

Mathematics and Further Mathematics Pre-U June 2010

The following question papers for Mathematics and Further Mathematics are the first papers to be taken by Pre-U students at the end of the two-year course. This also means that they are the first live question papers to be set for Pre-U candidates and in common with all new Pre-U examination questions were subjected to the same rigorous question paper setting procedure, involving subject experts and experienced teachers as well as assessment professionals.

Setting a new standard is always a challenging activity for those involved and CIE was aware that Mathematics in particular might be a difficult standard to gauge owing to the wide spread of ability in this subject. Extra work was therefore commissioned to assess the level of difficulty of the examination papers and the outcome reassured CIE that the papers were appropriate for the cohort.

However, it became evident as soon as Paper 1 had been sat that the level of difficulty might have been too high, and this was confirmed after Paper 2 had been taken. The grade boundaries for these papers were set at a level that reflected A level grades A and E at D3 and P3 respectively. Additional statistical analysis led to further work to ensure fairness to all candidates.

CIE is releasing these papers according to its usual practice but would like to point out that future examination papers will include more accessible marks for those in the middle of the distribution.



FURTHER MATHEMATICS

9795/01

Paper 1 Further Pure Mathematics

May/June 2010

3 hours

Additional Materials: Answer Booklet/Paper
 Graph Paper
 List of Formulae (MF20)



READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 120.

This document consists of **5** printed pages and **3** blank pages.



- 1 The equation $x^3 + 2x^2 + x - 7 = 0$ has roots α , β and γ . Use the substitution $y = 1 + x^2$ to find an equation, with integer coefficients, whose roots are $1 + \alpha^2$, $1 + \beta^2$ and $1 + \gamma^2$. [4]

- 2 Use the method of differences to express $\sum_{r=1}^n \frac{1}{4r^2 - 1}$ in terms of n , and hence deduce the sum of the infinite series

$$\frac{1}{3} + \frac{1}{15} + \frac{1}{35} + \dots + \frac{1}{4n^2 - 1} + \dots \quad [5]$$

- 3 The points $A(1, 3)$, $B(4, 36)$ and $C(9, 151)$ lie on the curve with equation $y = p + qx + rx^2$.

(i) Using this information, write down three simultaneous equations in p , q and r . [1]

(ii) Re-write this system of equations in the matrix form $\mathbf{C}\mathbf{x} = \mathbf{a}$, where \mathbf{C} is a 3×3 matrix, \mathbf{x} is an unknown vector, and \mathbf{a} is a fixed vector. [1]

(iii) By finding \mathbf{C}^{-1} , determine the values of p , q and r . [4]

- 4 (i) Using the definitions of \sinh and \cosh in terms of exponentials, prove that

$$\cosh A \cosh B + \sinh A \sinh B \equiv \cosh(A + B). \quad [1]$$

(ii) Solve the equation $5 \cosh x + 3 \sinh x = 12$, giving your answers in the form $\ln(p \pm q\sqrt{2})$ for rational numbers p and q to be determined. [5]

- 5 A curve has equation $y = \frac{x^2 + 5x - 6}{x + 3}$ for $x \neq -3$.

(i) Show that $\frac{dy}{dx} > 1$ at all points on the curve. [3]

(ii) Sketch the curve, justifying all significant features. [8]

- 6 (i) The set S consists of all 2×2 matrices of the form $\begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}$, where $n \in \mathbb{Z}$.

(a) Show that S , under the operation of matrix multiplication, forms a group G . [You may assume that matrix multiplication is associative.] [3]

(b) State, giving a reason, whether G is abelian. [1]

(c) The group H is the set \mathbb{Z} together with the operation of addition. Explain why G is isomorphic to H . [1]

(ii) The plane transformation T is given by the matrix $\begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}$, where n is a non-zero integer. Describe T fully. [3]

- 7 A curve C has polar equation $r = 2 + \cos \theta$ for $-\pi < \theta \leq \pi$.
- (i) The point P on C corresponds to $\theta = \alpha$, and the point Q on C is such that POQ is a straight line, where O is the pole. Show that the length PQ is independent of α . [3]
- (ii) Find, in an exact form, the area of the region enclosed by C . [3]
- (iii) Show that $(x^2 + y^2 - x)^2 = 4(x^2 + y^2)$ is a cartesian equation for C . Identify the coordinates of the point which is included in this cartesian equation but is not on C . [3]
- 8 For the differential equation $t^2 \frac{d^2x}{dt^2} - 4t \frac{dx}{dt} + (6 - 4t^2)x = 0$, use the substitution $x = t^2u$ to find a differential equation involving t and u only. Hence solve the above differential equation, given that $x = e - 1$ and $\frac{dx}{dt} = 4e$ when $t = 1$. [10]
- 9 Three non-collinear points A , B and C have position vectors \mathbf{a} , \mathbf{b} and \mathbf{c} respectively, relative to the origin O . The plane through A , B and C is denoted by Π .
- (i) (a) Prove that the area of triangle ABC is $\frac{1}{2}|\mathbf{a} \times \mathbf{b} + \mathbf{b} \times \mathbf{c} + \mathbf{c} \times \mathbf{a}|$. [3]
- (b) Describe the significance of the vector $\mathbf{a} \times \mathbf{b} + \mathbf{b} \times \mathbf{c} + \mathbf{c} \times \mathbf{a}$ in relation to Π . [1]
- (ii) (a) In the case when $\mathbf{a} = a\mathbf{i}$, $\mathbf{b} = b\mathbf{j}$ and $\mathbf{c} = c\mathbf{k}$, where a , b and c are positive scalar constants, determine the equation of Π in the form $\mathbf{r} \cdot \mathbf{n} = d$, where the components of \mathbf{n} and the value of the scalar constant d are to be given in terms of a , b and c . [4]
- (b) Deduce the shortest distance from the origin O to Π in this case. [2]
- 10 One root of the equation $z^5 - 1 = 0$ is the complex number $\omega = e^{\frac{2\pi i}{5}}$.
- (i) Show that
- (a) $\omega^5 = 1$, [1]
- (b) $\omega + \omega^2 + \omega^3 + \omega^4 = -1$, [2]
- (c) $\omega + \omega^4 = 2 \cos \frac{2\pi}{5}$, and write down a similar expression for $\omega^2 + \omega^3$. [3]
- (ii) Using these results, find the values of $\cos \frac{2\pi}{5} + \cos \frac{4\pi}{5}$ and $\cos \frac{2\pi}{5} \times \cos \frac{4\pi}{5}$, and deduce a quadratic equation, with integer coefficients, which has roots
- $$\cos \frac{2\pi}{5} \quad \text{and} \quad \cos \frac{4\pi}{5}. \quad [5]$$

11 (i) At all points (x, y) on the curve C , $\frac{dy}{dx} + xy = 0$.

(a) Prove by induction that, for all integers $n \geq 1$,

$$\frac{d^{n+1}y}{dx^{n+1}} + x \frac{d^n y}{dx^n} + n \frac{d^{n-1}y}{dx^{n-1}} = 0$$

where $\frac{d^0 y}{dx^0} = y$. [5]

(b) Given that $y = 1$ when $x = 0$, determine the Maclaurin expansion of y in ascending powers of x , up to and including the term in x^6 . [5]

(ii) Solve the differential equation $\frac{dy}{dx} + xy = 0$ given that $y = 1$ when $x = 0$. [4]

(iii) Given that $Z \sim N(0, 1)$, use your answers to parts (i) and (ii) to find an approximation, to 4 decimal places, to the probability $P(Z \leq 1)$. [4]

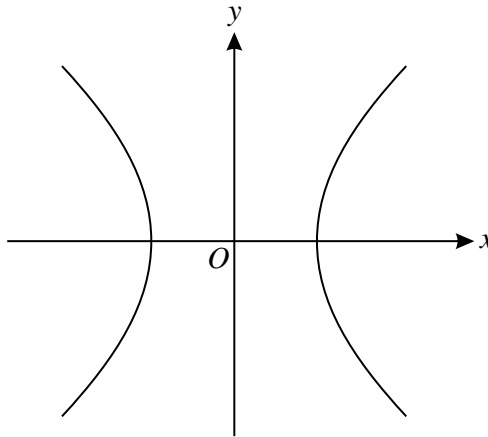
[Note that the probability density function of the standard normal distribution is $f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$.]

12 (i) Let $I_n = \int \frac{x^n}{\sqrt{x^2 + 1}} dx$, for integers $n \geq 0$.

By writing $\frac{x^n}{\sqrt{x^2 + 1}}$ as $x^{n-1} \times \frac{x}{\sqrt{x^2 + 1}}$, or otherwise, show that, for $n \geq 2$,

$$nI_n = x^{n-1}\sqrt{x^2 + 1} - (n-1)I_{n-2}. \quad [5]$$

(ii) The diagram shows a sketch of the hyperbola H with equation $\frac{x^2}{4} - \frac{y^2}{16} = 1$.



- (a) Find the coordinates of the points where H crosses the x -axis. [1]
- (b) The curve J has parametric equations $x = 2 \cosh \theta$, $y = 4 \sinh \theta$, for $\theta \geq 0$. Show that these parametric equations satisfy the cartesian equation of H , and indicate on a copy of the above diagram which part of H is J . [3]
- (c) The arc of the curve J between the points where $x = 2$ and $x = 34$ is rotated once completely about the x -axis to form a surface of revolution with area S . Show that

$$S = 16\pi \int_{\alpha}^{\beta} \sinh \theta \sqrt{5 \cosh^2 \theta - 1} d\theta$$

for suitable constants α and β . [4]

- (d) Use the substitution $u^2 = 5 \cosh^2 \theta - 1$ to show that

$$S = \frac{8\pi}{\sqrt{5}} (644\sqrt{5} - \ln(9 + 4\sqrt{5})). \quad [9]$$

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