Cambridge Pre-U

Cambridge International Examinations
Cambridge Pre-U Certificate Principal Subject

FURTHER MATHEMATICS **9795/02**

Paper 2 Further Applications of Mathematics **May/June 2014**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF20)

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READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value for the acceleration due to gravity is needed, use $10\,\mathrm{m\,s^{-2}}$.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 120.

This document consists of **5** printed pages and **3** blank pages.

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Section A: Probability (60 marks)

1 A machine is selecting independently and at random long rods and short rods. The length of the long rods, X cm, is normally distributed with mean 25 cm and variance 3 cm^2 and the length of the short rods, Y cm, is normally distributed with mean 15 cm and variance 2 cm^2. Assume that X and Y are independent random variables.

(i) One long rod and one short rod are chosen at random. Find the probability that the difference in the lengths, $X - Y$, is between 8 cm and 11 cm. [4]

(ii) Two long rods and two short rods are chosen at random and are assembled into an approximately rectangular frame. Find the probability that the perimeter of the resulting frame is more than 75 cm. [4]

2 The mean of a random sample of n observations drawn from a normal distribution with mean μ and variance σ^2 is denoted by $\overline{X}$. It is given that $\mathrm{P}(\mu - 0.5\sigma < \overline{X} < \mu + 0.5\sigma) > 0.95$.

(i) Find the smallest possible value of n. [5]

(ii) With this value of n, find $\mathrm{P}(\overline{X} > \mu - 0.1\sigma)$. [3]

3 A random sample of 400 seabirds is taken from a colony, ringed, and returned, unharmed, to the colony. After a suitable period of time has elapsed, a second random sample of 400 seabirds is taken, and 20 of this second sample are found to be ringed. You may assume that the probability that a seabird is captured is independent of whether or not it has been ringed and that the colony remains unchanged at the time of the second sampling.

(i) Estimate the number of seabirds in the colony. [1]

(ii) Find a 98% confidence interval for the proportion of seabirds in the colony which are ringed. [5]

(iii) Deduce a 98% confidence interval for the number of seabirds in the colony. [2]

4 The continuous random variable X has probability density function given by

$$\mathrm{f}(x) = \begin{cases} 3\mathrm{e}^{-x} & 0 \leqslant x \leqslant k, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $\mathrm{e}^{-k} = \frac{2}{3}$. [2]

(ii) Show that the moment generating function of X is given by $\mathrm{M}_X(t) = \dfrac{3}{1-t}\left(1 - \frac{2}{3}\mathrm{e}^{kt}\right)$. [4]

(iii) By expanding $\mathrm{M}_X(t)$ as a power series in t, up to and including the term in t^2, show that

$$M_X(t) = 1 + (1 - 2k)t + (1 - 2k - k^2)t^2 + \dots \, . \qquad [3]$$

[You may use the standard series for $(1 - t)^{-1}$ and e^{kt} without proof.]

(iv) Deduce that the exact value of $\mathrm{E}(X)$ is $1 - 2\ln\left(\frac{3}{2}\right)$. [1]

5 **(i)** The discrete random variable X has a Poisson distribution with mean λ. Use the probability generating function for X to show that both the mean and the variance have the value λ. [5]

(ii) The number of eggs laid by a certain insect has a Poisson distribution with variance 250. Find, using a suitable approximation, the probability that between 230 and 260 (inclusive) eggs are laid. [5]

(iii) An insect lays 250 eggs. The probability that any egg that is laid survives to maturity is 0.1. Use a suitable approximation to find the probability that more than 30 eggs survive to maturity. [3]

6 The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} \dfrac{4}{\pi(1+x^2)} & 0 \leqslant x \leqslant 1, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Verify that the median value of X lies between 0.41 and 0.42. [3]

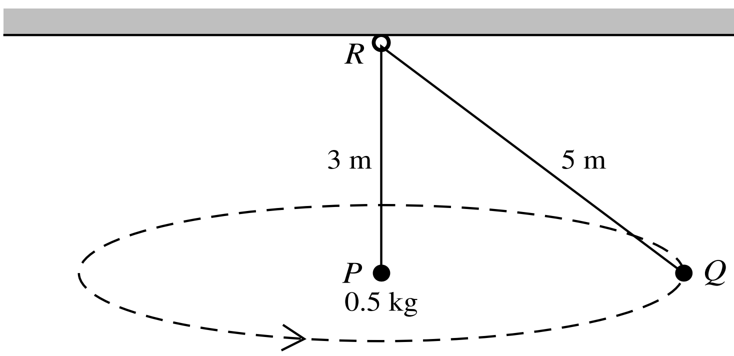
(ii) Show that $\mathrm{E}(X) = \dfrac{2}{\pi}\ln 2$. [2]

(iii) Find $\mathrm{Var}(X)$. [5]

(iv) Given that $\tan \frac{1}{8}\pi = \sqrt{2} - 1$, find the exact value of $\mathrm{P}(X > \frac{1}{3}\sqrt{3} \mid X > \sqrt{2} - 1)$. [3]

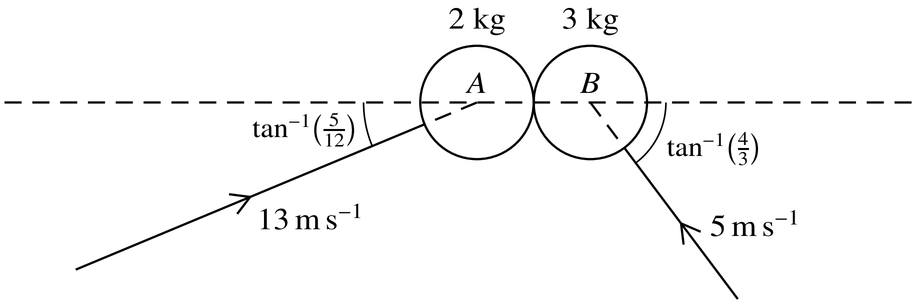
Section B: Mechanics (60 marks)

7



A light inextensible string of length 8 m is threaded through a smooth fixed ring, R, and carries a particle at each end. One particle, P, of mass 0.5 kg is at rest at a distance 3 m below R. The other particle, Q, is rotating in a horizontal circle whose centre coincides with the position of P (see diagram). Find the angular speed and the mass of Q. [8]

8



A smooth sphere with centre A and of mass 2 kg, moving at $13\,\mathrm{m\,s^{-1}}$ on a smooth horizontal plane, strikes a smooth sphere with centre B and of mass 3 kg moving at $5\,\mathrm{m\,s^{-1}}$ on the same smooth horizontal plane. The spheres have equal radii. The directions of motion immediately before impact are at angles $\tan^{-1}\left(\frac{5}{12}\right)$ to $\overrightarrow{AB}$ and $\tan^{-1}\left(\frac{4}{3}\right)$ to $\overrightarrow{BA}$ respectively (see diagram). Given that the coefficient of restitution is $\frac{2}{3}$, find the speeds of the spheres after impact. [9]

9 An engine is travelling along a straight horizontal track against negligible resistances. In travelling a distance of 750 m its speed increases from $5\,\mathrm{m\,s^{-1}}$ to $15\,\mathrm{m\,s^{-1}}$. Find the time taken if the engine was

(i) exerting a constant tractive force, [2]

(ii) working at constant power. [9]

10 One end of a light spring of length 0.5 m is attached to a fixed point F. A particle P of mass 2.5 kg is attached to the other end of the spring and hangs in equilibrium 0.55 m below F. Another particle Q, of mass 1.5 kg, is attached to P, without moving it, and both particles are then released.

(i) Show that the modulus of elasticity of the spring is 250 N. [2]

(ii) Prove that the motion is simple harmonic. [4]

(iii) Find

(a) the amplitude of the motion, [1]

(b) the greatest speed of the particles, [1]

(c) the period of the motion, [1]

(d) the time taken for the spring to be extended by 0.1 m for the first time. [3]

11 It is given that the trajectory of a projectile which is launched with speed V at an angle α above the horizontal has equation

$$y = x\tan\alpha - \frac{gx^2}{2V^2}(1 + \tan^2\alpha),$$

where the point of projection is the origin, and the x- and y-axes are horizontal and vertically upwards respectively.

(i) Express the above equation as a quadratic equation in $\tan\alpha$ and deduce that the boundary of all accessible points for this projectile has equation

$$y = \frac{1}{2gV^2}(V^4 - g^2x^2).$$ [4]

(ii) A stone is thrown with speed $\sqrt{gh}$ from the top of a vertical tower, of height h, which stands on horizontal ground. Find

(a) the maximum distance, from the foot of the tower, at which the stone can land, [3]

(b) the angle of elevation at which the stone must be thrown to achieve this maximum distance. [3]

12 A cyclist, when travelling due west at $15\,\mathrm{km\,h^{-1}}$, finds that the wind appears to be blowing **from** a bearing of 150°. When the cyclist is travelling due west at $10\,\mathrm{km\,h^{-1}}$, the wind appears to be blowing **from** a bearing of 135°. Find the velocity of the wind. [10]

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