	0.1869	0.0572
7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9998	0.9993	0.9983	0.9961	0.9916	0.9832	0.9681	0.9610	0.9424	0.8999	0.8322	0.7674	0.7275	0.5695	0.3366
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

CUMULATIVE BINOMIAL PROBABILITIES

n = 9

p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
x = 0	0.6302	0.3874	0.2316	0.1938	0.1342	0.0751	0.0404	0.0260	0.0207	0.0101	0.0046	0.0020	0.0008	0.0003	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.9288	0.7748	0.5995	0.5427	0.4362	0.3003	0.1960	0.1431	0.1211	0.0705	0.0385	0.0195	0.0091	0.0038	0.0014	0.0010	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.9916	0.9470	0.8591	0.8217	0.7382	0.6007	0.4628	0.3772	0.3373	0.2318	0.1495	0.0898	0.0498	0.0250	0.0112	0.0083	0.0043	0.0013	0.0003	0.0001	0.0000	0.0000	0.0000
3	0.9994	0.9917	0.9661	0.9520	0.9144	0.8343	0.7297	0.6503	0.6089	0.4826	0.3614	0.2539	0.1658	0.0994	0.0536	0.0424	0.0253	0.0100	0.0031	0.0011	0.0006	0.0001	0.0000
4	1.0000	0.9991	0.9944	0.9910	0.9804	0.9511	0.9012	0.8552	0.8283	0.7334	0.6214	0.5000	0.3786	0.2666	0.1717	0.1448	0.0988	0.0489	0.0196	0.0090	0.0056	0.0009	0.0000
5	1.0000	0.9999	0.9994	0.9989	0.9969	0.9900	0.9747	0.9576	0.9464	0.9006	0.8342	0.7461	0.6386	0.5174	0.3911	0.3497	0.2703	0.1657	0.0856	0.0480	0.0339	0.0083	0.0006
6	1.0000	1.0000	1.0000	0.9999	0.9997	0.9987	0.9957	0.9917	0.9888	0.9750	0.9502	0.9102	0.8505	0.7682	0.6627	0.6228	0.5372	0.3993	0.2618	0.1783	0.1409	0.0530	0.0084
7	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9990	0.9986	0.9962	0.9909	0.9805	0.9615	0.9295	0.8789	0.8569	0.8040	0.6997	0.5638	0.4573	0.4005	0.2252	0.0712
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997	0.9992	0.9980	0.9954	0.9899	0.9793	0.9740	0.9596	0.9249	0.8658	0.8062	0.7684	0.6126	0.3698
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

n = 10

p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
x = 0	0.5987	0.3487	0.1969	0.1615	0.1074	0.0563	0.0282	0.0173	0.0135	0.0060	0.0025	0.0010	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.9139	0.7361	0.5443	0.4845	0.3758	0.2440	0.1493	0.1040	0.0860	0.0464	0.0233	0.0107	0.0045	0.0017	0.0005	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.9885	0.9298	0.8202	0.7752	0.6778	0.5256	0.3828	0.2991	0.2616	0.1673	0.0996	0.0547	0.0274	0.0123	0.0048	0.0034	0.0016	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000
3	0.9990	0.9872	0.9500	0.9303	0.8791	0.7759	0.6496	0.5593	0.5138	0.3823	0.2660	0.1719	0.1020	0.0548	0.0260	0.0197	0.0106	0.0035	0.0009	0.0003	0.0001	0.0000	0.0000
4	0.9999	0.9984	0.9901	0.9845	0.9672	0.9219	0.8497	0.7869	0.7515	0.6331	0.5044	0.3770	0.2616	0.1662	0.0949	0.0766	0.0473	0.0197	0.0064	0.0024	0.0014	0.0001	0.0000
5	1.0000	0.9999	0.9986	0.9976	0.9936	0.9803	0.9527	0.9234	0.9051	0.8338	0.7384	0.6230	0.4956	0.3669	0.2485	0.2131	0.1503	0.0781	0.0328	0.0155	0.0099	0.0016	0.0001
6	1.0000	1.0000	0.9999	0.9997	0.9991	0.9965	0.9894	0.9803	0.9740	0.9452	0.8980	0.8281	0.7340	0.6177	0.4862	0.4407	0.3504	0.2241	0.1209	0.0697	0.0500	0.0128	0.0010
7	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9984	0.9966	0.9952	0.9877	0.9726	0.9453	0.9004	0.8327	0.7384	0.7009	0.6172	0.4744	0.3222	0.2248	0.1798	0.0702	0.0115
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9995	0.9983	0.9955	0.9893	0.9767	0.9536	0.9140	0.8960	0.8507	0.7560	0.6242	0.5155	0.4557	0.2639	0.0861
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9990	0.9975	0.9940	0.9865	0.9827	0.9718	0.9437	0.8926	0.8385	0.8031	0.6513	0.4013
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

n = 12

p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
x = 0	0.5404	0.2824	0.1422	0.1122	0.0687	0.0317	0.0138	0.0077	0.0057	0.0022	0.0008	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.8816	0.6590	0.4435	0.3813	0.2749	0.1584	0.0850	0.0540	0.0424	0.0196	0.0083	0.0032	0.0011	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.9804	0.8891	0.7358	0.6774	0.5583	0.3907	0.2528	0.1811	0.1513	0.0834	0.0421	0.0193	0.0079	0.0028	0.0008	0.0005	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.9978	0.9744	0.9078	0.8748	0.7946	0.6488	0.4925	0.3931	0.3467	0.2253	0.1345	0.0730	0.0356	0.0153	0.0056	0.0039	0.0017	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000
4	0.9998	0.9957	0.9761	0.9636	0.9274	0.8424	0.7237	0.6315	0.5833	0.4382	0.3044	0.1938	0.1117	0.0573	0.0255	0.0188	0.0095	0.0028	0.0006	0.0002	0.0001	0.0000	0.0000
5	1.0000	0.9995	0.9954	0.9921	0.9806	0.9456	0.8822	0.8223	0.7873	0.6652	0.5269	0.3872	0.2607	0.1582	0.0846	0.0664	0.0386	0.0143	0.0039	0.0013	0.0007	0.0001	0.0000
6	1.0000	0.9999	0.9993	0.9987	0.9961	0.9857	0.9614	0.9336	0.9154	0.8418	0.7393	0.6128	0.4731	0.3348	0.2127	0.1777	0.1178	0.0544	0.0194	0.0079	0.0046	0.0005	0.0000
7	1.0000	1.0000	0.9999	0.9998	0.9994	0.9972	0.9905	0.9812	0.9745	0.9427	0.8883	0.8062	0.6956	0.5618	0.4167	0.3685	0.2763	0.1576	0.0726	0.0364	0.0239	0.0043	0.0002
8	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9983	0.9961	0.9944	0.9847	0.9644	0.9270	0.8655	0.7747	0.6533	0.6069	0.5075	0.3512	0.2054	0.1252	0.0922	0.0256	0.0022
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9995	0.9992	0.9972	0.9921	0.9807	0.9579	0.9166	0.8487	0.8189	0.7472	0.6093	0.4417	0.3226	0.2642	0.1109	0.0196
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9989	0.9968	0.9917	0.9804	0.9576	0.9460	0.9150	0.8416	0.7251	0.6187	0.5565	0.3410	0.1184
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9992	0.9978	0.9943	0.9923	0.9862	0.9683	0.9313	0.8878	0.8578	0.7176	0.4596
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

CUMULATIVE BINOMIAL PROBABILITIES

$n = 14$																							
p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
$x = 0$	0.4877	0.2288	0.1028	0.0779	0.0440	0.0178	0.0068	0.0034	0.0024	0.0008	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.8470	0.5846	0.3567	0.2960	0.1979	0.1010	0.0475	0.0274	0.0205	0.0081	0.0029	0.0009	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.9699	0.8416	0.6479	0.5795	0.4481	0.2811	0.1608	0.1053	0.0839	0.0398	0.0170	0.0065	0.0022	0.0006	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.9958	0.9559	0.8535	0.8063	0.6982	0.5213	0.3552	0.2612	0.2205	0.1243	0.0632	0.0287	0.0114	0.0039	0.0011	0.0007	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.9996	0.9908	0.9533	0.9310	0.8702	0.7415	0.5842	0.4755	0.4227	0.2793	0.1672	0.0898	0.0426	0.0175	0.0060	0.0040	0.0017	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000
5	1.0000	0.9985	0.9885	0.9809	0.9561	0.8883	0.7805	0.6898	0.6405	0.4859	0.3373	0.2120	0.1189	0.0583	0.0243	0.0174	0.0083	0.0022	0.0004	0.0001	0.0000	0.0000	0.0000
6	1.0000	0.9998	0.9978	0.9959	0.9884	0.9617	0.9067	0.8505	0.8164	0.6925	0.5461	0.3953	0.2586	0.1501	0.0753	0.0576	0.0315	0.0103	0.0024	0.0007	0.0003	0.0000	0.0000
7	1.0000	1.0000	0.9997	0.9993	0.9976	0.9897	0.9685	0.9424	0.9247	0.8499	0.7414	0.6047	0.4539	0.3075	0.1836	0.1495	0.0933	0.0383	0.0116	0.0041	0.0022	0.0002	0.0000
8	1.0000	1.0000	1.0000	0.9999	0.9996	0.9978	0.9917	0.9826	0.9757	0.9417	0.8811	0.7880	0.6627	0.5141	0.3595	0.3102	0.2195	0.1117	0.0439	0.0191	0.0115	0.0015	0.0000
9	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9983	0.9960	0.9940	0.9825	0.9574	0.9102	0.8328	0.7207	0.5773	0.5245	0.4158	0.2585	0.1298	0.0690	0.0467	0.0092	0.0004
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9993	0.9989	0.9961	0.9886	0.9713	0.9368	0.8757	0.7795	0.7388	0.6448	0.4787	0.3018	0.1937	0.1465	0.0441	0.0042
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9994	0.9978	0.9935	0.9830	0.9602	0.9161	0.8947	0.8392	0.7189	0.5519	0.4205	0.3521	0.1584	0.0301
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9991	0.9971	0.9919	0.9795	0.9726	0.9525	0.8990	0.8021	0.7040	0.6433	0.4154	0.1530
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9992	0.9976	0.9966	0.9932	0.9822	0.9560	0.9221	0.8972	0.7712	0.5123
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$n = 16$																							
p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
$x = 0$	0.4401	0.1853	0.0743	0.0541	0.0281	0.0100	0.0033	0.0015	0.0010	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.8108	0.5147	0.2839	0.2272	0.1407	0.0635	0.0261	0.0137	0.0098	0.0033	0.0010	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.9571	0.7892	0.5614	0.4868	0.3518	0.1971	0.0994	0.0594	0.0451	0.0183	0.0066	0.0021	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.9930	0.9316	0.7899	0.7291	0.5981	0.4050	0.2459	0.1659	0.1339	0.0651	0.0281	0.0106	0.0035	0.0009	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.9991	0.9830	0.9209	0.8866	0.7982	0.6302	0.4499	0.3391	0.2892	0.1666	0.0853	0.0384	0.0149	0.0049	0.0013	0.0008	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.9999	0.9967	0.9765	0.9622	0.9183	0.8103	0.6598	0.5469	0.4900	0.3288	0.1976	0.1051	0.0486	0.0191	0.0062	0.0040	0.0016	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000
6	1.0000	0.9995	0.9944	0.9899	0.9733	0.9204	0.8247	0.7374	0.6881	0.5272	0.3660	0.2272	0.1241	0.0583	0.0229	0.0159	0.0071	0.0016	0.0002	0.0000	0.0000	0.0000	0.0000
7	1.0000	0.9999	0.9989	0.9979	0.9930	0.9729	0.9256	0.8735	0.8406	0.7161	0.5629	0.4018	0.2559	0.1423	0.0671	0.0500	0.0257	0.0075	0.0015	0.0004	0.0002	0.0000	0.0000
8	1.0000	1.0000	0.9998	0.9996	0.9985	0.9925	0.9743	0.9500	0.9329	0.8577	0.7441	0.5982	0.4371	0.2839	0.1594	0.1265	0.0744	0.0271	0.0070	0.0021	0.0011	0.0001	0.0000
9	1.0000	1.0000	1.0000	1.0000	0.9998	0.9984	0.9929	0.9841	0.9771	0.9417	0.8759	0.7728	0.6340	0.4728	0.3119	0.2626	0.1753	0.0796	0.0267	0.0101	0.0056	0.0005	0.0000
10	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9984	0.9960	0.9938	0.9809	0.9514	0.8949	0.8024	0.6712	0.5100	0.4531	0.3402	0.1897	0.0817	0.0378	0.0235	0.0033	0.0001
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9992	0.9987	0.9951	0.9851	0.9616	0.9147	0.8334	0.7108	0.6609	0.5501	0.3698	0.2018	0.1134	0.0791	0.0170	0.0009
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9991	0.9965	0.9894	0.9719	0.9349	0.8661	0.8341	0.7541	0.5950	0.4019	0.2709	0.2101	0.0684	0.0070
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9979	0.9934	0.9817	0.9549	0.9406	0.9006	0.8029	0.6482	0.5132	0.4386	0.2108	0.0429
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9990	0.9967	0.9902	0.9863	0.9739	0.9365	0.8593	0.7728	0.7161	0.4853	0.1892
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9990	0.9985	0.9967	0.9900	0.9719	0.9459	0.9257	0.8147	0.5599
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

n = 18

p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
x = 0	0.3972	0.1501	0.0536	0.0376	0.0180	0.0056	0.0016	0.0007	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.7735	0.4503	0.2241	0.1728	0.0991	0.0395	0.0142	0.0068	0.0046	0.0013	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.9419	0.7338	0.4797	0.4027	0.2713	0.1353	0.0600	0.0326	0.0236	0.0082	0.0025	0.0007	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.9891	0.9018	0.7202	0.6479	0.5010	0.3057	0.1646	0.1017	0.0783	0.0328	0.0120	0.0038	0.0010	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.9985	0.9718	0.8794	0.8318	0.7164	0.5187	0.3327	0.2311	0.1886	0.0942	0.0411	0.0154	0.0049	0.0013	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.9998	0.9936	0.9581	0.9347	0.8671	0.7175	0.5344	0.4122	0.3550	0.2088	0.1077	0.0481	0.0183	0.0058	0.0014	0.0009	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
6	1.0000	0.9988	0.9882	0.9794	0.9487	0.8610	0.7217	0.6085	0.5491	0.3743	0.2258	0.1189	0.0537	0.0203	0.0062	0.0039	0.0014	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
7	1.0000	0.9998	0.9973	0.9947	0.9837	0.9431	0.8593	0.7767	0.7283	0.5634	0.3915	0.2403	0.1280	0.0576	0.0212	0.0144	0.0061	0.0012	0.0002	0.0000	0.0000	0.0000	0.0000
8	1.0000	1.0000	0.9995	0.9989	0.9957	0.9807	0.9404	0.8924	0.8609	0.7368	0.5778	0.4073	0.2527	0.1347	0.0597	0.0433	0.0210	0.0054	0.0009	0.0002	0.0001	0.0000	0.0000
9	1.0000	1.0000	0.9999	0.9998	0.9991	0.9946	0.9790	0.9567	0.9403	0.8653	0.7473	0.5927	0.4222	0.2632	0.1391	0.1076	0.0596	0.0193	0.0043	0.0011	0.0005	0.0000	0.0000
10	1.0000	1.0000	1.0000	1.0000	0.9998	0.9988	0.9939	0.9856	0.9788	0.9424	0.8720	0.7597	0.6085	0.4366	0.2717	0.2233	0.1407	0.0569	0.0163	0.0053	0.0027	0.0002	0.0000
11	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9986	0.9961	0.9938	0.9797	0.9463	0.8811	0.7742	0.6257	0.4509	0.3915	0.2783	0.1390	0.0513	0.0206	0.0118	0.0012	0.0000
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9991	0.9986	0.9942	0.9817	0.9519	0.8923	0.7912	0.6450	0.5878	0.4656	0.2825	0.1329	0.0653	0.0419	0.0064	0.0002
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9987	0.9951	0.9846	0.9589	0.9058	0.8114	0.7689	0.6673	0.4813	0.2836	0.1682	0.1206	0.0282	0.0015
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9990	0.9962	0.9880	0.9672	0.9217	0.8983	0.8354	0.6943	0.4990	0.3521	0.2798	0.0982	0.0109
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9993	0.9975	0.9918	0.9764	0.9674	0.9400	0.8647	0.7287	0.5973	0.5203	0.2662	0.0581
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9987	0.9954	0.9932	0.9858	0.9605	0.9009	0.8272	0.7759	0.5497	0.2265
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9993	0.9984	0.9944	0.9820	0.9624	0.9464	0.8499	0.6028
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

n = 20

p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
x = 0	0.3585	0.1216	0.0388	0.0261	0.0115	0.0032	0.0008	0.0003	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.7358	0.3917	0.1756	0.1304	0.0692	0.0243	0.0076	0.0033	0.0021	0.0005	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.9245	0.6769	0.4049	0.3287	0.2061	0.0913	0.0355	0.0176	0.0121	0.0036	0.0009	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.9841	0.8670	0.6477	0.5665	0.4114	0.2252	0.1071	0.0604	0.0444	0.0160	0.0049	0.0013	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.9974	0.9568	0.8298	0.7687	0.6296	0.4148	0.2375	0.1515	0.1182	0.0510	0.0189	0.0059	0.0015	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.9997	0.9887	0.9327	0.8982	0.8042	0.6172	0.4164	0.2972	0.2454	0.1256	0.0553	0.0207	0.0064	0.0016	0.0003	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
6	1.0000	0.9976	0.9781	0.9629	0.9133	0.7858	0.6080	0.4793	0.4166	0.2500	0.1299	0.0577	0.0214	0.0065	0.0015	0.0009	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
7	1.0000	0.9996	0.9941	0.9887	0.9679	0.8982	0.7723	0.6615	0.6010	0.4159	0.2520	0.1316	0.0580	0.0210	0.0060	0.0037	0.0013	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
8	1.0000	0.9999	0.9987	0.9972	0.9900	0.9591	0.8867	0.8095	0.7624	0.5956	0.4143	0.2517	0.1308	0.0565	0.0196	0.0130	0.0051	0.0009	0.0001	0.0000	0.0000	0.0000	0.0000
9	1.0000	1.0000	0.9998	0.9994	0.9974	0.9861	0.9520	0.9081	0.8782	0.7553	0.5914	0.4119	0.2493	0.1275	0.0532	0.0376	0.0171	0.0039	0.0006	0.0001	0.0000	0.0000	0.0000
10	1.0000	1.0000	1.0000	0.9999	0.9994	0.9961	0.9829	0.9624	0.9468	0.8725	0.7507	0.5881	0.4086	0.2447	0.1218	0.0919	0.0480	0.0139	0.0026	0.0006	0.0002	0.0000	0.0000
11	1.0000	1.0000	1.0000	1.0000	0.9999	0.9991	0.9949	0.9870	0.9804	0.9435	0.8692	0.7483	0.5857	0.4044	0.2376	0.1905	0.1133	0.0409	0.0100	0.0028	0.0013	0.0001	0.0000
12	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9987	0.9963	0.9940	0.9790	0.9420	0.8684	0.7480	0.5841	0.3990	0.3385	0.2277	0.1018	0.0321	0.0113	0.0059	0.0004	0.0000
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9991	0.9985	0.9935	0.9786	0.9423	0.8701	0.7500	0.5834	0.5207	0.3920	0.2142	0.0867	0.0371	0.0219	0.0024	0.0000
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9997	0.9984	0.9936	0.9793	0.9447	0.8744	0.7546	0.7028	0.5836	0.3828	0.1958	0.1018	0.0673	0.0113	0.0003
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9985	0.9941	0.9811	0.9490	0.8818	0.8485	0.7625	0.5852	0.3704	0.2313	0.1702	0.0432	0.0026
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9987	0.9951	0.9840	0.9556	0.9396	0.8929	0.7748	0.5886	0.4335	0.3523	0.1330	0.0159
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9991	0.9964	0.9879	0.9824	0.9645	0.9087	0.7939	0.6713	0.5951	0.3231	0.0755
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9979	0.9967	0.9924	0.9757	0.9308	0.8696	0.8244	0.6083	0.2642
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9997	0.9992	0.9968	0.9885	0.9739	0.9612	0.8784	0.6415
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

CUMULATIVE BINOMIAL PROBABILITIES

$n = 25$ p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
$x = 0$	0.2774	0.0718	0.0172	0.0105	0.0038	0.0008	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.6424	0.2712	0.0931	0.0629	0.0274	0.0070	0.0016	0.0005	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.8729	0.5371	0.2537	0.1887	0.0982	0.0321	0.0090	0.0035	0.0021	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.9659	0.7636	0.4711	0.3816	0.2340	0.0962	0.0332	0.0149	0.0097	0.0024	0.0005	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.9928	0.9020	0.6821	0.5937	0.4207	0.2137	0.0905	0.0462	0.0320	0.0095	0.0023	0.0005	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.9988	0.9666	0.8385	0.7720	0.6167	0.3783	0.1935	0.1120	0.0826	0.0294	0.0086	0.0020	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
6	0.9998	0.9905	0.9305	0.8908	0.7800	0.5611	0.3407	0.2215	0.1734	0.0736	0.0258	0.0073	0.0016	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
7	1.0000	0.9977	0.9745	0.9553	0.8909	0.7265	0.5118	0.3703	0.3061	0.1536	0.0639	0.0216	0.0058	0.0012	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
8	1.0000	0.9995	0.9920	0.9843	0.9532	0.8506	0.6769	0.5376	0.4668	0.2735	0.1340	0.0539	0.0174	0.0043	0.0008	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
9	1.0000	0.9999	0.9979	0.9953	0.9827	0.9287	0.8106	0.6956	0.6303	0.4246	0.2424	0.1148	0.0440	0.0132	0.0029	0.0016	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
10	1.0000	1.0000	0.9995	0.9988	0.9944	0.9703	0.9022	0.8220	0.7712	0.5858	0.3843	0.2122	0.0960	0.0344	0.0093	0.0056	0.0018	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
11	1.0000	1.0000	0.9999	0.9997	0.9985	0.9893	0.9558	0.9082	0.8746	0.7323	0.5426	0.3450	0.1827	0.0778	0.0255	0.0164	0.0060	0.0009	0.0001	0.0000	0.0000	0.0000	0.0000
12	1.0000	1.0000	1.0000	0.9999	0.9996	0.9966	0.9825	0.9585	0.9396	0.8462	0.6937	0.5000	0.3063	0.1538	0.0604	0.0415	0.0175	0.0034	0.0004	0.0001	0.0000	0.0000	0.0000
13	1.0000	1.0000	1.0000	1.0000	0.9999	0.9991	0.9940	0.9836	0.9745	0.9222	0.8173	0.6550	0.4574	0.2677	0.1254	0.0918	0.0442	0.0107	0.0015	0.0003	0.0001	0.0000	0.0000
14	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9982	0.9944	0.9907	0.9656	0.9040	0.7878	0.6157	0.4142	0.2288	0.1780	0.0978	0.0297	0.0056	0.0012	0.0005	0.0000	0.0000
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9995	0.9984	0.9971	0.9868	0.9560	0.8852	0.7576	0.5754	0.3697	0.3044	0.1894	0.0713	0.0173	0.0047	0.0021	0.0001	0.0000
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9992	0.9957	0.9826	0.9461	0.8660	0.7265	0.5332	0.4624	0.3231	0.1494	0.0468	0.0157	0.0080	0.0005	0.0000
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9988	0.9942	0.9784	0.9361	0.8464	0.6939	0.6297	0.4882	0.2735	0.1091	0.0447	0.0255	0.0023	0.0000
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9984	0.9927	0.9742	0.9264	0.8266	0.7785	0.6593	0.4389	0.2200	0.1092	0.0695	0.0095	0.0002
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9980	0.9914	0.9706	0.9174	0.8880	0.8065	0.6217	0.3833	0.2280	0.1615	0.0334	0.0012
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9977	0.9905	0.9680	0.9538	0.9095	0.7863	0.5793	0.4063	0.3179	0.0980	0.0072
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9976	0.9903	0.9851	0.9668	0.9038	0.7660	0.6184	0.5289	0.2364	0.0341
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9979	0.9965	0.9910	0.9679	0.9018	0.8113	0.7463	0.4629	0.1271
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9995	0.9984	0.9930	0.9726	0.9371	0.9069	0.7288	0.3576
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9992	0.9962	0.9895	0.9828	0.9282	0.7226
25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

CUMULATIVE BINOMIAL PROBABILITIES

$n = 30$																							
p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
$x = 0$	0.2146	0.0424	0.0076	0.0042	0.0012	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.5535	0.1837	0.0480	0.0295	0.0105	0.0020	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.8122	0.4114	0.1514	0.1028	0.0442	0.0106	0.0021	0.0007	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.9392	0.6474	0.3217	0.2396	0.1227	0.0374	0.0093	0.0033	0.0019	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.9844	0.8245	0.5245	0.4243	0.2552	0.0979	0.0302	0.0122	0.0075	0.0015	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.9967	0.9268	0.7106	0.6164	0.4275	0.2026	0.0766	0.0355	0.0233	0.0057	0.0011	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
6	0.9994	0.9742	0.8474	0.7765	0.6070	0.3481	0.1595	0.0838	0.0586	0.0172	0.0040	0.0007	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
7	0.9999	0.9922	0.9302	0.8863	0.7608	0.5143	0.2814	0.1668	0.1238	0.0435	0.0121	0.0026	0.0004	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
8	1.0000	0.9980	0.9722	0.9494	0.8713	0.6736	0.4315	0.2860	0.2247	0.0940	0.0312	0.0081	0.0016	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
9	1.0000	0.9995	0.9903	0.9803	0.9389	0.8034	0.5888	0.4317	0.3575	0.1763	0.0694	0.0214	0.0050	0.0009	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
10	1.0000	0.9999	0.9971	0.9933	0.9744	0.8943	0.7304	0.5848	0.5078	0.2915	0.1350	0.0494	0.0138	0.0029	0.0004	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
11	1.0000	1.0000	0.9992	0.9980	0.9905	0.9493	0.8407	0.7239	0.6548	0.4311	0.2327	0.1002	0.0334	0.0083	0.0014	0.0007	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
12	1.0000	1.0000	0.9998	0.9995	0.9969	0.9784	0.9155	0.8340	0.7802	0.5785	0.3592	0.1808	0.0714	0.0212	0.0045	0.0025	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
13	1.0000	1.0000	1.0000	0.9999	0.9991	0.9918	0.9599	0.9102	0.8737	0.7145	0.5025	0.2923	0.1356	0.0481	0.0124	0.0072	0.0021	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
14	1.0000	1.0000	1.0000	1.0000	0.9998	0.9973	0.9831	0.9565	0.9348	0.8246	0.6448	0.4278	0.2309	0.0971	0.0301	0.0188	0.0064	0.0008	0.0001	0.0000	0.0000	0.0000	0.0000
15	1.0000	1.0000	1.0000	1.0000	0.9999	0.9992	0.9936	0.9812	0.9699	0.9029	0.7691	0.5722	0.3552	0.1754	0.0652	0.0435	0.0169	0.0027	0.0002	0.0000	0.0000	0.0000	0.0000
16	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9979	0.9928	0.9876	0.9519	0.8644	0.7077	0.4975	0.2855	0.1263	0.0898	0.0401	0.0082	0.0009	0.0001	0.0000	0.0000	0.0000
17	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9975	0.9955	0.9788	0.9286	0.8192	0.6408	0.4215	0.2198	0.1660	0.0845	0.0216	0.0031	0.0005	0.0002	0.0000	0.0000
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9993	0.9986	0.9917	0.9666	0.8998	0.7673	0.5689	0.3452	0.2761	0.1593	0.0507	0.0095	0.0020	0.0008	0.0000	0.0000
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9996	0.9971	0.9862	0.9506	0.8650	0.7085	0.4922	0.4152	0.2696	0.1057	0.0256	0.0067	0.0029	0.0001	0.0000
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9991	0.9950	0.9786	0.9306	0.8237	0.6425	0.5683	0.4112	0.1966	0.0611	0.0197	0.0097	0.0005	0.0000
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9984	0.9919	0.9688	0.9060	0.7753	0.7140	0.5685	0.3264	0.1287	0.0506	0.0278	0.0020	0.0000
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9996	0.9974	0.9879	0.9565	0.8762	0.8332	0.7186	0.4857	0.2392	0.1137	0.0698	0.0078	0.0001
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9993	0.9960	0.9828	0.9414	0.9162	0.8405	0.6519	0.3930	0.2235	0.1526	0.0258	0.0006
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9989	0.9943	0.9767	0.9645	0.9234	0.7974	0.5725	0.3836	0.2894	0.0732	0.0033
25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9985	0.9925	0.9878	0.9698	0.9021	0.7448	0.5757	0.4755	0.1755	0.0156
26	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9981	0.9967	0.9907	0.9626	0.8773	0.7604	0.6783	0.3526	0.0608
27	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9993	0.9979	0.9894	0.9558	0.8972	0.8486	0.5886	0.1878
28	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9980	0.9895	0.9705	0.9520	0.8163	0.4465
29	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9988	0.9958	0.9924	0.9576	0.7854
30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

CUMULATIVE POISSON PROBABILITIES

λ		0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
x = 0		0.9900	0.9802	0.9704	0.9608	0.9512	0.9418	0.9324	0.9231	0.9139
1		1.0000	0.9998	0.9996	0.9992	0.9988	0.9983	0.9977	0.9970	0.9962
2		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999
3		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

λ		0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
x = 0		0.9048	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493	0.4066
1		0.9953	0.9825	0.9631	0.9384	0.9098	0.8781	0.8442	0.8088	0.7725
2		0.9998	0.9989	0.9964	0.9921	0.9856	0.9769	0.9659	0.9526	0.9371
3		1.0000	0.9999	0.9997	0.9992	0.9982	0.9966	0.9942	0.9909	0.9865
4		1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9986	0.9977
5		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9997
6		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

λ	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90
x = 0	0.3679	0.3329	0.3012	0.2725	0.2466	0.2231	0.2019	0.1827	0.1653	0.1496
1	0.7358	0.6990	0.6626	0.6268	0.5918	0.5578	0.5249	0.4932	0.4628	0.4337
2	0.9197	0.9004	0.8795	0.8571	0.8335	0.8088	0.7834	0.7572	0.7306	0.7037
3	0.9810	0.9743	0.9662	0.9569	0.9463	0.9344	0.9212	0.9068	0.8913	0.8747
4	0.9963	0.9946	0.9923	0.9893	0.9857	0.9814	0.9763	0.9704	0.9636	0.9559
5	0.9994	0.9990	0.9985	0.9978	0.9968	0.9955	0.9940	0.9920	0.9896	0.9868
6	0.9999	0.9999	0.9997	0.9996	0.9994	0.9991	0.9987	0.9981	0.9974	0.9966
7	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9997	0.9996	0.9994	0.9992
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

λ	2.00	2.10	2.20	2.30	2.40	2.50	2.60	2.70	2.80	2.90
x = 0	0.1353	0.1225	0.1108	0.1003	0.0907	0.0821	0.0743	0.0672	0.0608	0.0550
1	0.4060	0.3796	0.3546	0.3309	0.3084	0.2873	0.2674	0.2487	0.2311	0.2146
2	0.6767	0.6496	0.6227	0.5960	0.5697	0.5438	0.5184	0.4936	0.4695	0.4460
3	0.8571	0.8386	0.8194	0.7993	0.7787	0.7576	0.7360	0.7141	0.6919	0.6696
4	0.9473	0.9379	0.9275	0.9162	0.9041	0.8912	0.8774	0.8629	0.8477	0.8318
5	0.9834	0.9796	0.9751	0.9700	0.9643	0.9580	0.9510	0.9433	0.9349	0.9258
6	0.9955	0.9941	0.9925	0.9906	0.9884	0.9858	0.9828	0.9794	0.9756	0.9713
7	0.9989	0.9985	0.9980	0.9974	0.9967	0.9958	0.9947	0.9934	0.9919	0.9901
8	0.9998	0.9997	0.9995	0.9994	0.9991	0.9989	0.9985	0.9981	0.9976	0.9969
9	1.0000	0.9999	0.9999	0.9999	0.9998	0.9997	0.9996	0.9995	0.9993	0.9991
10	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9998	0.9998
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

λ	3.00	3.10	3.20	3.30	3.40	3.50	3.60	3.70	3.80	3.90
x = 0	0.0498	0.0450	0.0408	0.0369	0.0334	0.0302	0.0273	0.0247	0.0224	0.0202
1	0.1991	0.1847	0.1712	0.1586	0.1468	0.1359	0.1257	0.1162	0.1074	0.0992
2	0.4232	0.4012	0.3799	0.3594	0.3397	0.3208	0.3027	0.2854	0.2689	0.2531
3	0.6472	0.6248	0.6025	0.5803	0.5584	0.5366	0.5152	0.4942	0.4735	0.4532
4	0.8153	0.7982	0.7806	0.7626	0.7442	0.7254	0.7064	0.6872	0.6678	0.6484
5	0.9161	0.9057	0.8946	0.8829	0.8705	0.8576	0.8441	0.8301	0.8156	0.8006
6	0.9665	0.9612	0.9554	0.9490	0.9421	0.9347	0.9267	0.9182	0.9091	0.8995
7	0.9881	0.9858	0.9832	0.9802	0.9769	0.9733	0.9692	0.9648	0.9599	0.9546
8	0.9962	0.9953	0.9943	0.9931	0.9917	0.9901	0.9883	0.9863	0.9840	0.9815
9	0.9989	0.9986	0.9982	0.9978	0.9973	0.9967	0.9960	0.9952	0.9942	0.9931
10	0.9997	0.9996	0.9995	0.9994	0.9992	0.9990	0.9987	0.9984	0.9981	0.9977
11	0.9999	0.9999	0.9999	0.9998	0.9998	0.9997	0.9996	0.9995	0.9994	0.9993
12	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

CUMULATIVE POISSON PROBABILITIES

λ	4.00	4.10	4.20	4.30	4.40	4.50	4.60	4.70	4.80	4.90
$x=0$	0.0183	0.0166	0.0150	0.0136	0.0123	0.0111	0.0101	0.0091	0.0082	0.0074
1	0.0916	0.0845	0.0780	0.0719	0.0663	0.0611	0.0563	0.0518	0.0477	0.0439
2	0.2381	0.2238	0.2102	0.1974	0.1851	0.1736	0.1626	0.1523	0.1425	0.1333
3	0.4335	0.4142	0.3954	0.3772	0.3594	0.3423	0.3257	0.3097	0.2942	0.2793
4	0.6288	0.6093	0.5898	0.5704	0.5512	0.5321	0.5132	0.4946	0.4763	0.4582
5	0.7851	0.7693	0.7531	0.7367	0.7199	0.7029	0.6858	0.6684	0.6510	0.6335
6	0.8893	0.8786	0.8675	0.8558	0.8436	0.8311	0.8180	0.8046	0.7908	0.7767
7	0.9489	0.9427	0.9361	0.9290	0.9214	0.9134	0.9049	0.8960	0.8867	0.8769
8	0.9786	0.9755	0.9721	0.9683	0.9642	0.9597	0.9549	0.9497	0.9442	0.9382
9	0.9919	0.9905	0.9889	0.9871	0.9851	0.9829	0.9805	0.9778	0.9749	0.9717
10	0.9972	0.9966	0.9959	0.9952	0.9943	0.9933	0.9922	0.9910	0.9896	0.9880
11	0.9991	0.9989	0.9986	0.9983	0.9980	0.9976	0.9971	0.9966	0.9960	0.9953
12	0.9997	0.9997	0.9996	0.9995	0.9993	0.9992	0.9990	0.9988	0.9986	0.9983
13	0.9999	0.9999	0.9999	0.9998	0.9998	0.9997	0.9997	0.9996	0.9995	0.9994
14	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

λ	5.00	5.50	6.00	6.50	7.00	7.50	8.00	8.50	9.00	9.50
$x=0$	0.0067	0.0041	0.0025	0.0015	0.0009	0.0006	0.0003	0.0002	0.0001	0.0001
1	0.0404	0.0266	0.0174	0.0113	0.0073	0.0047	0.0030	0.0019	0.0012	0.0008
2	0.1247	0.0884	0.0620	0.0430	0.0296	0.0203	0.0138	0.0093	0.0062	0.0042
3	0.2650	0.2017	0.1512	0.1118	0.0818	0.0591	0.0424	0.0301	0.0212	0.0149
4	0.4405	0.3575	0.2851	0.2237	0.1730	0.1321	0.0996	0.0744	0.0550	0.0403
5	0.6160	0.5289	0.4457	0.3690	0.3007	0.2414	0.1912	0.1496	0.1157	0.0885
6	0.7622	0.6860	0.6063	0.5265	0.4497	0.3782	0.3134	0.2562	0.2068	0.1649
7	0.8666	0.8095	0.7440	0.6728	0.5987	0.5246	0.4530	0.3856	0.3239	0.2687
8	0.9319	0.8944	0.8472	0.7916	0.7291	0.6620	0.5925	0.5231	0.4557	0.3918
9	0.9682	0.9462	0.9161	0.8774	0.8305	0.7764	0.7166	0.6530	0.5874	0.5218
10	0.9863	0.9747	0.9574	0.9332	0.9015	0.8622	0.8159	0.7634	0.7060	0.6453
11	0.9945	0.9890	0.9799	0.9661	0.9467	0.9208	0.8881	0.8487	0.8030	0.7520
12	0.9980	0.9955	0.9912	0.9840	0.9730	0.9573	0.9362	0.9091	0.8758	0.8364
13	0.9993	0.9983	0.9964	0.9929	0.9872	0.9784	0.9658	0.9486	0.9261	0.8981
14	0.9998	0.9994	0.9986	0.9970	0.9943	0.9897	0.9827	0.9726	0.9585	0.9400
15	0.9999	0.9998	0.9995	0.9988	0.9976	0.9954	0.9918	0.9862	0.9780	0.9665
16	1.0000	0.9999	0.9998	0.9996	0.9990	0.9980	0.9963	0.9934	0.9889	0.9823
17	1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9984	0.9970	0.9947	0.9911
18	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997	0.9993	0.9987	0.9976	0.9957
19	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9995	0.9989	0.9980
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9991
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

CUMULATIVE POISSON PROBABILITIES

λ	10.00	11.00	12.00	13.00	14.00	15.00	16.00	17.00	18.00	19.00
x = 0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0005	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0028	0.0012	0.0005	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.0103	0.0049	0.0023	0.0011	0.0005	0.0002	0.0001	0.0000	0.0000	0.0000
4	0.0293	0.0151	0.0076	0.0037	0.0018	0.0009	0.0004	0.0002	0.0001	0.0000
5	0.0671	0.0375	0.0203	0.0107	0.0055	0.0028	0.0014	0.0007	0.0003	0.0002
6	0.1301	0.0786	0.0458	0.0259	0.0142	0.0076	0.0040	0.0021	0.0010	0.0005
7	0.2202	0.1432	0.0895	0.0540	0.0316	0.0180	0.0100	0.0054	0.0029	0.0015
8	0.3328	0.2320	0.1550	0.0998	0.0621	0.0374	0.0220	0.0126	0.0071	0.0039
9	0.4579	0.3405	0.2424	0.1658	0.1094	0.0699	0.0433	0.0261	0.0154	0.0089
10	0.5830	0.4599	0.3472	0.2517	0.1757	0.1185	0.0774	0.0491	0.0304	0.0183
11	0.6968	0.5793	0.4616	0.3532	0.2600	0.1848	0.1270	0.0847	0.0549	0.0347
12	0.7916	0.6887	0.5760	0.4631	0.3585	0.2676	0.1931	0.1350	0.0917	0.0606
13	0.8645	0.7813	0.6815	0.5730	0.4644	0.3632	0.2745	0.2009	0.1426	0.0984
14	0.9165	0.8540	0.7720	0.6751	0.5704	0.4657	0.3675	0.2808	0.2081	0.1497
15	0.9513	0.9074	0.8444	0.7636	0.6694	0.5681	0.4667	0.3715	0.2867	0.2148
16	0.9730	0.9441	0.8987	0.8355	0.7559	0.6641	0.5660	0.4677	0.3751	0.2920
17	0.9857	0.9678	0.9370	0.8905	0.8272	0.7489	0.6593	0.5640	0.4686	0.3784
18	0.9928	0.9823	0.9626	0.9302	0.8826	0.8195	0.7423	0.6550	0.5622	0.4695
19	0.9965	0.9907	0.9787	0.9573	0.9235	0.8752	0.8122	0.7363	0.6509	0.5606
20	0.9984	0.9953	0.9884	0.9750	0.9521	0.9170	0.8682	0.8055	0.7307	0.6472
21	0.9993	0.9977	0.9939	0.9859	0.9712	0.9469	0.9108	0.8615	0.7991	0.7255
22	0.9997	0.9990	0.9970	0.9924	0.9833	0.9673	0.9418	0.9047	0.8551	0.7931
23	0.9999	0.9995	0.9985	0.9960	0.9907	0.9805	0.9633	0.9367	0.8989	0.8490
24	1.0000	0.9998	0.9993	0.9980	0.9950	0.9888	0.9777	0.9594	0.9317	0.8933
25	1.0000	0.9999	0.9997	0.9990	0.9974	0.9938	0.9869	0.9748	0.9554	0.9269
26	1.0000	1.0000	0.9999	0.9995	0.9987	0.9967	0.9925	0.9848	0.9718	0.9514
27	1.0000	1.0000	0.9999	0.9998	0.9994	0.9983	0.9959	0.9912	0.9827	0.9687
28	1.0000	1.0000	1.0000	0.9999	0.9997	0.9991	0.9978	0.9950	0.9897	0.9805
29	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9989	0.9973	0.9941	0.9882
30	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9994	0.9986	0.9967	0.9930
31	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9993	0.9982	0.9960
32	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9990	0.9978
33	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9995	0.9988
34	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9994
35	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997
36	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998
37	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
38	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

THE NORMAL DISTRIBUTION FUNCTION

If Z has a normal distribution with mean 0 and variance 1 then, for each value of z, the table gives the value of $\Phi(z)$, where

$$\Phi(z) = P(Z \le z).$$

For negative values of z use $\Phi(-z) = 1 - \Phi(z)$.
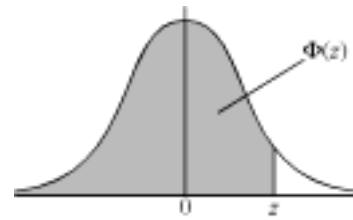
z	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
															ADD				
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	4	8	12	16	20	24	28	32	36
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	4	8	12	16	20	24	28	32	36
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	4	8	12	15	19	23	27	31	35
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	4	7	11	15	19	22	26	30	34
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	4	7	11	14	18	22	25	29	32
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	3	7	10	14	17	20	24	27	31
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	3	7	10	13	16	19	23	26	29
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	3	6	9	12	15	18	21	24	27
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	3	5	8	11	14	16	19	22	25
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	3	5	8	10	13	15	18	20	23
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621	2	5	7	9	12	14	16	19	21
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830	2	4	6	8	10	12	14	16	18
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015	2	4	6	7	9	11	13	15	17
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177	2	3	5	6	8	10	11	13	14
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	1	3	4	6	7	8	10	11	13
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441	1	2	4	5	6	7	8	10	11
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545	1	2	3	4	5	6	7	8	9
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633	1	2	3	4	4	5	6	7	8
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706	1	1	2	3	4	4	5	6	6
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767	1	1	2	2	3	4	4	5	5
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817	0	1	1	2	2	3	3	4	4
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857	0	1	1	2	2	2	3	3	4
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890	0	1	1	1	2	2	2	3	3
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916	0	1	1	1	1	2	2	2	2
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936	0	0	1	1	1	1	1	2	2
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952	0	0	0	1	1	1	1	1	1
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964	0	0	0	0	1	1	1	1	1
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974	0	0	0	0	0	1	1	1	1
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981	0	0	0	0	0	0	0	1	1
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986	0	0	0	0	0	0	0	0	0

Critical values for the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that

$$P(Z \le z) = p.$$

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

CRITICAL VALUES FOR THE *t*-DISTRIBUTION

If T has a *t*-distribution with ν degrees of freedom then,
for each pair of values of p and ν, the table gives the value
of t such that:

$P(T \le t) = p$.
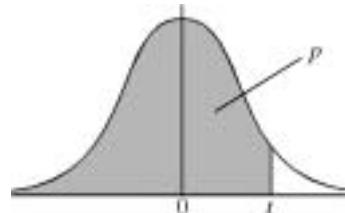
p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
$\nu = 1$	1.000	3.078	6.314	12.71	31.82	63.66	127.3	318.3	636.6
2	0.816	1.886	2.920	4.303	6.965	9.925	14.09	22.33	31.60
3	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.21	12.92
4	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.894	6.869
6	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.768
24	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.689
28	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.660
30	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Mathematical notation

Examinations for the syllabus in this booklet may use relevant notation from the following list.

1 Set notation

$\in$	is an element of
$\notin$	is not an element of
$\{x_1, x_2, ...\}$	the set with elements $x_1, x_2, ...$
$\{x : ...\}$	the set of all x such that ...
$n(A)$	the number of elements in set A
$\emptyset$	the empty set
$\mathscr{E}$	the universal set
A'	the complement of the set A
$\mathbb{N}$	the set of natural numbers, $\{1, 2, 3, ...\}$
$\mathbb{Z}$	the set of integers, $\{0, \pm 1, \pm 2, \pm 3, ...\}$
$\mathbb{Z}^+$	the set of positive integers, $\{1, 2, 3, ...\}$
$\mathbb{Z}_n$	the set of integers modulo n, $\{0, 1, 2, ..., n-1\}$
$\mathbb{Q}$	the set of rational numbers, $\left\{ \frac{p}{q} : p \in \mathbb{Z}, q \in \mathbb{Z}^+ \right\}$
$\mathbb{Q}^+$	the set of positive rational numbers, $\{x \in \mathbb{Q} : x > 0\}$
$\mathbb{Q}_0^+$	set of positive rational numbers and zero, $\{x \in \mathbb{Q} : x \geq 0\}$
$\mathbb{R}$	the set of real numbers
$\mathbb{R}^+$	the set of positive real numbers, $\{x \in \mathbb{R} : x > 0\}$
$\mathbb{R}_0^+$	the set of positive real numbers and zero, $\{x \in \mathbb{R} : x \geq 0\}$
$\mathbb{C}$	the set of complex numbers
(x, y)	the ordered pair x, y
$A \times B$	the cartesian product of sets A and B, i.e. $A \times B = \{(a, b) : a \in A, b \in B\}$
$\subseteq$	is a subset of
$\subset$	is a proper subset of
$\cup$	union
$\cap$	intersection
$[a, b]$	the closed interval $\{x \in \mathbb{R} : a \leq x \leq b\}$
$[a, b)$	the interval $\{x \in \mathbb{R} : a \leq x < b\}$
$(a, b]$	the interval $\{x \in \mathbb{R} : a < x \leq b\}$
(a, b)	the open interval $\{x \in \mathbb{R} : a < x < b\}$
$y \, R \, x$	y is related to x by the relation R
$y \sim x$	y is equivalent to x, in the context of some equivalence relation

2 Miscellaneous symbols

$=$	is equal to
$\neq$	is not equal to
$\equiv$	is identical to or is congruent to
$\approx$	is approximately equal to
$\cong$	is isomorphic to
$\propto$	is proportional to
$<$	is less than
$\leq$	is less than or equal to, is not greater than
$>$	is greater than
$\geq$	is greater than or equal to, is not less than
∞	infinity
$p \wedge q$	p and q
$p \vee q$	p or q (or both)
$\sim p$	not p
$p \Rightarrow q$	p implies q (if p then q)
$p \Leftarrow q$	p is implied by q (if q then p)
$p \Leftrightarrow q$	p implies and is implied by q (p is equivalent to q)
$\exists$	there exists
$\forall$	for all

3 Operations

$a + b$	a plus b
$a - b$	a minus b
$a \times b,\ ab,\ a.b$	a multiplied by b
$a \div b,\ \dfrac{a}{b},\ a / b$	a divided by b
$\displaystyle\sum_{i=1}^{n} a_i$	$a_1 + a_2 + ... + a_n$
$\displaystyle\prod_{i=1}^{n} a_i$	$a_1 \times a_2 \times ... \times a_n$
$\sqrt{a}$	the positive square root of a
$\lvert a \rvert$	the modulus of a
$n!$	n factorial
$\dbinom{n}{r}$	the binomial coefficient $\dfrac{n!}{r!\,(n-r)!}$ for $n \in \mathbb{Z}^+$ or $\dfrac{n(n-1)...(n-r+1)}{r!}$ for $n \in \mathbb{Q}$

4 Functions

$\mathrm{f}(x)$	the value of the function f at x
$\mathrm{f} : A \rightarrow B$	f is a function under which each element of set A has an image in set B
$\mathrm{f} : x \mapsto y$	the function f maps the element x to the element y
f^{-1}	the inverse function of the function f
gf	the composite function of f and g which is defined by $\mathrm{gf}(x) = \mathrm{g}(\mathrm{f}(x))$
$\displaystyle\lim_{x \to a} \mathrm{f}(x)$	the limit of $\mathrm{f}(x)$ as x tends to a

$\Delta x, \delta x$	an increment of x
$\dfrac{dy}{dx}$	the derivative of y with respect to x
$\dfrac{d^n y}{dx^n}$	the nth derivative of y with respect to x
f$'(x)$, f$''(x)$, …, f$^{(n)}(x)$	the first, second, ..., nth derivatives of f(x) with respect to x
$\displaystyle\int y\,dx$	the indefinite integral of y with respect to x
$\displaystyle\int_a^b y\,dx$	the definite integral of y with respect to x between the limits $x = a$ and $x = b$
$\dfrac{\partial V}{\partial x}$	the partial derivative of V with respect to x
$\dot{x}, \ddot{x}, ...$	the first, second, ... derivatives of x with respect to t

5 Exponential and logarithmic functions

e	base of natural logarithms
e^x, exp x	exponential function of x
$\log_a x$	logarithm to the base a of x
$\ln x$, $\log_e x$	natural logarithm of x
$\lg x$, $\log_{10} x$	logarithm of x to base 10

6 Circular and hyperbolic functions

sin, cos, tan, cosec, sec, cot	the circular functions
$\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$, cosec^{-1}, $\sec^{-1}$, $\cot^{-1}$	the inverse circular functions
sinh, cosh, tanh, cosech, sech, coth	the hyperbolic functions
$\sinh^{-1}$, $\cosh^{-1}$, $\tanh^{-1}$, cosech^{-1}, sech^{-1}, $\coth^{-1}$	the inverse hyperbolic functions

7 Complex numbers

i	square root of -1
z	a complex number, $z = x + i\,y = r(\cos\theta + i\sin\theta)$
Re z	the real part of z, Re $z = x$
Im z	the imaginary part of z, Im $z = y$
$|z|$	the modulus of z, $|z| = \sqrt{x^2 + y^2}$
arg z	the argument of z, arg $z = \theta$, $-\pi < \theta \le \pi$
z^*	the complex conjugate of z, $x - i\,y$

8 Matrices

$\mathbf{M}$	a matrix $\mathbf{M}$
$\mathbf{M}^{-1}$	the inverse of the matrix $\mathbf{M}$
$\mathbf{M}^{\mathrm{T}}$	the transpose of the matrix $\mathbf{M}$
det $\mathbf{M}$ or $|\mathbf{M}|$	the determinant of the square matrix $\mathbf{M}$

9 Vectors

$\mathbf{a}$	the vector $\mathbf{a}$
$\overrightarrow{AB}$	the vector represented in magnitude and direction by the directed line segment AB
$\hat{\mathbf{a}}$	a unit vector in the direction of $\mathbf{a}$
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	unit vectors in the directions of the cartesian coordinate axes
$|\mathbf{a}|, a$	the magnitude of $\mathbf{a}$
$|\overrightarrow{AB}|, AB$	the magnitude of $\overrightarrow{AB}$
$\mathbf{a}.\mathbf{b}$	the scalar product of $\mathbf{a}$ and $\mathbf{b}$
$\mathbf{a} \times \mathbf{b}$	the vector product of $\mathbf{a}$ and $\mathbf{b}$

10 Probability and statistics

$A, B, C,$ etc.	events
$A \cup B$	union of the events A and B
$A \cap B$	intersection of the events A and B
P(A)	probability of the event A
A'	complement of the event A
P($A|B$)	probability of the event A conditional on the event B
$X, Y, R,$ etc.	random variables
$x, y, r,$ etc.	values of the random variables $X, Y, R,$ etc.
$x_1, x_2, ...$	observations
$f_1, f_2, ...$	frequencies with which the observations $x_1, x_2, ...$ occur
p(x)	probability function P($X = x$) of the discrete random variable X
$p_1, p_2, ...$	probabilities of the values $x_1, x_2, ...$ of the discrete random variable X
f(x), g(x), ...	the value of the probability density function of a continuous random variable X
F(x), G(x), ...	the value of the (cumulative) distribution function P($X \leq x$) of a continuous random variable X
E(X)	expectation of the random variable X
E(g(X))	expectation of g(X)
Var(X)	variance of the random variable X
G(t)	probability generating function for a random variable which takes the values $0, 1, 2, ...$
B(n, p)	binomial distribution with parameters n and p
Geo(p)	geometric distribution with parameter p
Po(λ)	Poisson distribution with parameter λ
N(μ, σ^2)	normal distribution with mean μ and variance σ^2
μ	population mean
σ^2	population variance
σ	population standard deviation
$\bar{x}$, m	sample mean
s^2, σ^2	unbiased estimate of population variance from a sample, $$s^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2$$
ϕ	probability density function of the standardised normal variable with distribution N(0, 1)
Φ	corresponding cumulative distribution function

Appendix 1: Grade descriptors

The following grade descriptors indicate the level of attainment characteristic of the middle of the given grade. They give a general indication of the required standard at each specified grade. The descriptors should be interpreted in relation to the content outlined in the syllabus; they are not designed to define that content.

The grade awarded will depend in practice upon the extent to which the candidate has met the assessment objectives overall. Shortcomings in some aspects of the examination may be balanced by better performance in others.

Distinction (D2)

Candidates manipulate mathematical expressions and use graphs, sketches and diagrams, all with high accuracy and skill. They use mathematical language correctly and proceed logically and rigorously through extended arguments or proofs. When confronted with unstructured problems they can usually devise and implement an effective solution strategy. If errors are made in their calculations or logic, these are usually noticed and corrected.

Candidates recall or recognise almost all the mathematical facts, concepts and techniques that are needed, and select appropriate ones to use in a wide variety of contexts.

Candidates recall or recognise almost all the standard models that are needed, and select appropriate ones to represent a wide variety of situations in the real world. They comprehend or understand the meaning of almost all translations into mathematics of common realistic contexts. They correctly refer results from calculations using a mathematical model to the original situation; they give sensible interpretations of their results in context and usually make sensible comments or predictions. They make intelligent comments on any modelling assumptions. They often are able to solve problems in less familiar contexts.

Candidates make appropriate and efficient use of calculators and other permitted resources, and are aware of any limitations to their use. They present results to an appropriate degree of accuracy.

Merit (M2)

Candidates manipulate mathematical expressions and use graphs, sketches and diagrams, all with a reasonable level of accuracy and skill. They use mathematical language with some skill and often proceed logically through extended arguments or proofs. When confronted with unstructured problems they often devise and implement an effective solution strategy. They sometimes notice and correct errors in their calculations.

Candidates recall or recognise most of the mathematical facts, concepts and techniques that are needed, and usually select appropriate ones to use in a variety of contexts.

Candidates recall or recognise most of the standard models that are needed, and usually select appropriate ones to represent a variety of situations in the real world. They comprehend or understand the meaning of most translations into mathematics of common realistic contexts. They often correctly refer results from calculations using a mathematical model to the original situation; they often give sensible interpretations of their results in context and sometimes make sensible comments or predictions. They sometimes make intelligent comments on any modelling assumptions. They sometimes are able to solve problems in less familiar contexts.

Candidates usually make appropriate and efficient use of calculators and other permitted resources, and are often aware of any limitations to their use. They usually present results to an appropriate degree of accuracy.

Pass (P2)

Candidates manipulate mathematical expressions and use graphs, sketches and diagrams, all with some accuracy and skill. They sometimes use mathematical language correctly and occasionally proceed logically through extended arguments or proofs.

Candidates recall or recognise some of the mathematical facts, concepts and techniques that are needed, and sometimes select appropriate ones to use in some contexts.

Candidates recall or recognise some of the standard models that are needed, and sometimes select appropriate ones to represent a variety of situations in the real world. They sometimes comprehend or understand the meaning of translations into mathematics of common realistic contexts. They sometimes correctly refer results from calculations using a mathematical model to the original situation; they sometimes interpret their results in context and attempt to make sensible comments or predictions. They try to solve problems in less familiar contexts.

Candidates usually make appropriate and efficient use of calculators and other permitted resources, and are often aware of any limitations to their use. They often present results to an appropriate degree of accuracy.

Appendix 2: Additional information

Guided learning hours

It is intended that each Principal Subject should be delivered through 380 hours of guided learning. This is a notional measure of the substance of the qualification. It includes an estimate of the time that might be allocated to direct teaching or instruction, together with other structured learning time such as directed assignments or supported individual study and practice. It excludes learner-initiated private study.

Certification title

This qualification is shown on the certificate as:

• Cambridge International Level 3 Pre-U Certificate in **Further Mathematics (Principal)**

The qualification is accredited at Level 3 of the UK National Qualifications Framework and provides a solid grounding for candidates to pursue a variety of progression pathways.

Entries

For entry information please refer to the *UK E3 Booklet*.

Grading and reporting

The Cambridge International Level 3 Pre-U Certificates in the Principal Subjects are qualifications in their own right. They are acceptable as an alternative to A Level (or other Level 3 qualifications) for entry into higher education or employment. Each individual Principal Subject is graded separately on a scale of nine grades: Distinction 1, Distinction 2, Distinction 3, Merit 1, Merit 2, Merit 3, Pass 1, Pass 2, Pass 3.

Subjects can also be combined with two core components to meet the requirements for eligibility for the Cambridge International Level 3 Pre-U Diploma. More details about the Diploma requirements and the core components can be found in a separate Diploma syllabus. The results of the individual Principal Subjects are reported on a separate certificate to the Diploma result.

Classification code for UK Centres

In the UK, every syllabus is assigned to a national classification code that indicates the subject area to which it belongs. UK Centres should be aware that candidates who enter for more than one qualification with the same classification code will have only one grade (the highest) counted for the purpose of the School and College Performance Tables.

The classification code for this syllabus is **2330**.

Language

This syllabus and the associated assessment materials are currently available in English only.

Procedures and regulations

This syllabus complies with the *CIE Code of Practice* and *The Statutory Regulation of External Qualifications 2004*.

Further information about the administration of Cambridge Pre-U qualifications can be found in the *CIE Handbook for UK Centres* available from CIE Publications or by contacting **international@cie.org.uk**.

Spiritual, moral, ethical, social, legislative, economic and cultural issues

Candidates are required to examine arguments critically and so to distinguish between truth and falsehood. They are also expected to interpret the results of modelling exercises and there are times when this could raise some of the above issues. Such issues are not assessed in examination questions.

Sustainable development, environmental education, health and safety considerations, European dimension and international agreements

Candidates are expected to interpret the results of modelling exercises and there are times when this could raise some of the above issues. Such issues are not assessed in examination questions.

Avoidance of bias

CIE has taken great care in the preparation of this syllabus and assessment materials to avoid bias of any kind.

Key Skills

This syllabus provides opportunities for the development of the Key Skills of: *Communication*, *Application of Number*, *Information Technology*, *Working with Others*, *Improving Own Learning and Performance* and *Problem Solving* at Level 3. However, the extent to which this evidence fulfils the Key Skills criteria at this level will be totally dependent on the style of teaching and learning adopted for each section.

The Key Skills awarding bodies and the regulatory authorities have produced a suite of example portfolios that will help to give candidates and practitioners a clear understanding of the requirements for the Key Skills portfolio. These are available on the QCDA Key Skills website (**www.qcda.org.uk/keyskills**). Full details of the requirements for certification can be obtained from the awarding bodies that are approved to offer Key Skills. For further information about Key Skills assessment, including the current standards, please see the document *The Key Skills Qualifications Standards and Guidance* published by the Qualifications and Curriculum Authority 2004 (ISBN 1 85838 548 2).

The following table indicates where opportunities may exist for at least some coverage of the various Key Skills criteria at Level 3 for each section.

Component	Communication	Application of Number	IT	Working with Others	Improving own Learning and Performance	Problem Solving
1		✓			✓	✓
2	✓	✓	✓	✓	✓	✓

PART OF THE
CAMBRIDGE ASSESSMENT
GROUP