Eton College King's Scholarship Examination 201

(One and a half h

MATHEMATICS A

Answer Question 1 and as many of the other five questions as you can.
Question 1 is worth 50 marks. All other questions are worth 10 marks each.

Show all of your working. The use of calculators is permitted.

1. This question is compulsory.

(a) If $x = -5$ and $y = 12$, evaluate the following, leaving your answers as exact fracti

(i) $\dfrac{8x^2 - y^2}{y - 12x}$ [3]

(ii) $\dfrac{y}{2x} + \dfrac{x}{y}$ [2]

(b) Solve the following inequalities:

(i) $\dfrac{2}{3}(x - 3) < 18$ [2]

(ii) $8 - 3x < 2x - 2$ [2]

(c) Simon and Terry are both told to draw an isosceles triangle which has two angles differing by 15^o. They both draw a triangle but find they have drawn ones with different angles from each other. Can they both be correct? [3]

(d) Calculate

(i) 15% of £40.00 [1]

(ii) 89% of £111.00 [1]

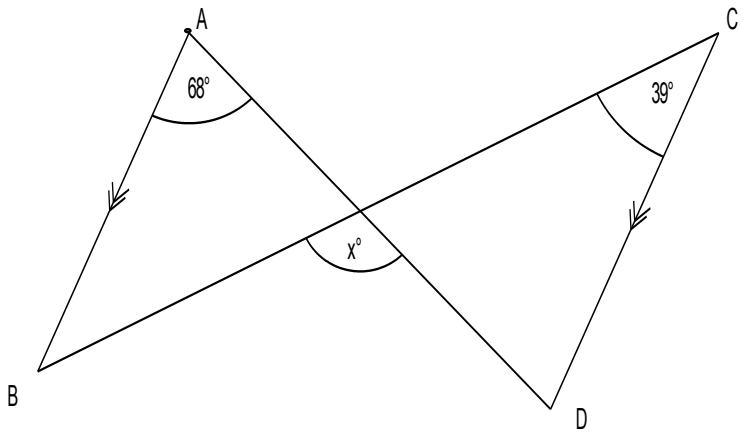
(e) A triangle of base length 29.7 cm has area 8.9 cm^2. Find the height of the triangle, giving your answer correct to 2 significant figures. [3]

(f) Solve the following equations, leaving your answers as mixed numbers where appropriate:

(i) $\dfrac{4 - 3x}{5} = 9$ [2]

(ii) $\dfrac{2 - 5x}{3} = \dfrac{5 - 3x}{2}$ [3]

(g) In the diagram below, AB and CD are parallel. Calculate the value of x. [2]



(h) Solve the following simultaneous equations

$4x - 3y = 15$

$5x + 7y = 8$ [4]

(i) Simplify the following as far as possible:

(i) $\left(3ab^4\right)^3$ [1]

(ii) $\dfrac{8d^2-3d^2}{20d^2}$ [2]

(j) A rectangular field has length 30 m. Its width is half its length.

(i) Find, correct to 3 significant figures, the distance between the opposite corners of the field. [3]

(ii) A second field is 9% less wide but 9% longer. Find the distance, correct to 3 significant figures, between the opposite corners of the second field. [3]

(k) (i) By what do you multiply 8 to get $2\frac{1}{2}$? Give your answer as an exact fraction. [2]

(ii) By what do you multiply ab to get $\dfrac{2b^2}{3}$? [2]

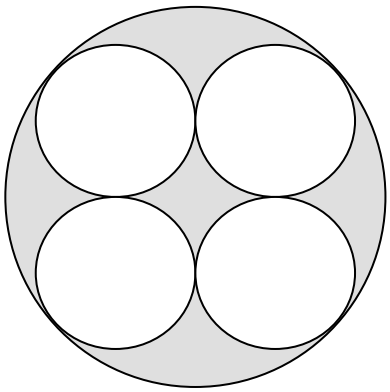
(l) A large company insists that each shareholder invests at least £12,000 in the company. At present, the company has two thousand shareholders and their average investment is £13,040.

(i) How much is their total investment? [2]

(ii) Suppose that 100 new people become shareholders in the company. What is the lowest level to which the average investment could drop? Give your answer to the nearest pound. [2]

(iii) In fact a further n new people become shareholders and on average they invest £12,320. If the average investment across all the shareholders is now £12,960, find n. [4]

2. The diagram shows four identical circles inside one large circle. The radius of each of the smaller circles is $\sqrt{2}$ cm.



(a) Prove that the radius of the large circle is $2+\sqrt{2}$ cm. [3]

(b) Show that the shaded area is $2\pi\left(2\sqrt{2}-1\right)$ cm^2. [4]

(c) Find an exact expression for the t[redacted]eter of the shaded area. [3]

3. This question is about factors of numbers.

(a) Write down all of the 6 factors of 45.

(b) Two whole numbers multiply to give 45. Explain why their sum must be even. [1]

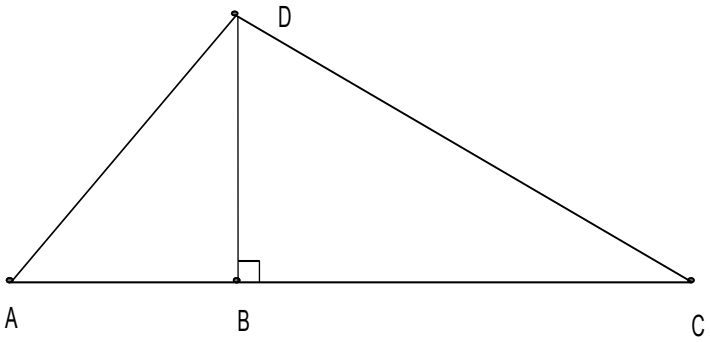
(c) Two whole numbers multiply to give 32. Explain why if their sum is odd, the two numbers must add to 33. [2]

(d) Two whole numbers multiply to give 81. What are the possible values for their sum? [2]

(e) Two whole numbers multiply to give 1,417,176 and add to make 354,298. Find the two numbers. [3]

4. In this question, the diagrams are not drawn to scale.

(a) In the diagram below, the points A, B and C all lie on a straight line and the angle CBD is a right angle. AD and CD are 3 cm and 8.2 cm respectively and AB is 2.4cm.

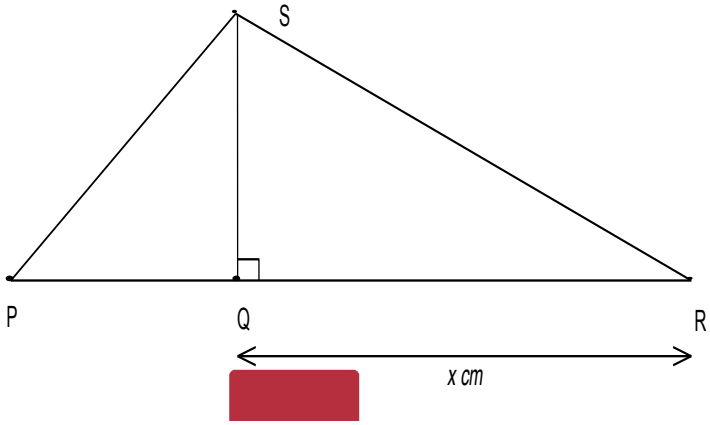


(i) Calculate the length BD.
(ii) Calculate the length BC. [4]

(b) Show that $(c-x)^2 = c^2 - 2cx + x^2$. [2]

(c) In the diagram below, P, Q and R lie on a straight line and angle SQR is a right angle. The lengths PS, SR and RP are a, b and c cm respectively and QR is x cm. Use algebra to show that

$$x = \frac{b^2 + c^2 - a^2}{2c}$$ [4]



5. (a) A postman has ten letters to deliver. Each letter is addressed to exactly one of [illegible] addresses. Explain why at least two letters have the same address on them. [1]

(b) 230 Etonians have their birthday during the week starting Monday 30th July. Explain why at least 33 of them must have their birthday on the same day. [3]

(c) At a birthday party, thirty five sweets are shared between eight children. Given that each child receives at least one sweet, is it possible for them all to receive a different number of sweets? [3]

(d) A rectangle has width 6 cm and height 12 cm. If 9 points are chosen from within the rectangle, explain why two of the points must be at most $\sqrt{18}$ cm away from each other. [Hint: divide the rectangle into squares of equal area.] [3]

6. (a) Show by multiplying out that if $(x-A)(y-A) \le 0$, then

$$xy \le A(x+y-A)$$ [2]

For any set of numbers, the R algorithm is as follows:

- Work out the mean of the numbers: call the answer A
- Replace x, the smallest number in the set, and y, the largest number in the set, by A and $x+y-A$ respectively. (If there is more than one smallest number, replace the first. If there is more than one greatest number, replace the first.)

For example, for the number set $\{1, 4, 6, 9\}$, $A = 5$ and so 1 is replaced by 5 and 9 is replaced by 5. Thus the R algorithm replaces $\{1, 4, 6, 9\}$ with $\{5, 4, 6, 5\}$.

Furthermore, the R algorithm replaces $\{5, 4, 6, 5\}$ by $\{5, 5, 5, 5\}$.

(b) Show that if the R algorithm is applied twice to $\{10, 13, 23, 29, 35\}$, the resulting set of numbers is $\{22, 22, 23, 20, 23\}$. [2]

(c) Show further that if the R algorithm is applied 4 times to $\{10, 13, 23, 29, 35\}$, the resulting set of numbers is $\{22, 22, 22, 22, 22\}$. [2]

(d) Use part (a) to explain why when you apply the R algorithm to a set of numbers, then the new set of numbers will not multiply to give a smaller answer than the original set of numbers. [2]

(e) Explain why if you repeat the R algorithm, A remains unchanged. [1]

(f) Use your answers to the earlier parts to explain why $10 \times 13 \times 23 \times 29 \times 35 \le 22^5$. [1]

[END OF PAPER]