UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

Pre-U Certificate

MARK SCHEME for the May/June 2012 question paper

for the guidance of teachers

9795 FURTHER MATHEMATICS

9795/02 Paper 2 (Further Application of Mathematics), maximum raw mark 120

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- Cambridge will not enter into discussions or correspondence in connection with these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2012 question papers for most IGCSE, Pre-U, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

1	**(i)**	$\mathrm{M}_X(t) = \int_0^\infty \mathrm{e}^{tx} k \mathrm{e}^{-kx} \mathrm{d}x$ (Limits Required)	M1	
		$= k\int_0^\infty \mathrm{e}^{(t-k)x}\mathrm{d}x = k\int_0^\infty \mathrm{e}^{-(k-t)x}\mathrm{d}x$ (Limits not required)	M1	
		$= \frac{-k}{k-t}\left[\mathrm{e}^{-(k-t)x}\right]_0^\infty = \frac{k}{k-t}$ (AG)	A1 [3]	
	(ii)	$\mathrm{M}_X{}'(t) = \frac{k}{(k-t)^2} \Rightarrow \mathrm{E}(X) = \mathrm{M}_X{}'(0) = \frac{1}{k}$	M1A1	
		$\mathrm{M}_X{}''(t) = \frac{2k}{(k-t)^3} \Rightarrow \mathrm{E}(X^2) = \mathrm{M}_X{}''(0) = \frac{2}{k^2}$	M1A1	
		(A1 ft if double sign error when differentiating twice, but CA0)	A1 [5]	
		Alternatively:		
		$\mathrm{M}_X(t) = \left(1-\frac{t}{k}\right)^{-1} = 1+\frac{t}{k}+\frac{t^2}{k^2}+\ldots = 1+\frac{1}{k}t+\frac{2}{k^2}t^2+\ldots$	M1A1	
		$\mathrm{E}(X) = \frac{1}{k}$	A1	
		$\mathrm{E}(X^2) = \frac{2}{k^2} \Rightarrow \mathrm{Var}(X) = \frac{2}{k^2} - \left(\frac{1}{k}\right)^2 = \frac{1}{k^2}$	M1A1 [5]	**[8]**
2	**(i)**	$\mathrm{E}(a\overline{X} + b\overline{Y}) = \mu$	M1	
		$\mathrm{E}(a\overline{X} + b\overline{Y}) = a\mathrm{E}(\overline{X}) + b\mathrm{E}(\overline{Y})$	M1	
		$\Rightarrow a\mu + 3b\mu = \mu \Rightarrow a + 3b = 1$	A1 [3]	
	(ii)	$\mathrm{Var}(a\overline{X} + b\overline{Y}) = a^2\mathrm{Var}(\overline{X}) + b^2\mathrm{Var}(\overline{Y}) = a^2\frac{\sigma^2}{n} + 4b^2\frac{\sigma^2}{n}$	M1	
		$= \frac{\sigma^2}{n}(a^2+4b^2) = \frac{\sigma^2}{n}(1-6b+9b^2+4b^2) = \frac{\sigma^2}{n}(1-6b+13b^2)$ (AG)	M1A1 [3]	
	(iii)	$\frac{\mathrm{d}}{\mathrm{d}b}\mathrm{Var}(a\overline{X} + b\overline{Y}) = -6 + 26b = 0 \Rightarrow b = \frac{3}{13}$	M1A1	
		$\Rightarrow \mathrm{Var}_{\min}(a\overline{X} + b\overline{Y}) = \frac{\sigma^2}{n}\left(1 - 6\times\frac{3}{13} + 13\times\frac{9}{13}\right) = \frac{4\sigma^2}{13n}$	A1 [3]	**[9]**

3	(i)	$\bar{x} = 1.675$	B1	
		99% confidence limits are $1.675 \pm 2.576 \times \frac{0.1}{\sqrt{6}}$ (ft on wrong mean)	M1A1√	
		99% confidence interval is (1.57, 1.78) (AWRT)	A1 [4]	
	(ii)	$s_n = 0.09215$ or $s_{n-1} = 0.1009$	B1	
		$v = 5 \Rightarrow t_5(0.99) = 4.032$	B1	
		99% confidence limits are $1.675 \pm 4.032 \times \frac{0.1009}{\sqrt{6}}$ or $1.675 \pm 4.032 \times \frac{0.09215}{\sqrt{5}}$	M1A1	
		99% confidence interval is (1.51, 1.84) (AWRT)	A1 [5]	
	(iii)	Sensible comment referring to the fact that 1.8 is outside the 1st interval but inside 2nd. (ft on their CI's.)	B1√ [1]	[10]
4	(i)	$\mathrm{P}(X = r) = \mathrm{e}^{-\lambda}\frac{\lambda^r}{r!}$ (may be implied by next line.)	B1	
		$\mathrm{G}_X(t) = \sum_0^\infty p_r t^r = \sum_0^\infty \mathrm{e}^{-\lambda}\frac{(\lambda t)^r}{r!} = \mathrm{e}^{-\lambda}\mathrm{e}^{\lambda t} = \mathrm{e}^{\lambda(t-1)}$ (AG)	M1A1 [3]	
	(ii)	$\mathrm{G}_{X+Y}(t) = \mathrm{e}^{\lambda(t-1)}.\mathrm{e}^{\mu(t-1)} = \mathrm{e}^{(\lambda+\mu)(t-1)} \Rightarrow (X+Y) \sim \mathrm{Po}(\lambda+\mu)$	M1A1 [2]	
	(iii)	P([0, 4] or [1, 3] or [2, 2]) = 0.2231 × 0.1336 + 0.3347 × 0.2138 + 0.2510 × 0.2565	B2,1,0	
		$\mathrm{P}(X \le 2 \mid X + Y = 4) = \frac{0.1657}{0.1954}$	M1A1√	
		= 0.848 (AWRT)	A1	
		(S.C. If no marks earned in **(iii)**, award B1 for $\mathrm{P}(X \le 2) = 0.1657$ if seen.)	[5]	[10]
5	(i)	$np = 100 \times \frac{1}{5} = 20$ and $npq = 20 \times \frac{4}{5} = 16$	B1	
		Standardisation in either **(a)** or **(b)**	M1	
	(a)	$z = \frac{14.5 - 20}{4} = -1.375 \Rightarrow \mathrm{P}(\ge 15) = 0.915$ (AWFW [0.915,0.916]) (ft on variance of 20)	A1√A1	
	(b)	$z = \frac{12.5 - 20}{4} = -1.875 \Rightarrow \mathrm{P}(<12) = 1 - 0.9696 = 0.0304$ (AWFW [0.0303,0.0304])	A1A1 [6]	
	(ii)	mean = variance = 36 $\Rightarrow$ S.D. = 6	B1	
		$z = 1.645$	B1	
		$\frac{\left(N + \frac{1}{2}\right) - 36}{6} > 1.645$ (Allow working with equality, but must be $\left(N + \frac{1}{2}\right)$.)	M1A1	
		$\Rightarrow N > 45.37 \quad \therefore$ least $N = 46$	A1 [5]	[11]

6	**(i)**	**Above** x-axis between (0, 0) to (3, 0) Correct concavity. (Do not condone parabolas.)	B1 B1 [2]	
	(ii)	$\mu = \frac{4}{27}\int_0^3 \left(3x^3 - x^4\right)\mathrm{d}x$ (Limits required.) $= \frac{4}{27}\left[3\frac{x^4}{4} - \frac{x^5}{5}\right]_0^3 = 1.8$ $\mathrm{f}'(x) = \frac{4}{27}(6x - 3x^2) = 0$ $\Rightarrow x = 0, 2 \quad \therefore$ Mode = 2	M1 A1A1 M1A1 A1 [6]	
	(iii)	Mean less than mode in **(ii)** matches negative skew in sketch.	B1 [1]	
	(iv)	$\mathrm{P}(\lvert X - \mu\rvert < \sigma) = \frac{4}{27}\int_{1.2}^{2.4}\left(3x^2 - x^3\right)\mathrm{d}x$ (Limits required.) $= \frac{4}{27}\left[x^3 - \frac{x^4}{4}\right]_{1.2}^{2.4} = 0.64$	M1 A1A1 [3]	**[12]**
7	**(i)**	Tractive force = Resistance at maximum speed $\Rightarrow \frac{75}{10} = 10k \Rightarrow k = \frac{3}{4}$. (AG)	B1 [1]	
	(ii)	$F = ma \Rightarrow \frac{75}{v} - \frac{3}{4}v = 90\frac{\mathrm{d}v}{\mathrm{d}t} \Rightarrow \frac{25}{v} - \frac{1}{4}v = 30\frac{\mathrm{d}v}{\mathrm{d}t}$ (AG) (3 terms req. for M1)	M1A1 [2]	
	(iii)	$\int_0^t \mathrm{d}t = \int_3^7 \frac{120v}{100 - v^2}\,\mathrm{d}v$ $t = -60\int_3^7 \frac{-2v}{100 - v^2}\,\mathrm{d}v = \left[-60\ln\lvert 100 - v^2\rvert\right]_3^7$ (Limits not required) $= -60\ln 51 + 60\ln 91 = 60\ln\left(\frac{91}{51}\right)$ (= 34.7) seconds.	M1 M1A1 M1A1 [5]	**[8]**

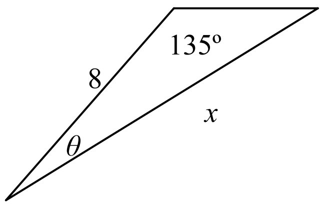
8

$x^2 = 3^2 + 8^2 - 2\times3\times8\cos135° \Rightarrow x = 10.341$ M1A1

$\frac{\sin\theta}{3} = \frac{\sin135°}{8} \Rightarrow \theta = 11.837°$ M1A1

(*θ* as in distance diagram.)

Distances

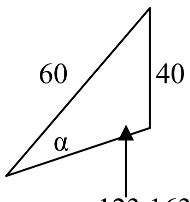


$180° - (45° + 11.837°) = 123.163°$ B1

$\frac{\sin\alpha}{40} = \frac{\sin123.163°}{60} \Rightarrow \alpha = 33.923°$ M1A1

(*α* as in velocities diagram.)

Velocities



123.163º Course is : (45º + 11.837º) – 33.923º = 22.914º

Course is 023º (Bearing notation not required.) A1

[8] **[8]**

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8 **Alternative solution:**

Equate easterly and northerly displacements at time *t* hours, where *θ* is required bearing:

Attempt at either equation M1

$60\cos\theta\, t = 4\sqrt{2} + 40t$ A1

$60\sin\theta\, t = 4\sqrt{2} + 3$ A1

Attempt to eliminate *t*. M1

Obtain e.g. $(4\sqrt{2}+3)\cos\theta - 4\sqrt{2}\sin\theta = \frac{2}{3}(4\sqrt{2}+3)$ A1

or $8.656...\cos\theta - 5.656...\sin\theta = 5.7712....$

Rearrange to obtain $\cos(\theta + 33.16) = 0.5580\ldots$ A1

Solve: $\theta + 33.16 = 56.076\ldots \Rightarrow \theta = 22.9$ M1

Course is 023º (Bearing notation not required.) A1

[8] **[8]**

9	(i)	$T\cos\theta = mg$ (Must be seen to score in part **(i)**.) $\theta = 90° \Rightarrow mg = 0 \quad \therefore AP$ can never be horizontal. $\therefore 0 < \cos\theta < 1 \quad \Rightarrow T\left(=\dfrac{mg}{\cos\theta}\right) > mg$	B1 B1 B1 [3]	
	(ii)	$T\sin\theta = ml\sin\theta\varpi^2 \Rightarrow \mathrm{T} = ml\varpi^2$	M1A1 [2]	
	(iii)	$T\dfrac{h}{l} = mg \Rightarrow T = \dfrac{mgl}{h}$ $\Rightarrow ml\varpi^2 = \dfrac{mgl}{h}$ $\Rightarrow \omega^2 h = g$. $h = 0.5 \Rightarrow \varpi = \sqrt{20} = 4.47$ Angular speed is 4.47 rad/s. (Allow 4.43 from g = 9.8.)	B1 B1 B1 B1 [4]	**[9]**
10	(i)	Let u denote speed of sphere Q before impact, v_1 and v_2 the speeds of spheres Q and P, respectively, after impact and α the angle between Q's initial direction of motion and the line of centres. After impact, if moving perpendicularly, Q moves perpendicular to line of centres and P moves along line of centres. (Stated or implied.) CLM: $mu\cos\alpha = 0 + 3mv_2$ or $mu_x = 3mv$ NEL: $eu\cos\alpha = v_2$ or $eu_x = v$. $\therefore e = \dfrac{1}{3}$	B1 M1A1 A1 A1 [5]	
	(ii)	$v_1 = u\sin\alpha$ and $v_2 = \dfrac{1}{3}u\cos\alpha$. Loss in KE is $\dfrac{1}{2}mu^2 - \dfrac{1}{2}mu^2\sin^2\alpha - \dfrac{1}{2}.3m\dfrac{u^2\cos^2\alpha}{9}$ $= \dfrac{1}{12}mu^2$ (Or remaining KE is 5/6 of initial KE etc.) But $\cos^2\alpha + \sin^2\alpha = 1$ (used) $\Rightarrow ... \Rightarrow \sin^2\alpha = \dfrac{3}{4} \quad \Rightarrow \sin\alpha = \dfrac{\sqrt{3}}{2} \Rightarrow \alpha = 60°$	B1 (both) M1A1 A1 M1 M1A1 [7]	**[12]**

11	**(i)**	$T = \dfrac{6mg\left(\sqrt{9a^2+x^2}-a\right)}{a}$	M1A1	
		Let θ be the angle between each string and line of motion of particle.	M1	
		$m\ddot{x} = -2T\cos\theta = -\dfrac{12mg}{a}\left(\sqrt{9a^2+x^2}-a\right)\times\dfrac{x}{\sqrt{9a^2+x^2}}$	A1A1	
		$\Rightarrow \ddot{x} = -\dfrac{12gx}{a}\left(1-\dfrac{a}{\sqrt{9a^2+x^2}}\right)$	A1 [6]	
	(ii)	$\therefore \ddot{x} \approx (-12g+4g)\dfrac{x}{a} = -\dfrac{8g}{a}x$	M1A1	
		Which is SHM of period $2\pi\sqrt{\dfrac{a}{8g}}$ or $\pi\sqrt{\dfrac{a}{2g}}$.	A1 [3]	
	(iii)	$v_{\max} = \varpi a \Rightarrow \dfrac{ga}{200} = \dfrac{8g}{a}A^2 \Rightarrow A^2 = \dfrac{a^2}{1600} \Rightarrow A = \dfrac{a}{40}$ where A is the amplitude.	M1A1 [2]	**[11]**
12	**(i)**	$x = 20\cos\alpha t \quad y = 20\sin\alpha t - 5t^2$ (Allow g or 9.8 here for B mark).	B1B1	
		$y = 20\sin\alpha\cdot\dfrac{x}{20\cos\alpha} - 5\left(\dfrac{x}{20\cos\alpha}\right)^2 = x\tan\alpha - \dfrac{x^2}{80}(1+\tan^2\alpha)$ (AG)	M1A1 [4]	
	(ii)	$x^2\tan^2\alpha - 80x\tan\alpha + x^2 + 80y = 0$ (Can be implied by what follows.)	B1	
		Real roots $\Rightarrow 6400x^2 - 4x^2(x^2+80y) \geq 0$	M1A1	
		$\Rightarrow 1600 - x^2 - 80y \geq 0 \Rightarrow y \leq 20 - \dfrac{x^2}{80}$, $(x \neq 0)$.	A1 [4]	
	(iii)	$x = R\cos\beta$ and $y = R\sin\beta \Rightarrow y = x\tan\beta$.		
		In $y = 20 - \dfrac{x^2}{80} \Rightarrow R\sin\beta = 20 - \dfrac{R^2(1-\sin^2\beta)}{80}$	M1A1	
		$\therefore$ $R^2(1-\sin^2\beta) + 80R\sin\beta - 1600 = 0 \Rightarrow (R[1-\sin\beta]+40)(R[1+\sin\beta]-40)$	M1	
		$\Rightarrow R = \dfrac{40}{1+\sin\beta}$ (up) or $\dfrac{-40}{1-\sin\beta}$ (down)	A1 [4]	
12	**(iii)**	**Alternative solution:**		
		$x^2 + 80\tan\beta x - 1600 = 0$	B1	
		$\Rightarrow x = -40\tan\beta \pm 40\sec\beta$	B1	
		$\Rightarrow R_{up} = \dfrac{-40\sin\beta+40}{\cos^2\beta} = \dfrac{40(1-\sin\beta)}{1-\sin^2\beta} = \dfrac{40}{1+\sin\beta}$	B1	
		$\Rightarrow R_{down} = \left\lvert\dfrac{-40\sin\beta-40}{\cos^2\beta}\right\rvert = \dfrac{40(1+\sin\beta)}{1-\sin^2\beta} = \dfrac{40}{1+\sin\beta}$	B1 [4]	**[12]**