FURTHER MATHEMATICS

The following question papers for Further Mathematics are the first papers to be taken by Pre-U students at the end of the two-year course. This also means that they are the first live question papers to be set for Pre-U candidates and in common with all new Pre-U examination questions were subjected to the same rigorous question paper setting procedure, involving subject experts and experienced teachers as well as assessment professionals.

Setting a new standard is always a challenging activity for those involved and CIE was aware that Mathematics and Further Mathematics in particular might be a difficult standard to gauge owing to the wide spread of ability in this subject. Extra work was therefore commissioned to assess the level of difficulty of the examination papers and the outcome reassured CIE that the papers were appropriate for the cohort.

However, it became evident as soon as Paper 1 had been sat that the level of difficulty might have been too high, and this was confirmed after Paper 2 had been taken. The grade boundaries for these papers were set at a level that reflected A level grades A and E at D3 and P3 respectively. Additional statistical analysis led to further work to ensure fairness to all candidates.

CIE is releasing these papers according to its usual practice but would like to point out that future examination papers will include more accessible marks for those in the middle of the distribution.

Paper 9795/01

Further Pure Mathematics

General Comments

It was pleasing to see that so many candidates coped admirably with the questions on this first Pre-U Further Mathematics paper, producing some extraordinarily high-quality mathematics, with several candidates achieving really impressive total scores near, or above, the 100 mark (out of 120).

In general, candidates found the paper challenging both in terms of time and content. One of the principal motivating features of the Pre-U is that candidates get to study the subject holistically, and are thus examined in a way that reflects this goal. Candidates were often left to decide for themselves what they should be doing and the methods they needed to employ in order to achieve the answer. Although almost all candidates seemed to have made attempts at all questions within their scope, it was clear that many found little time at the end of the examination to go back and review their working and answers.

Comments on Individual Questions

Question 1

The algebraic demands of this question caught many candidates by surprise, and they were not always handled comfortably. The key was to isolate the $\sqrt{y-1}$ terms and then square both sides: candidates who tried the latter before the former generally did poorly. A handful of candidates attempted the question using the relationships between roots and coefficients. These candidates found it a lot harder to get to the finish intact and they were, subsequently, less successful.

Answer: $y^3 - 5y^2 + 36y - 81 = 0$

Question 2

The essential first step in this question was to split $\dfrac{1}{4r^2-1}$ into partial fractions. Thereafter, it was a straightforward question and the majority of candidates coped very well with it.

Answer: $S_\infty = \frac{1}{2}$

Question 3

This was found to be the easiest question on the paper, with only occasional arithmetic slips to prevent candidates from getting full marks on it.

Answers: **(i)** $\begin{aligned} 3 &= p+q+r \\ 36 &= p+4q+16r \\ 151 &= p+9q+81r \end{aligned}$ **(ii)** $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 4 & 16 \\ 1 & 9 & 81 \end{pmatrix}\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} 3 \\ 36 \\ 151 \end{pmatrix}$ **(iii)** $p = -2$, $q = 3.5$, $r = 1.5$

Question 4

Part **(i)** was generally managed without difficulty. There were two main approaches to part **(ii)**: to use the result of part **(i)** to express the LHS of the given equation using the "Compound Angle" formula for trigonometric functions, or to turn to exponentials, obtain a quadratic in e^x, and solve that. The majority of candidates took the second route which tended to be the easier one.

Answer: **(ii)** $\ln(1.5 \pm \sqrt{2})$

Question 5

This question proved to be very well-received, with almost all candidates at least knowing what they should be attempting to do. There were a few long and unclear arguments about why $\dfrac{dy}{dx} > 1$, which were marked generously on this occasion (since, fortunately, the denominator of the derivative is a perfect square). The only general loss of marks arose when candidates overlooked the existence of an oblique asymptote or did not find its equation correctly.

Question 6

Some candidates appeared unsure of how to deal with groups. A considerable proportion of the entry lost 3 or 4 "easy" marks due to a lack of certainty as to how to express themselves: several tried to prove closure by multiplying $\begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}$ by $\begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}$ for instance, or by using something similarly unhelpful or not general. The same occurred with the simple commutativity proof in part **(i)(b)**. In part **(c)**, a simple statement that $\begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix} \to n$ would have gained the single mark available (although one should, of course, confirm that the mapping maintains the group structure of the respective operations; i.e.

$$f\left\{\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} \times_M \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix}\right\} = f(a) + f(b) = a + b \ ).$$

In part **(ii)**, the transformation is a shear. A number of candidates described it as a "skew" which, on this occasion, was only penalised by the loss of one mark.

Answer: **(ii)** Shear, parallel to the x-axis mapping e.g. $\begin{pmatrix} 0 \\ 1 \end{pmatrix} \to \begin{pmatrix} n \\ 1 \end{pmatrix}$

Question 7

Candidates who realised in part **(i)** that Q had polar coordinates ($2 + \cos\phi$, ϕ), where $\phi = \theta + \pi$ (or something similar), managed to show that $PQ = 4$ relatively easily; those who did not generally gave up on this part. Part **(ii)** involved the more routine work and nearly all candidates spotted that one of the double-angle formulae for cos was required. Part **(iii)** received answers of a more variable quality. The simplest approach was to multiply the polar equation by r first (which, incidentally, gives away the origin as the “naughty” extra point that gets introduced); many candidates chose instead to substitute for x and y and then had to cope with some trigonometry work and, in some cases, some very cumbersome squaring. A few candidates worked backwards, finding themselves having to explain the choice of the positive sign when square-rooting $(r - \cos\theta)^2 = 4$. It was also the case that the more complicated the approach employed by a candidate, the less likely they were to choose (0, 0) at the end.

Answer: **(ii)** $\frac{9}{2}\pi$

Question 8

This question proved difficult, with many candidates scoring fewer than half marks. The standard differential equations work was out of reach to some candidates because of the substitution. Some candidates did not realise that $x = t^2u$ meant that both t *and* u were variables, and many more could not find $\frac{\mathrm{d}u}{\mathrm{d}t}$ and $\frac{\mathrm{d}^2u}{\mathrm{d}t^2}$ correctly, preventing them from making any substantial progress.

Answer: $x = t^2\left(\mathrm{e}^{2t-1} - \mathrm{e}^{2-2t}\right)$

Question 9

The greatest difficulty for candidates in this question lay in the lack of any numbers: this question was both theoretical and algebraic. Quite a few candidates gave up on it early on. Even amongst the highest-scoring candidates, few realised that $(\mathbf{b} - \mathbf{a})\times(\mathbf{c} - \mathbf{a})$ would ultimately lead to $\mathbf{a}\times\mathbf{b} + \mathbf{b}\times\mathbf{c} + \mathbf{c}\times\mathbf{a}$, and that it was the properties of the cross-product that were being tested. However, it is worth noting that many candidates continued, and proceeded to score up to 7 of the 10 marks on the question by skipping onto part **(ii)**, where they coped successfully despite the algebraic tone to the question.

Answers: **(ii)(a)** $\mathbf{n} = \begin{pmatrix} bc \\ ca \\ ab \end{pmatrix}$, $d = abc$

Question 10

There were a variety of approaches that could be employed to tackle this question, and these had varied success. Whilst using de Moivre’s theorem was clearly a popular method, the rather more obvious algebraic (polynomial) results were seldom to be seen; e.g. if ω is a root of the equation $z^5 - 1 = 0$ then $\omega^5 - 1 = 0$ and the first result follows, as indeed does the second when the factor of $(\omega - 1)$ is extracted. Trigonometric approaches did not work quite so easily, and most candidates using such an approach had to give up. The trigonometry worked well in part **(c)** since $\omega^4 = \frac{1}{\omega} = \cos\frac{2\pi}{5} - \mathrm{i}.\sin\frac{2\pi}{5}$ but some candidates struggled over this idea, often at considerable length. For the last result of part **(i)**, it was hoped that candidates would have seen what was going on and just write down the answer. Those candidates who got this far had no difficulty in attempting the equation. It should be noted for future reference that several candidates overlooked the request for integer coefficients to the quadratic, and several others only wrote down an expression, not an equation.

Answers: **(ii)** $-\frac{1}{2}$, $-\frac{1}{4}$, $4x^2 + 2x - 1 = 0$

Question 11

Many of the candidates found the structured later questions more accessible. Most managed a good attempt at the induction, although their working was often not well presented; for instance, the validity of the "n = 1" case arises from differentiating the given first-order differential equation; many candidates just seemed to write down what the given formula yielded when n = 1 was substituted, and this was insufficiently clear to earn this mark. At the other end of the proof, there was a mark for rounding it off and explaining the induction logic. This was very poorly done indeed, often overlooked or attempted via untrue statements such as, "Since we have proved the result is true for n = 1, n = k and n = k + 1, then …". Most candidates did attempt to use this result to find the coefficients of y in the Maclaurin expansion, though careless mistakes were quite common. Part **(ii)** was usually done correctly by the method of separation of variables, although a few candidates managed it by the integrating-factor method instead. About half the candidates realised what to do in part **(iii)**, although they often overlooked the fact that they should be integrating their polynomial approximation for y *before* substituting in the limits. Only a small number of candidates actually arrived at a correct final answer.

Answers: **(i)(b)** $y = 1 - \frac{1}{2}x^2 + \frac{1}{8}x^4 - \frac{1}{48}x^6 + \ldots$ **(ii)** $y = \mathrm{e}^{-\frac{1}{2}x^2}$ **(iii)** 0.8412

Question 12

Despite the hint in part **(i)**, there were very few attempts that got beyond the first line of an integration by parts. The real difficulty seemed to lie in the fact that so few candidates appreciated that their second integral should have the $\sqrt{x^2+1}$ in the denominator, and that to get it required writing it as $\frac{x^2+1}{\sqrt{x^2+1}}$.

Candidates generally tended to realise that they did not know how to proceed and, showing excellent exam technique, carried on with the rest of the question. In part **(ii)**, the one mark for part **(a)** was almost invariably scored. Part **(b)** was handled less well. Most candidates opted for either the right-hand-side or the upper-half of H. A few noted that the cosh function is always ≥ 1. However, the question also specified that $\theta \geq 0$, and this was almost never noticed. The final two parts of this question were hard, and only the brave and the best managed to cope with it with much success.

Answer: **(ii)(a)** $(\pm 2, 0)$

FURTHER MATHEMATICS

Paper 9795/02

Further Application of Mathematics

General Comments

The scripts for this inaugural Pre-U paper produced a wide range of responses. There were a number of outstanding scripts and a small number of poor scripts. Many candidates presented their work well, displaying their arguments in an orderly and logical fashion, which made it easy to follow their line of thinking. Arithmetical accuracy was, mostly, quite good.

Most candidates seemed to have time to complete the paper. A high proportion of scripts had substantial attempts at all twelve questions. Possibly there was not much time to check answers, as a number of rather implausible answers followed substantially correct work.

Candidates seemed to have covered most areas of the syllabus and good work was seen on both the mechanics section and the probability section of the paper. As might be expected, some candidates proved stronger on one section rather than the other, but the differences in performance were, by no means, all in one direction.

Comments on specific questions

Section A: Mechanics

Question 1

This first question proved to be rather more of a challenge than was expected. A number of candidates had the wrong sign for the resistance term in their differential equation. A small number of candidates either had $v\dfrac{\mathrm{d}v}{\mathrm{d}x}$, or simply a, for the acceleration, in which case no progress was possible. The next step was to separate the variables, and some candidates did not use the correct algebra here. There were some incorrect statements involving reciprocals and a poor understanding of fractions exhibited by some candidates. Generally there was a propensity for candidates to persevere with fractions, and in some case decimals, rather than writing equations with integer coefficients, even among those who obtained the correct result. The final hurdle was to get the integrand into a form in which it could be integrated. Again, algebraic frailties proved to be some candidates' undoing.

Answer: $2.5(20\ln 2 - 5)$ seconds (or 22.2 seconds)

Question 2

Part **(i)** of this question produced a number of different approaches. The use of a vector triangle of velocities was popular. The sine rule readily gave the angle between *A*'s velocity and the direction of the velocity of *A* relative to *B*. It was then straightforward to find the bearing and a good many candidates did so. By calculating the speed of *A* relative to *B*, from the same triangle, the time to interception was obtained correctly by many using this approach. Others used components of velocity and distance in the east-west and north-south directions, in some cases setting their calculations out as vectors. This approach tended to produce sign errors and was somewhat less successful than the previous approach. There were relatively few correct answers to part **(ii)**. Candidates found difficulty in deciding upon the correct vector triangle. Some texts indicate that the moving object should steer perpendicular to the relative path. Those aware of this produced the correct result without trouble.

Answers: **(i) (a)** 322° or $\mathrm{N}\,38^\circ\,\mathrm{W}$ **(ii)** 1209 or 1210 **(b)** 300 kmh^{-1}

Question 3

This question also brought a variety of responses. Essentially they fell into two categories: these were taking axes horizontally and vertically, or taking axes parallel and perpendicular to the plane. Both produced a reasonable share of correct work. The usual problems with this sort of work were evident in the work of some candidates: forgetting to resolve g, if resolving parallel and perpendicular to the plane, interchanging sines and cosines of angles, getting wrong signs. Less confident candidates who found there were insufficient equations to solve the problem, had usually overlooked the fact that the horizontal and vertical displacements were equal at the point of landing, since the plane was inclined at 45° to the horizontal. The more successful candidates tended to apply the compound angle formula earlier in their solution and thus deal with less complicated expressions. When candidates did not simplify expressions they often did not obtain the required result even though they had all the necessary information. Those who obtained the result, but with an overall opposite sign, were not penalised. Many picked up the final 2 marks in part **(ii)** since they realised that the expression in part **(i)** took the value 1, when the particle landed horizontally.

Answer: **(ii)** $\tan^{-1} 2$ or 63.4°

Question 4

Part **(i)** was done poorly by many candidates, largely because they did not resolve the tension in the string horizontally. The absence of $\sin\theta$ on both sides of the equation was extremely frequent. Part **(ii)(a)**, by contrast, was often done well. Only the occasional good candidate was able to solve part **(b)**. There were some good answers to Part **(iii).**

Answer: **(iii)** $\frac{\sqrt{3}}{5}$ or 0.346 m

Question 5

The majority of candidates were able to write a correct energy equation in part **(i)** and thus obtain the modulus of elasticity correctly. In part **(ii)** there were two distinct groups, those who had learnt the necessary bookwork and those who had not, the former being slightly more numerous. The weaker candidates did not appreciate the need to consider the equilibrium and general position of the particle. In part **(iii),** many correctly found the amplitude. Only the stronger candidates correctly found the required time, the work of many candidates showing some confusion at this stage.

Question 6

Weaker candidates had considerable difficulty setting up the initial momentum equation. To do so, they had to appreciate that after the impact *B* moved along the line of centres and, since it struck the wall at an angle α, *A*, which was initially travelling parallel to the wall, must have been travelling at α to the line of centres. The usual problems in this question were wrong signs, wrong masses and weak algebra which appeared from time to time. There were few attempts at part **(ii),** but those who did attempt it, generally understood the concepts required and some obtained the correct result.

Answers: **(i)** $v_A = 0$ $\quad v_B = 0.5u\cos\alpha$ **(ii)** 54.7°

Section B: Probability

Question 7

Part **(i)(a)** required the candidate to state the probability of 0 goals being scored in the interval (0,*t*) and hence to deduce the cumulative distribution function. Since a surprising number of candidates could not make the initial statement, many lost both marks here. Similarly, in part **(i)(b),** there were almost as many candidates who did not recall, correctly, that the probability density function was the derivative of the cumulative distribution function. There were much better attempts at part **(ii)** and there was no penalty for giving a decimal value, correct to 3 significant figures, for the final answer.

Answers: **(i) (b)** $\mathrm{f}(t) = \frac{1}{24}e^{-\frac{1}{24}t}$, $\quad t \geq 0$ **(ii)** 24ln3

Question 8

There were many good answers to this question. Most candidates were able to obtain the estimate of the common variance. A few did not realise that the given data could immediately be inserted into the numerator and, consequently, there was multiplication and division by the same number on both terms. A few candidates thought that a *z*-value rather than a *t*-value was required and inevitably a few candidates had the wrong number of degrees of freedom. In part **(ii)** the majority realised that a comment based on the presence of 0 in the confidence interval was required. Here both marks could be earned if the comment related correctly to the candidate's confidence interval.

Answers: **(i)** (-2.11,14.1) **(ii)** $0 \in$ CI $\Rightarrow$ insufficient evidence of a difference.

Question 9

The candidates who approached part **(i)** via multiplying probability generating functions were successful, whereas those who tried to find $\mathrm{P}(X+Y=r)$ were unsuccessful, since they did not recognise that, in most cases, *r* could be obtained in several ways, so summations were required. Most candidates were able to obtain the correct values in part **(ii)**. The majority of candidates recognised that the mean was 15 in part **(iii)**, only the better candidates correctly answered this part.

Answers: **(ii) (a)** 4.54×10^{-5} **(b)** 0.0901 **(iii)** 27.

Question 10

This question was answered well by many candidates. A frequent error was to omit a continuity correction. If this was the case and no other errors occurred, leading to the incorrect value for *n* of 420, then only 2 marks, overall, were deducted. A sign error on the lower *z*-value was also quite common. This was another question where some poor algebra was again evident towards the end of the question, particularly amongst those who worked with *np* and *np*(1 – *p*) from the beginning of the question. Those who began with *μ* and *σ*, initially, and evaluated them, before inserting *np* and *np*(1 – *p*), were generally more successful.

Answers: $n = 225$ $p = 0.2$

Question 11

There were many good and substantially correct answers to this question. A common error was to get the wrong variance in parts **(ii)** and/or **(iii),** particularly part **(ii),** where candidates used $2P+4H$, rather than $P_1+P_2+H_1+H_2+H_3+H_4$. Occasionally the complementary probability was given as the answer to part **(iii).**

Answers: **(i)** 0.876 **(ii)** 0.109 **(iii)** 0.669

Question 12

There were substantially correct and complete answers to this question by a good number of candidates. Part **(i)** caused few problems, as did part **(ii)(b)**. In part **(ii)(a)**, those who wrote down a few terms of the probability generating function sometimes saw that it could be summed by summing alternate terms using the sum to infinity of a GP formula.

Answers: **(i)(a)** $\frac{3}{20}$ **(b)** $\frac{3}{20}$ **(c)** $\frac{5}{8}$ **(ii)(b)** $\frac{35}{8}$