Mathematics and Further Mathematics Pre-U June 2010

The following question papers for Mathematics and Further Mathematics are the first papers to be taken by Pre-U students at the end of the two-year course. This also means that they are the first live question papers to be set for Pre-U candidates and in common with all new Pre-U examination questions were subjected to the same rigorous question paper setting procedure, involving subject experts and experienced teachers as well as assessment professionals.

Setting a new standard is always a challenging activity for those involved and CIE was aware that Mathematics in particular might be a difficult standard to gauge owing to the wide spread of ability in this subject. Extra work was therefore commissioned to assess the level of difficulty of the examination papers and the outcome reassured CIE that the papers were appropriate for the cohort.

However, it became evident as soon as Paper 1 had been sat that the level of difficulty might have been too high, and this was confirmed after Paper 2 had been taken. The grade boundaries for these papers were set at a level that reflected A level grades A and E at D3 and P3 respectively. Additional statistical analysis led to further work to ensure fairness to all candidates.

CIE is releasing these papers according to its usual practice but would like to point out that future examination papers will include more accessible marks for those in the middle of the distribution.

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
Cambridge International Level 3 Pre-U Certificate
Principal Subject

FURTHER MATHEMATICS **9795/02**

Paper 2 Further Applications of Mathematics **May/June 2010**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF20)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value for the acceleration due to gravity is needed, use $10\,\mathrm{m\,s^{-2}}$.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 120.

This document consists of **5** printed pages and **3** blank pages.

Section A: Mechanics (60 marks)

1 A lorry moves along a straight horizontal road. The engine of the lorry produces a constant power of 80 kW. The mass of the lorry is 10 tonnes and the resistance to motion is constant at 4000 N.

(i) Express the driving force of the lorry in terms of its velocity and hence, using Newton's second law, write down a differential equation which connects the velocity of the lorry and the time for which it has been moving. [2]

(ii) Hence find the time taken, in seconds, for the lorry to accelerate from $10\,\text{m}\,\text{s}^{-1}$ to $15\,\text{m}\,\text{s}^{-1}$. [5]

2 At 12 00 hours an aircraft, A, sets out to intercept a second aircraft, B, which is 200 km away on a bearing of 300° and is flying due east at $600\,\text{km}\,\text{h}^{-1}$. Both aircraft are at the same altitude and continue to fly horizontally.

(i) (a) Find the bearing on which A should fly when travelling at $800\,\text{km}\,\text{h}^{-1}$. [3]

(b) Find the time at which A intercepts B in this case. [4]

(ii) Find the least steady speed at which A can fly to intercept B. [2]

3 A particle is projected at an angle θ above the horizontal from the foot of a plane which is inclined at 45° to the horizontal. Subsequently the particle impacts on the plane at a higher point.

(i) Prove that the angle at which the particle strikes the plane is ϕ, where

$$\tan\phi = \frac{\tan\theta - 1}{3 - \tan\theta}.$$ [9]

(ii) Find the angle to the horizontal at which the particle would have to be projected if it were to strike the plane horizontally. [2]

4 One end of a light elastic string of natural length 0.2 m and modulus of elasticity 100 N is attached to a fixed point A. The other end is attached to a particle of mass 5 kg. The particle moves with angular speed ω radians per second in a horizontal circle with the centre vertically below A. The string makes an angle θ with the vertical.

(i) By considering the horizontal component of the tension in the string, show that the tension in the string is $(1 + 5x)\omega^2$ N, where x is the extension, in metres, of the string. [2]

(ii) (a) By considering vertical forces and also Hooke's law, deduce that $\cos\theta = \dfrac{1}{10x}$. [4]

(b) Show that $\omega > \dfrac{10\sqrt{3}}{3}$. [2]

(iii) When the value of ω is $5\sqrt{2}$, find the radius of the circular motion. [3]

5 A particle of mass m is attached by a light elastic string of natural length l and modulus of elasticity λ to a fixed point A, from which it is allowed to fall freely. The particle first comes to rest, instantaneously, at B, where $AB = 2l$. Prove that

(i) $\lambda = 4mg$, [2]

(ii) while the string is taut, $\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\dfrac{4g}{l}x$, where x is the displacement from the equilibrium position at time t, [4]

(iii) the time taken between the first occasion when the string becomes taut and the next occasion when it becomes slack is

$$\left[\tfrac{1}{2}\pi + \sin^{-1}\left(\tfrac{1}{3}\right)\right]\sqrt{\frac{l}{g}}.$$ [5]

6 Two smooth spheres, A and B, have masses m and $2m$ respectively and equal radii. Sphere B is at rest on a smooth horizontal floor. Sphere A is projected with speed u along the floor in a direction parallel to a smooth vertical wall and strikes B obliquely. Subsequently B strikes the wall at an angle α with the wall. The coefficient of restitution between A and B and between B and the wall is 0.5. After B has struck the wall, A and B are moving parallel to each other.

(i) Write down a momentum equation and a restitution equation **along the line of centres** for the impact between A and B. Hence find the components of velocity of A and B in this direction after this first impact. [6]

(ii) Find the value of α, giving your answer in degrees. [5]

Section B: Probability (60 marks)

7 The number of goals scored by a hockey team in an interval of time of length t minutes follows a Poisson distribution with mean $\frac{1}{24}t$. The random variable T is defined as the length of time, in minutes, between successive goals.

(i) (a) Show that $\mathrm{P}(T < t) = 1 - \mathrm{e}^{-\frac{1}{24}t}$ for $t \geqslant 0$. [2]

(b) Hence find the probability density function of T. [2]

(ii) Find the exact value of the interquartile range of T. [4]

8 Two groups of Year 12 pupils, one at each of schools A and B, are given the same mathematics test. The scores, x and y, of pupils at schools A and B respectively are summarised as follows.

School A	$n_A = 15$	$\bar{x} = 53$	$\Sigma(x - \bar{x})^2 = 925$
School B	$n_B = 12$	$\bar{y} = 47$	$\Sigma(y - \bar{y})^2 = 850$

(i) Assuming that the two groups are random samples from independent normal populations with means μ_A and μ_B respectively and a common, but unknown, variance, construct a 98% confidence interval for $\mu_A - \mu_B$. [6]

(ii) Comment, with a reason, on any difference in ability between the two schools. [2]

9 **(i)** Two independent discrete random variables X and Y follow Poisson distributions with means λ and μ respectively. Prove that the discrete random variable $Z = X + Y$ follows a Poisson distribution with mean $\lambda + \mu$. [3]

A garage has a white limousine and a green limousine for hire. Demands to hire the white limousine occur at a constant mean rate of 3 per week and demands to hire the green limousine occur at a constant mean rate of 2 per week. Demands for hire are received independently and randomly.

(ii) Calculate the probability that in a period of **two** weeks

(a) no demands for hire are received, giving your answer to 3 significant figures, [2]

(b) seven demands for hire are received. [2]

(iii) Find the least value of n such that the probability of at least n demands for hire in a period of **three** weeks is less than 0.005. [3]

10 A box contains a large number, n, of identical dice, which are thought to be biased. The probability that one of these dice will show a six in a single roll is p. The n dice are rolled many times and the number of sixes obtained in each trial is recorded. In 4.01% of these trials 56 or more dice showed a six. In 10.56% of these trials 37 or fewer dice showed a six. Using a suitable normal approximation, find the values of n and p. [11]

11 The thickness of a randomly chosen paperback book is P cm and the thickness of a randomly chosen hardback is H cm, where P and H have distributions $N(2.0, 0.75)$ and $N(5.0, 2.25)$ respectively. When more than one book is selected, any book is selected independently of all other books.

(i) Calculate the probability that a randomly chosen hardback is more than 1 cm thicker than a randomly chosen paperback. [4]

(ii) Calculate the probability that 2 paperbacks and 4 hardbacks, randomly chosen, have a combined thickness of less than 20 cm. [4]

(iii) Find the probability that a randomly chosen hardback is more than twice the thickness of a randomly chosen paperback. [4]

12 Two players, A and B, are taking turns to shoot at a basket with a basketball. The winner of this game is the first player to score a basket. The probability that A scores a basket with any shot is $\frac{1}{4}$ and the probability that B scores a basket with any shot is $\frac{1}{5}$. Each shot is independent of all other shots. A shoots first.

(i) Find

(a) the probability that B wins with his first shot, [1]

(b) the probability that A wins with his second shot, [2]

(c) the probability that A wins the game. [3]

(ii) R is the total number of shots taken by A and B up to and including the shot that scores a basket.

(a) Show that the probability generating function of R is given by

$$\mathrm{G}(t) = \frac{5t + 3t^2}{4(5 - 3t^2)}.$$ [3]

(b) Hence find $\mathrm{E}(R)$. [2]

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